PLENARY SESSION

(AP-01)

Several Episodes in Recent Studies on the KKM theory

Sehie Park
(Seoul National University, **Korea**)
(e-mail: shpark@math.snu.ac.kr)

Abstract

In the last decade, there have appeared a very large number of papers on the KKM theory. Since the appearance of generalized convex (simply, G-convex) spaces in 1993, the concept has been challenged by several authors who aimed to obtain more general concepts. In fact, a number of modifications or imitations of the concept has followed; for example, L-spaces, spaces having property (H), FC-spaces, pseudo-H-spaces, another L-spaces, M-spaces, GFC-spaces, simplicial spaces, and others. It is known that most of such examples belong to the class of ϕ_A-spaces and are particular forms of G-convex spaces. Recently all of the above mentioned spaces are unified to the class of abstract convex spaces.

In our previous reviews, we presented quite critical comments on many of recent works on the KKM theory in order to improve the theory itself. Continuing this line, in the present review,, we give comments on some of them, namely, Lin et al., Chen et al., Wen, Fang et al., Hou, Xiang et al., Gonzáles et al., Kulpa et al., and Park. Finally, in order to make even, we introduce two reviewers' comments on the original version of the author's paper related to the concept of generalized convex spaces.

2000 Mathematics Subject Classification: 47H04, 47H10, 49J27, 49J35, 54H25, 91B50.

Keywords and Phrases: Abstract convex space, generalized (G-) convex space, ϕ_A-space, L-space, KKM theorem, (partial) KKM principle.

(DP-02)

Some New Grüss' Type Inequalities for Functions of Selfadjoint Operators in Hilbert Spaces

S. S. Dragomir
(Victoria University, **Australia**)
(e-mail: sever.dragomir@vu.edu.au)
(URL: http://www.staff.vu.edu.au/rgmia/dragomir)

Abstract

The following lemmas, that are of interest in their own right, collect some Grüss type inequalities for vectors in inner product spaces obtained earlier by the author:

Lemma 1. *Let $(H, \langle \cdot, \cdot \rangle)$ be an inner product space over the real or complex number field $\mathbb{K}$, $u, v, e \in H$, $\|e\| = 1$, and $\alpha, \beta, \gamma, \delta \in \mathbb{K}$ such that*

$$Re \langle \beta e - u, u - \alpha e \rangle \geq 0, \quad Re \langle \delta e - v, v - \gamma e \rangle \geq 0 \tag{1.1}$$

or equivalently,

$$\left\| u - \frac{\alpha + \beta}{2} e \right\| \leq \frac{1}{2} |\beta - \alpha|, \quad \left\| v - \frac{\gamma + \delta}{2} e \right\| \leq \frac{1}{2} |\delta - \gamma|. \tag{1.2}$$

Then

$$\begin{aligned}
&\left|\langle u,v\rangle - \langle u,e\rangle \langle e,v\rangle\right| \\
&\qquad \leq \frac{1}{4}\cdot|\beta-\alpha|\,|\delta-\gamma| \\
&\qquad\quad - \begin{cases} \left[Re\,\langle \beta e-u, u-\alpha e\rangle\, Re\,\langle \delta e-v, v-\gamma e\rangle\right]^{\frac{1}{2}}, \\ \left|\langle u,e\rangle - \dfrac{\alpha+\beta}{2}\right|\left|\langle v,e\rangle - \dfrac{\gamma+\delta}{2}\right|. \end{cases}
\end{aligned} \tag{1.3}$$

The first inequality has been obtained in [2] (see also [8, p.44]) while the second result was established in [3] (see also [8, p.90]). They provide refinements of the earlier result from [1] where only the first part of the bound, i.e., $\frac{1}{4}|\beta-\alpha|\,|\delta-\gamma|$ has been given. Notice that, as pointed out in [3], the upper bounds for the Grüss functional incorporated in (1.3) cannot be compared in general, meaning that one is better than the other depending on appropriate choices of the vectors and scalars involved.

Another result of this type is the following one:

Lemma 2. *With the assumptions in Lemma 1 and if* $Re\,(\beta\overline{\alpha}) > 0, Re\,(\delta\overline{\gamma}) > 0$ *then*

$$\begin{aligned}
&\left|\langle u,v\rangle - \langle u,e\rangle \langle e,v\rangle\right| \\
&\leq \begin{cases} \dfrac{1}{4}\cdot\dfrac{|\beta-\alpha|\,|\delta-\gamma|}{\left[Re\,(\beta\overline{\alpha})\, Re\,(\delta\overline{\gamma})\right]^{\frac{1}{2}}}\,|\langle u,e\rangle \langle e,v\rangle|, \\ \left[\left(|\alpha+\beta| - 2\left[Re\,(\beta\overline{\alpha})\right]^{\frac{1}{2}}\right)\left(|\delta+\gamma| - 2\left[Re\,(\delta\overline{\gamma})\right]^{\frac{1}{2}}\right)\right]^{\frac{1}{2}} \\ \qquad \cdot\left[|\langle u,e\rangle \langle e,v\rangle|\right]^{\frac{1}{2}}. \end{cases}
\end{aligned} \tag{1.4}$$

The first inequality has been established in [4] (see [8, p.62]SSDM) while the second one can be obtained in a canonical manner from the reverse of the Schwarz inequality given in [5]. The details are omitted.

Finally, another inequality of Grüss type that has been obtained in [6] (see also [8, p.65] can be stated as:

Lemma 3. *With the assumptions in Lemma 1 and if $\beta \neq -\alpha$, $\delta \neq -\gamma$ then*

$$\begin{aligned}&\left|\langle u,v\rangle - \langle u,e\rangle\langle e,v\rangle\right| \\ &\leq \frac{1}{4}\cdot\frac{|\beta-\alpha|\,|\delta-\gamma|}{[|\beta+\alpha|\,|\delta+\gamma|]^{\frac{1}{2}}}\left[(\|u\|+|\langle u,e\rangle|)(\|v\|+|\langle v,e\rangle|)\right]^{\frac{1}{2}}.\end{aligned} \tag{1.5}$$

The following results obtained by the use of the above lemmas incorporates some new inequalities of Grüss' type for two functions of a selfadjoint operator.

Theorem 1. *Let A be a selfadjoint operator on the Hilbert space $(H;\langle .,.\rangle)$ and assume that $Sp(A)\subseteq[m,M]$ for some scalars $m<M$. If f and g are continuous on $[m,M]$ and $\gamma := \min_{t\in[m,M]} f(t)$, $\Gamma := \max_{t\in[m,M]} f(t)$, $\delta := \min_{t\in[m,M]} g(t)$ and $\Delta := \max_{t\in[m,M]} g(t)$ then*

$$\begin{aligned}&\left|\langle f(A)g(A)x,x\rangle - \langle f(A)x,x\rangle\langle g(A)x,x\rangle\right| \\ &\leq \frac{1}{4}\cdot(\Gamma-\gamma)(\Delta-\delta)\| \\ &\quad - \begin{cases}\left[\langle \Gamma x - f(A)x, f(A)x-\gamma x\rangle\langle \Delta x - g(A)x, g(A)x-\delta x\rangle\right]^{\frac{1}{2}}, \\ \left|\langle f(A)x,xt\rangle - \dfrac{\Gamma+\gamma}{2}\right|\left|\langle g(A)x,x\rangle - \dfrac{\Delta+\delta}{2}\right|.\end{cases}\end{aligned} \tag{1.6}$$

for each $x\in H$ with $\|x\|=1$.

Moreover if γ and δ are positive, then we also have

$$\begin{aligned}&\left|\langle f(A)g(A)x,x\rangle - \langle f(A)x,x\rangle\langle g(A)x,x\rangle\right| \\ &\leq \begin{cases}\dfrac{1}{4}\cdot\dfrac{(\Gamma-\gamma)(\Delta-\delta)}{\sqrt{\Gamma\gamma\Delta\delta}}\langle f(A)x,x\rangle\langle g(A)x,x\rangle, \\ \left(\sqrt{\Gamma}-\sqrt{\gamma}\right)\left(\sqrt{\Delta}-\sqrt{\delta}\right)\left[\langle f(A)x,x\rangle\langle g(A)x,x\rangle\right]^{\frac{1}{2}}.\end{cases}\end{aligned} \tag{1.7}$$

while for $\Gamma + \gamma, \Delta + \delta \neq 0$ *we have*

$$\begin{aligned} &\left| \langle f(A) g(A) x, x\rangle - \langle f(A) x, x\rangle \langle g(A) x, x\rangle \right| \\ &\leq \frac{1}{4} \cdot \frac{(\Gamma - \gamma)(\Delta - \delta)}{[|\Gamma + \gamma| \, |\Delta + \delta|]^{\frac{1}{2}}} \\ &\quad \left[(\|f(A) x\| + |\langle f(A) x, x\rangle|) (\|g(A) x\| + |\langle g(A) x, x\rangle|) \right]^{\frac{1}{2}} \end{aligned} \tag{1.8}$$

for each $x \in H$ *with* $\|x\| = 1$.

2000 Mathematics Subject Classification: 47A63, 47A99.

Keywords and Phrases: Selfadjoint operators, Grüss inequality, functions of selfadjoint operators.

(CP-01)

Systems of Quasi-Variational Relations with Applications

Lai-Jiu Lin
(National Changhua University of Education, **Taiwan**)
(e-mail: maljlin@math.ncue.edu.tw)
Qamrul Hasan Ansari
(Aligarh Muslim University, **India**)
(e-mail: qhansari@gmail.com)

Abstract

In this paper, we introduce system of quasi-variational relations (in short, SQVR) and present several examples which show that it is a very general and unified model of several problems. We establish the existence of solutions of SQVP, in general, and several other problems, in particular. As an application of our results, we de- rive maximal element theorems and a collectively fixed point theorem for a family

of multivalued maps. As further applications, we study Ky Fan type inequality / inclusion problem for vector valued bifunctions which include constrained Nash equilibrium problem as a special acse. We also present a common fixed point theo- rem for a family of multivalued maps. The results of this paper improve and gen- eralize several known results on (system of) quasi-equilibrium problems, (system of) quasi-variational inclusions, constrained Nash equilibrium problem, collectively fixed point theorem and KKM type theorems for a family of multivalued maps. Our results also contain several results appeared in the literature in the recent past.

(EP-01)

Convergence Theorem of Common Fixed Points for Lipschitzian Pseudocontraction Semigroups in Banach Spaces

Shih-sen Chang
(Yibin University, **P. R. China**)
(Sichuan University, **P. R. China**)
(e-mail: changss@yahoo.cn)

Abstract

The purpose of this paper is to study the weak convergence problems of the implicity iteration process for a Lipschitzian pseudocontractive semi-groups in general Banach spaces. The results presented in this paper extend and improve the corresponding results of Zhou [Nonlinear Anal. 68:10 (2008), 2977-1983.], Chen, Song and Zhou [J. Math. Anal. Appl., 314 (2006), 701-709], Xu and Ori [Numer. Funct. Anal. Optim, 22 (2001), 767-773] and Osilike [J. Math. Anal. Appl., 294 (2004), 73-81].

(DP-01)

Optimality Conditions and Duality in Nonsmooth Multiobjective Programs

Do Sang Kim
(Pukyong National University, **Korea**)
(e-mail: dskim@pknu.ac.kr)
Hyo Jung Lee
(Pukyong National University, **Korea**)

Abstract

In this talk, we introduce the nonsmooth multiobjective programming problem involving locally Lipschitz functions and support functions. Two types of Karush-Kuhn-Tucker optimality conditions are introduced. We present two types of Karush-Kuhn-Tucker conditions which differ only in the nonnegativity of the multipliers for the equality constraints and neither of which includes a complementary slackness condition, common in necessary optimality conditions. Sufficient Karush-Kuhn-Tucker optimality conditions are given by using generalized convexity assumptions and certain regularity conditions. In addition, we formulate the Wolfe type dual and Mond-Weir type dual problems and establish duality theorems for (weak) Pareto-optimal solutions under generalized convexity assumptions and regularity conditions.

2000 Mathematics Subject Classification: 90C29, 49J52, 90C46.

Keywords and Phrases: Multiobjective programming, Pareto-opti -mal solutions, Karush-Kuhn-Tucker conditions, duality relations, support functions.

(EP-02)

Iterative Approximation of Common Fixed Points for Two Quasi-ϕ-nonexpansive Mappings in Banach Spaces

Haiyun Zhou
(Shijiazhuang Mechanical Engineering College, **P. R. China**)
(e-mail: witman66@yahoo.com.cn)
Xinghui Gao
(Yanan University, **P. R. China**)

Abstract

In this paper, we introduce a new type of hybrid algorithm for a pair of quasi-ϕ-nonexpansive mappings. We establish strong convergence theorems of common fixed points in uniformly smooth and strictly convex Banach spaces with the property(K). Our results improve and extend the corresponding results announced by many others.

2000 Mathematics Subject Classification: 47H09; 47H10.

Keywords and Phrases: Strong convergence, hybrid projection algorithm, Quasi-ϕ-nonexpansive mapping, the property(K).

(EP-03)

Convergence Theorems on Iteration Methods for Nonexpansive Nonself-mappings in Banach Spaces

Jong Soo Jung
(Dong-A University, **Korea**)
(e-mail: jungjs@mail.donga.ac.kr)

Abstract

Strong convergence theorem of the explicit viscosity iterative scheme

involving the sunny nonexpansive retraction for nonexpansive nonself-mappings is established in a reflexive and strictly convex Banach spaces having a weakly sequentially continuous duality mapping. The main result improves the corresponding result of Song and Chen [Viscosity approximation methods for nonexpansive nonself-mappings, *J. Math. Anal. Appl.* 321 (2006) 316–326] to the more general class of mappings together with certain different control conditions.

2000 Mathematics Subject Classification: 47H09, 47H10, 47J20, 47J25, 49M05.

Keywords and Phrases: Viscosity explicit iterative scheme, nonexpansive nonself mapping, sunny and nonexpansive retraction, contraction, variational inequality, weakly sequential continuous duality mapping.

(BP-01)

Existence of Solutions of Fractional Evolution Integrodifferential Equations in Banach Spaces

K. Balachandran
(Bharathiar University, **India**)
(e-mail: balachandran_k@lycos.com)

Abstract

In this paper we prove the existence of solutions of fractional evolution integrodifferential equations of the form

$$\frac{d^q u(t)}{dt^q} = A(t)u(t) + \int_0^t B(t,s)u(s)ds + f(t,u(t), \int_0^t K(t,s)u(s)ds),$$
$$0 < q < 1,$$
$$u(0) = u_0,$$

in Banach spaces. Further the nonlocal Cauchy problem for this equation is also discussed. The results are obtained by using the fixed point argument.

2000 Mathematics Subject Classification: 34G20.

Keywords and Phrases: Evolution integrodifferential equation, fractional calculus, nonlocal condition, fixed point theorem.

(BP-02)

Global and Asymptotic Attractivity Results for A Nonlinear Functional Integral Equation

Bapurao C. Dhage
(Kasubai, Gurukul Colony, Ahmedpur-413 515, Dist: Latur Maharashtra, **India**)
(e-mail: bcdhage@yahoo.co.in)
Jong Kyu Kim
(Kyungnam University, **Korea**)
(e-mail: jongkyuk@kyungnam.ac.kr)

Abstract

In this paper two existence results concerning the global attractivity and global asymptotic attractivity for a certain functional nonlinear integral equation are proved. Our existence results include several existence and attractivity results earlier by Dar- wish [EJQTDE 2007(2007) No. 7, pp. 1-10] and Hu and Yan [J. Math. Anal. Appl. 321 (2006), 147-156] as special cases under weaker conditions. A measure theoretic fixed point theorem of Dhage [Comm. Appl. Nonlinear Anal. 15 (2) (2008), 89- 101] is used in formulating our main results and the characterizations of solutions are obtained in the space of functions defined, continuous and bounded on unbounded intervals.

I. SESSION A

(A-01)

Fixed Point Theorems of Some Type Contraction Mappings

Sahar Mohammed Ali
(Ain Shams University, **Egypt**)
(e-mail: saharm_ali@yahoo.com)

Abstract

Let X be a normed space, C be a closed convex and weakly Cauchy subset of X, T be a mapping from $C \times C$ into C. In this paper we proved that if T satisfies the following condition:

$$\|T(x,y) - T(y,z)\| \leq a\|x-y\| + b\|T(x,y) - x\| + c\|T(y,z) - y\|$$
$$\forall x, y, z \in C,$$

for some triple of real numbers (a,b,c), (a,b,c) is in the region inclosed by, the cub $a,b,c \in [0,1]$, the region left to the plane $a+b+c=1$, and the parabolic region $(b+1)^2 < -2(c-1)$, then T has a unique fixed point in the sense that there is a unique point $x_0 \in C$ such that $T(x_0,x_0) = x_0$. Moreover, if x_1, x_2 are two arbitrarily elements in C, then for $n > 2$, the sequence defined iteratively as: $x_n = T(x_{n-1}, x_{n-2})$ a subsequence converging strongly to the unique fixed point of such new defined mapping T. We also considered a contraction type mapping with a mixed monotone property called $\{a,b,c\}$ type contraction mapping defined on $C \times C$ into C, where C is a closed convex and weakly Cauchy subset of a sequentially ordered normed space X and proved the existence of first-anti-second and second-anti-first couple fixed points of such a mapping which leads to an extension of some previous fixed point theorem.

(A-02)

Generalized Weak Contraction Conditions and Common Fixed Point Theorems

G. V. Ravindranadh Babu
(Andhra University, **India**)
(e-mail: gvr_babu@hotmail.com)

Abstract

We define generalized weak contraction condition for two pairs of selfmaps and discuss the existence of common fixed points of these maps under the assumptions that these two pairs of maps are weakly compatible in a complete metric space. Also, we discuss the existence of common fixed points for two pairs of such selfmaps in which one pair is compatible, reciprocal continuous and the other one is weakly compatible. Further, we establish the existence of common fixed points for a pair of selfmaps in a convex metric space via the convergence of modified Mann iteration and as an application we prove an invariant approximation result.

(A-03)

Cone Metric Spaces and Fixed Point Theory

Mohsen Alimohammady
(University of Mazandaran, **Iran**)
(e-mail: m.alimohammady@gmail.com)
Mehdi Roohi
(University of Mazandaran, **Iran**)
(e-mail: mehdi.roohi@gmail.com)

Abstract

Huang and Zhang [2] introduced the notion of cone metric spaces as a generalization of metric spaces. In this paper some results for

normal and non-normal cone metric spaces are established. The main aim of this paper is to prove some fixed points theorems on cone metric spaces. Comparisons and examples are given.

2000 Mathematics Subject Classification: Primary 46A40, Secondary 46A55, 06F30.

Keywords and Phrases: Fixed point theorem, common fixed point theorems, contraction mappings, metric space, cone.

(A-04)

Fixed Points for Hybrid Mappings in Banach Spaces

Hemant Kumar Pathak
(Pt. Ravishankar Shukla University, **India**)
(e-mail: hkpathak05@gmail.com)
Yeol Je Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)
Shin Min Kang
(Gyeongsang National University, **Korea**)
(e-mail: smkang@gnu.ac.kr)

Abstract

In this paper, following Hu and Sun [S. Hu and Y. Sun, Fixed point index for weakly inward mappings, J. Math. Anal. Appl., 172 (1993), 266-273] and S. Xu et al. [S. Xu, B. Jia and G-z. Li, Fixed points for weakly inward mappings in Banach spaces, J. Math. Anal. Appl., (2005), doi:10.1016/j.jmaa.2005.07.034], we continue to investigate boundary conditions, under which the fixed point index for the hybrid mapping $\nu A+B-I_{\bar{\Omega}}$ of the sum and difference of scalar multiple of a completely continuous and weakly inward mapping, a continuous mapping and the identity mapping, denoted by $i(\nu A+B-I_{\bar{\Omega}}, \Omega, P)$, is equal to 1 or 0. As a special case, we can obtain some new fixed point

theorems of hybrid mapping and existence theorems of solutions for the equations $(\nu A + B - I_{\bar{\Omega}})x = x$, which extend many well known results in the literature such as Leray-Schauders theorem, Rothes two theorems, Krasnoselkiis theorem, Altmans theorem, Petryshyns theorem, etc., to the case of weakly inward mappings.

2000 Mathematics Subject Classification: 41A50, 46A03, 47H10.

Keywords and Phrases: Fixed point, fixed point index, weakly inward mapping, completely continuous operator, real Banach space, cone.

(A-05)

Common Fixed Point Theorem for Weakly Compatible Mappings without Continuity in Metric Spaces

Atul Pathak
(Mahakal Institute of Technology, **India**)
(e-mail: atulpathak71@yahoo.co.in)

Abstract

In 1998, Jungck and Rhoades [Fixed point for set valued functions without continuity, Ind. J. Pure and Appl. Math., 29(3)(1998), 227-238] introduced the notion of weakly compatible maps and showed that compatible maps are weakly compatible but converse need not be true . The aim of this paper is to prove some common fixed point theorems in metric spaces by removing the assumption of continuity, replacing the condition of compatibility of type (B) by weak compatibility and replacing the completeness of the space with a set of alternative conditions. We improve results of Djoudi [A common fixed point theorem for compatible mappings of type (B) in complete metric spaces, Demonstratio Math., Vol.36, No.2(2003),463-470].

We prove the following:

Theorem 1. *Let I, J, S and T be mappings from a metric space (X, d) into itself satisfying the conditions*

(1) *$S(X) \subset J(X)$ and $T(X) \subset I(X)$,*
(2) *$d(Sx, Ty) \leq \phi(d(Ix, Jy),\ d(Ix, Sx),\ d(Jy, Ty),\ d(Ix, Ty),\ d($ $Jy, Sx))$ for all $x, y \in X$.*
(3) *One of $I(X)$, $J(X)$, $S(X)$ or $T(X)$ is a complete subspace of X, then*
(i) S and I have a coincidence point,
(ii) J and T have a coincidence point.
Further if
(4) *The pairs $\{S, I\}$ and $\{J, T\}$ are weakly compatible, then*
(iii) I, J, S and T have a unique common fixed point in X.

Some useful examples and applications will be discussed.

(A-06)

A Common Fixed Point in Banach Space

Kanhaiya Jha
(Kathmandu University, **Nepal**)
(e-mail: jhaknh(at)yahoo.co.in, jhakn(at)ku.edu.np)

Abstract

The presence or absence of a fixed point is an intrinsic property of mapping. The study of common fixed points for four mappings and their generalizations for the sequence of mappings satisfying contractive type conditions and also the comparative study of various contractive definitions have attracted a great deal of research activities. The most commonly used contractive conditions are Banach type contractive condition, or, a Meir-Keeler type (ϵ, δ)-contractive condition, or, a Boyd-Wong type -contractive condition, or even Lipschitz type

plane contractive condition. Then, many authors have established several interesting common fixed point theorems for these contractive type mappings in metric space which are available in literature. Recently, V. Popa in 1999 proved a fixed point theorem in complete metric space satisfying suitable implicit relation.

In this paper, our main aim is to establish a common fixed point theorem for a sequence of self mappings in Banach space under an implicit relation. In this case, the continuity or reciprocal continuity condition on mappings is removed and weak contractive condition is employed. This result generalizes and improves various known results of fixed points.

2000 Mathematics Subject Classification: 54H25.

Keywords and Phrases: Common fixed point, weakly compatible maps, Banach space.

(A-07)

Common Fixed Point Results for a New Class of Noncommuting Mappings with Applications in Menger Convex Metric Space

Mujahid Abbas
(Lahore University of Management Sciences, **Pakistan**)
(e-mail: mujahid@lums.edu.pk)

Abstract

Sufficient conditions for the existence of a common fixed point for a new class of noncommuting mappings more general than uniformly $R-$ subweakly commuting mappings, satisfying a generalized contractive conditions in the framework of a Menger convex metric space are obtained. As an application, related results on best approximation are derived. Our results generalize various known results in the literature.

(A-08)

Ky Fan Best Approximation Theorem in Metrizable Topological Spaces

Hemant Kumar Nashine
(Disha Institute of Management and Technology, **India**)
(e-mail: hemantnashine@rediffmail.com, hemantnashine@gmail.com)
C. L. Dewangan
(Govt. J. Yoganandam Chhattisgarh College, **India**)
(e-mail: cldewangan@rediffmail.com)

Abstract

The purpose of this paper is to establish existence results on Ky Fan best approximation in the context of Metrizable topological vector space(MTVS).

2000 Mathematics Subject Classification: 41A50, 47H10, 54H25.

Keywords and Phrases: Almost quasi convex, best approximant, relative almost quasi convex, upper semicontinuous map, open inverse values, Metrizable topological vector space(MTVS).

(A-09)

Fixed Point Theorems of Contraction Multi-valued Mappings in Menger Space

Aradhana Sharma
(R-7 Vinoba Nagar, **India**)
(e-mail: aradhana_sharma14@yahoo.com)

Abstract

Fixed point theorem for Menger spaces on probabilistic metric spa

-ces were first proved by Menger which is generalization of metric space The aim of present paper is to extend results of Alaca, Turoglu and Yildiz [5] on the lines of Sharma and Deshpande [7] for multivalued mappings and solving various types of the improved results in the Menger spaces.

(A-10)

Common Fixed Point Theorem Satisfying Implicit Function Relation

Urmila Mishra
(Lakhmi Chand Institute of Technology, **India**)
(e-mail: mishra.urmila@gmail.com)

Abstract

In this paper we obtain a fixed point theorem using the concept of absorbing maps introduced by us [10], satisfying implicit relation and totally replaced the commutativity condition called weakly compatible maps used by Ali et al [1]. Further we generate independent examples for implicit function and give an example in support of our theorem. Our theorem generalizes the result of Ali et al [1] and similar results two(see for instance [3, 4]).

2000 Mathematics Subject Classification: 54H25, 47H10, 54E70.

Keywords and Phrases: Absorbing Maps, (E.A.) property, Common property (E.A.), compatible maps, non-compatible maps, weak compatible maps, common fixed point.

(A-11)

Common Fixed Points of Nonlinear Contraction in Menger Spaces

Javid Ali
(Indian Institute of Technology, **India**)
(e-mail: javid@math.com)
M. Imdad
(Aligarh Muslim University, **India**)
(e-mail: mhimdad@yahoo.co.in)

Abstract

In this paper, we introduce the notion of common property (E.A) in Menger spaces. We prove a result which shows the relation between property (E.A) and common property (E.A). Using common property (E.A), some common fixed point theorems proved for self mappings satisfying Ciric-type and f-type contractions in Menger PM spaces. Our results generalize many known results in Menger as well as metric spaces. Some related results and illustrative examples are also furnished.

2000 Mathematics Subject Classification: Primary 54H25, Secondary 47H10.

Keywords and Phrases: Menger spaces, common property (E.A), weakly compatible mappings and t-norm.

(A-12)

Fixed Points of Weak Contraction Mappings under Non-compatible Maps

P. P. Murthy

(Guru Ghasidas Vishwavidyalaya(A Central University), **India**)
(e-mail: ppmurthy@gmail.com)

Abstract

Alber and Guerre-Delabriere (Principles of weakly contractive maps in Hilbert spaces, in: I. Gohberg, Yu Lyubich(Eds), New Results in Operative Theory, in: Advances and appl. Vol 98, Birkhuser, Basel(1997), 7-22) introduced the concept of weak contraction in Hilbert spaces. Rhoades(Some theorems on weakly contractive maps, Nonlinear Analysis: Theory, Methods and Applicaton 47(2001), 2683-2693) has sown that the result which Alber and Guerre-Delabriere(Principles of weakly contractive maps in Hilbert spaces, in: I. Gohberg, Yu Lyubich(Eds), New Results in Operative Theory, in: Advances and appl. Vol 98, Birkhuser, Basel(1997), 7-22) had proved that the same is true in case of complete metric spaces. Recently, the concept of cone metric space introduced by Huang and Zhain(Cone Metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332(2)(2007), 1468-1476) in which the set of real numbers are replaced by real Banach space in the definition of metric space. Jungck, Radenovic, Radojevic and Rakocevic(Common fixed point theorems for weakly compatible pairs on cone metric spaces) omitted the concept of normality in the results for proving fixed point theorems. In the next step we shall study the concept of non-compatible maps in the setting of Cone Metric Spaces. We shall extend, generalize and improve some results of some fixed point theorems in the literature of Metric Fixed Point Theory.

(A-13)

Invariant Means and Fixed Point Properties for Semigroups of Non-expansive Mappings

Anthony To-Ming Lau
(University of Alberta, **Canada**)

(e-mail: tlau@math.ualberta.ca)

Abstract

Let S be a semigroup and $m(S)$ be the Banach space of bounded real-valued functions on S. There is a strong connection between the existence of an invariant mean on an invariant subspace of $m(S)$ and fixed point properties or ergodic type properties of S when S is represented as a semigroup of non-expansive mappings on a closed convex subset of a Banach space.

In this talk I shall describe some recent works in this direction connecting nonlinear analysis and amenability.

(A-14)

Some New Common Fixed Point Theorems under Strict Contraction in Menger PM Spaces

M. Tanveer
(Jawaharlal Nehru University, **India**)
(e-mail: tanveergouri@gmail.com)
M. Imdad
(Aligarh Muslim University, **India**)
(e-mail: mhimdad@yahoo.co.in)

Abstract

In this paper, we prove some existence results on coincidence and common fixed points of two pairs of self mappings without continuity in Menger PM Spaces under strict contractions. Our results generalize many known results in Menger as well as metric spaces. Some related results are also derived besides furnishing illustrative examples.

2000 Mathematics Subject Classification: Primary 54H25, Secondary 47H10.

Keywords and Phrases: Menger spaces, common property (E.A), weakly compatible mappings and t-norm.

(A-15)

Generalization Fixed Point Theorems in D^*-metric and F-metric Spaces

Shaban Sedghi
(Islamic Azad University-Ghaemshahr Branch, **Iran**)
(e-mail: sedghi_gh@yahoo.com)

Abstract

In this paper, we give some new definitions of D^*-metric spaces and we prove a common fixed point theorem for six mappings under the condition of weakly compatible mappings in complete D^*-metric spaces. We get some improved versions of several fixed point theorems in complete D^*-metric spaces.

2000 Mathematics Subject Classification: 54E40, 54E35, 54H25.

Keywords and Phrases: D^*-metric contractive mapping, complete D^*-metric space, common fixed point theorem.

(A-16)

The Rise and Decline of Generalized Convex Spaces

Sehie Park
(Seoul National University, **Korea**)
(e-mail: shpark@math.snu.ac.kr)

Abstract

The KKM theory, first called by the author, is the study on applications of equivalent formulations or generalizations of the KKM theorem due to Knaster, Kuratowski, and Mazurkiewicz in 1929. The KKM theorem provides the foundations for many of the modern essential results in diverse areas of mathematical sciences.

In the KKM theory, various types of ϕ_A-spaces $(X, D; \{\phi_A\}_{A\in\langle D\rangle})$ obtained by other authors are simply G-convex spaces. Various types of generalized KKM maps on ϕ_A-spaces are simply KKM maps on G-convex spaces. Therefore, our G-convex space theory can be applied to various types of ϕ_A-spaces. In 2006-09, G-convex spaces are extended to KKM spaces. In the present survey, we recall the recent transition from G-convex spaces to KKM spaces and introduce basic properties of KKM spaces.

2000 Mathematics Subject Classification: 47H04, 47H10, 49J27, 49J35, 54H25, 91B50.

Keywords and Phrases: Abstract convex space, generalized (G-) convex space, ϕ_A-space, L-space, KKM theorem, KKM space, (partial) KKM principle.

(A-17)

Applications of Fuzzy Set Theory and Recent Development in Fixed Point Theory in Fuzzy Metric Spaces

Sushil Sharma
(Vikram University, **India**)
(e-mail: sksharma2005@yahoo.com)

Abstract

The theory of fuzzy sets has evolved in many directions after inves-

tigation of notion of fuzzy sets by Zadeh, L. A. [Fuzzy Sets, Inform Control 8(1965), 338-353] and is finding applications in wide variety of fields in which the phenomenon under study are too complex or too ill defined to be analyzed by the conventional techniques. In applications of fuzzy set theory the field of engineering has undoubtedly been a leader. All engineering disciplines such as civil engineering, electrical engineering, mechanical engineering, robotics, industrial engineering, computer engineering, nuclear engineering etc. have already been affected to various degrees by the new methodological possibilities opened by fuzzy sets. Fuzzy set theory is involved in day-today life. In this lecture we will discuss examples, uses and a number of applications of fuzzy set theory. We will discuss recent developments in fixed point theory also. Of-course fixed point in fuzzy metric spaces and intuitionistic fuzzy metric spaces are very common these days. We will discuss some recent results of workers of this line and some of our new results will be presented in the talk.

2000 Mathematics Subject Classification: 47H10, 54H25.

Keywords and Phrases: Triangular norms, triangular co-norms, fuzzy metric spaces, common fixed points.

(A-18)

Some Common Fixed Point Theorems in Fuzzy Metric Spaces

Prashant Tilwankar
(Devi Ahilya University, **India**)
(e-mail: prashant_tilwankar@yahoo.com)

Abstract

Some common fixed point theorems in complete fuzzy metric spaces in sense of Song and Vasuki - Veeramani are proved which generalize

earlier results due to Vasuki, Chugh and Kumar and others. Imdad and Ali introduce the concept of R-weak commutativity of type (P) in fuzzy metric spaces. In this paper we prove some common fixed point theorems for six discontinuous mappings satisfying some conditions on fuzzy metric spaces.

2000 Mathematics Subject Classification: 47H10, 54H25.

Keywords and Phrases: Fuzzy metric space, R-weakly commuting mappings, common fixed point.

(A-19)

On Semi-inner-product Spaces

V. Lokesha
(Acharya Institute of technology, **India**)
(e-mail: lokiv@yahoo.com)

Abstract

In this talk, I will discuss briefly the semi inner product theory and some applications to different branches of Mathematics (Historical and general survey).

2000 Mathematics Subject Classification: 46C15.

Keywords and Phrases: Adjoint abelian operator, semi-inner-prod -uct, strictly convexity.

(A-20)

Mathematics to the World: In the Context of Fuzzy Logic and its Applications

P. P. Murthy
(Guru Ghasidas Vishwavidyalaya (A Central University), **India**)
(e-mail: ppmurthy@gmail.com)

Abstract

Non-Fuzzy Sets (Crisp)we mean those sets with mathematical logic i.e., the value of the membership function defined is 0 or 1 and **Fuzzy Sets** (Uncertain Sets), we mean the membership values lies between $[0, 1]$. The pioneer concept of **Fuzzy Sets and Fuzzy Logic** introduced initially by Prof. Lofti Zadeh (Fuzzy sets, Information and Control, 8 (1965), 338-353) of University of California, Berkeley, USA in 1965. In this talk, I shall try to give some results of Fuzzy Sets and Fuzzy Logic. Also we shall discuss some applications like of Fuzzy Logic in **Insulin Pump Technology** which is going to helpful for Diabetic people.

II. SESSION B

(B-01)

Singular Three-point Boundary Value Problems for Delay Higher Order Dynamic Equations on Time Scales

Liang-Gen Hu
(University of Science and Technology of China, **P. R. China**)
(e-mail: hulianggen@yahoo.cn)

Abstract

In this paper, we are concerned with singular three-point boundary value problems for delay higher-order dynamic equation on time scales:

$$\begin{aligned}(-1)^n u^{\Delta^{2n}}(t) &= w(t)f(t,u(t-c)), \quad t\in[a,b],\\ u(t) &= \psi(t), \quad t\in[a-c,a),\\ u^{\Delta^{2i}}(a)-\beta_{i+1}u^{\Delta^{2i+1}}(a) &= \alpha_{i+1}u^{\Delta^{2i}}(\omega),\\ \gamma_{2i+1}u^{\Delta^{2i}}(\omega) &= u^{\Delta^{2i}}(b), \quad 1\le i\le n-1,\end{aligned}$$

where α_i, β_i, γ_i and ω satisfy the certain conditions, $\psi\in C([a-c,a])$. $w:(a,b)\to[0,\infty)$ is continuous and is continuous and $f:[a,b]\times(0,\infty)\to[0,\infty)$ is continuous. Our nonlinearity may have singularity at $t=a$ and/or $t=b$ and f may have singularity at $u=0$. Theorems on the existence of positive solutions are obtained by utilizing the fixed point theorem of cone expansion and compression type. Examples is also given to illustrate our results.

(B-02)

Existence Theory for Quadratic Perturbations of Abstract Measure Differential Equations

Bapurao C. Dhage
(Kasubai, Gurukul Colony, Ahmedpur-413 515, Dist: Latur Maharashtra, **India**)
(e-mail: bcdhage@yahoo.co.in)
S. S. Bellale
(Kasubai, Gurukul Colony, Ahmedpur-413 515, Dist: Latur Maharashtra, **India**)

Abstract

In this paper, an existence theorem for quadratic perturbations of second type for a nonlinear abstract measure differential equation is proved via a hybrid nonlinear alternative of Dhage [Nonlinear Funct. Anal. & Appl. 13(2) (2008), 323352]. Two existence results are also proved for extremal solutions for Carathéodory as well as discontinuous cases of the nonlinearities involved in the equations under certain monotonic conditions. Our results includes the existence results for a nonlinear abstract measure differential equation of Sharma [Proc. Amer. Math. Soc. 32 (1972), 503-510] and Sharma [Proc. Amer. Math. Soc. 48 (1975), 87-97] as special cases under weaker conditions.

(B-03)

A Collocation Method for Solving Integral Equations with Algebraic-logarithmic Singular Kernels

Yaghoub Mahmoudi
(Islamic Azad University, **Iran**)
(e-mail: mahmoudi@iaut.ac.ir)

Abstract

In this paper, a collocation method based on Taylor polynomials is presented for approximation of an integral equation with singular kernel. The kernel has the form $K(x,t) = |t-x|^{\alpha}\log|t-x|$. We approximate the unknown function in terms of Taylor polynomials. After substitution, the integral equation is converted to a system of algebraic linear equations. Numerical examples illustrate the pertinent features of the method.

Keywords and Phrases: Indefinite integral, algebraic-logarithmic singularity, Taylor polynomials, Fredholm integral equation, Volterra integral equation.

(B-04)

Nonhomogeneous Boundary Value Problems for Nonlinear Equation with p-Laplacian Operator

Shijie Dong
(Mechanical Engineering College, **P. R. China**)
(e-mail: jie_ ds@sina.com)
Haiyun Zhou
(Mechanical Engineering College, **P. R. China**)
(e-mail: witman66@yahoo.com.cn)

Abstract

Using an upper and lower solutions method and the Schauder fixed point theorem , we provide sufficient conditions for the existence of solutions to nonhomogeneous boundary value problem (BVP) for nonlinear equation with p-Laplacian operator

$$\begin{cases} (\phi_p(u'))' + f(t,u,u') = 0,\ 0 < t < 1, \\ u'(0) = \lambda_1,\ u(1) = \lambda_2, \end{cases}$$

where $\phi_p(s) = |s|^{p-2}s,\ p > 1$, and $\lambda_1, \lambda_2 \in \mathbb{R} := (-\infty, +\infty)$. Using one of the results, under some assumptions, we obtain explicit ranges of values of λ_1 and λ_2 with which the BVP has solutions, has positive solutions, and has no solution, respectively. Furthermore, the whole plane for λ_1 and λ_2 can be divided into two disjoint connected regions Λ^E and Λ^N such that the BVP has solutions for $(\lambda_1, \lambda_2) \in \Lambda^E$ and has no solutions for $(\lambda_1, \lambda_2) \in \Lambda^N$.

Keywords and Phrases: p-Laplacian, nonhomogeneous boundary value problem, Schauder fixed point theorem, upper and lower solutions.

(B-05)

Regularity for Solutions of Nonlinear Second Order Evolution Equations

Han-Geul Kim
(Pukyong National University, **Korea**)
(e-mail: gry99@naver.com)
Jin-Mun Jeong
(Pukyong National University, **Korea**)
(e-mail: jmjeong@pknu.ac.kr)

Abstract

In this paper, we consider the existence and regularity of the solutions for the following semilinear wave equation:

$$\begin{cases} u''(t) + Au(t) = f(t, u(t)) + h(t) \\ u(0) = u_0, \quad u'(0) = u_1. \end{cases} \tag{SE}$$

First, we give some basic results on existence, uniqueness, and a representation formula of solutions for the given equation (SE). We will also obtain the regularity for solutions of (SE) by converting the problem

into the contraction mapping principle when the nonlinear mapping f is Lipschitz continuous from $\mathbb{R} \times V$ into H, and obtain the norm estimate of a solution of the above nonlinear equation on $L^2(0,T;V) \cap W^{1,2}(0,T;H) \cap W^{2,2}(0,T;V^*)$ by using the results of its corresponding the linear part.

2000 Mathematics Subject Classification: Primary 35F25, Secondary 35K55.

Keywords and Phrases: Approximate controllability, regularity, parabolic variational inequalities, subdifferential operator.

(B-06)

Differential Z-matrix Linear Complementarity Systems

Xiaojun Chen
(The Hong Kong Polytechnic University, **Hong Kong**)
(e-mail: maxjchen@polyu.edu.hk)
Shuhuang Xiang
(Central South University, **P. R. China**)
(e-mail: xiangsh@mail.csu.edu.cn)

Abstract

In a recent Pango Jong-Si et al's article, a least-norm time-stepping scheme for the differential positive semi-definite linear complementarity system was proposed and its convergence to a weak solution of the system was proved under passivity and broader assumptions. In this talk, we show that the least-norm time-stepping scheme for the differential Z-matrix linear complementarity system converges linearly to a solution (x^*, y^*) such that x^* is continuously differentiable and y^* is Lipschitz continuous. Moreover, instead of solving a quadratic program with linear complementarity constraints, a generalized Newton method with a special matrix in the generalized Jacobian is used to

solve a system of nonsmooth equations at each step of the scheme. Numerical results of differential Z-matrix linear complementarity systems are given to illustrate the existence of solutions and convergence of the scheme.

Keywords and Phrases: Differential linear complementarity problem, least-norm, least-element, Lipschitz continuity, Z-matrix, generalized Newton method.

(B-07)

Gradient Estimates for Nonlinear Elliptic Equations

Sun-Sig Byun
(Seoul National University, **Korea**)
(e-mail: byun@snu.ac.kr)

Abstract

We establish global Calderon-Zygmund type estimates for a class of elliptic equations with discontinuous nonlinearity on a nonsmooth domain whose model is a $p(x)$-Laplacian equation. Our assumption is that for each point and for each sufficiently small scale the nonlinearity has small mean oscillations in the variables for the fixed gradient, the boundary of the domain can be trapped by two hyperplanes, and the bounded function $p(x)$ satisfies some natural modulus of continuity.

(B-08)

Existence and Uniqueness of Solutions for A Fourth-Order Boundary Value Problem

Hanying Feng

(Shijiazhuang Mechanical Engineering College, **P. R. China**)
(e-mail: fhanying@yahoo.com.cn)

Abstract

In this paper, we study the fourth-order two-point boundary value problem

$$x''''(t) - f(t, x(t), x'(t), x''(t), x'''(t)) = 0, \quad t \in (0, 1),$$
$$x(0) = x'(1) = 0, \quad ax''(0) - bx'''(0) = 0, \quad cx''(1) + dx'''(1) = 0.$$

By means of lower and upper solution method, growth conditions on the nonlinear term f which guarantee the existence of solutions for the above boundary value problem are given. In particular, we obtain the uniqueness of the solution by imposing a monotone condition of the term f.

(B-09)

General Nonlinear Random Equations with Random Multi-valued Operator in Banach Spaces

H. Y. Lan
(Sichuan University of Science and Engineering, **P. R. China**)
(e-mail: lanhenggou@hotmail.com)

Abstract

The purpose of this paper is to introduce and study a new class of general nonlinear random multi-valued operator equations involving generalized m-accretive mappings in Banach spaces. By using the Chang's lemma and the resolvent operator technique for generalized m-accretive mapping due to Huang and Fang [Generalized m-accretive mappings in Banach spaces, *J. Sichuan Univ.* **38(4)** (2001), 591-592], we also prove the existence theorems of the solution and convergence theorems of the generalized random iterative procedures with errors for

this nonlinear random multi-valued operator equations in q-uniformly smooth Banach spaces. The results presented in this paper improve and generalize some known corresponding results in the literature.

(B-10)

Multiple Solutions for a Class of the Critical Growth Suspension Bridge Equations

Tacksun Jungn
(Kunsan National University, **Korea)**
(e-mail: tsjung@kunsan.ac.kr)

Q-Heung Choi
(Inha University, **Korea)**
(e-mail: qheung@inha.ac.kr)

Abstract

We consider the multiplicity of the solutions for the following class of systems of the critical growth nonlinear suspension bridge equations with Dirichlet boundary condition and periodic condition.

(B-11)

At Least Four Solutios to the Nonlinear Elliptic System

Tacksun Jungn
(Kunsan National University, **Korea)**
(e-mail: tsjung@kunsan.ac.kr)

Q-Heung Choi
(Inha University, **Korea)**
(e-mail: qheung@inha.ac.kr)

Abstract

In this paper we investigate the existence of solutions for the nonlinear elliptic system with Dirichlet boundary conditions.

(B-12)

Controllability of Neutral Stochastic Impulsive Systems with Distributed Control Delays

S. Karthikeyan
(Bharathiar University, **India**)
(e-mail: karthi82india@gmail.com)

Abstract

Many evolution processes, optimal control models in economics, stimulated neural networks, frequency modulated systems and some motions of missiles or aircrafts, automatic control systems, artificial intelligence and robotics are characterized by the fact that at certain moments of time they experience changes of states abruptly. The impulsive differential equations provide a natural description of this type of observed evolutionary processes which are subject to short term perturbations acting instantaneously in the form of impulses. Uncertainty can be incorporated either as an expression of our lack of precise knowledge or as a true driving force. In the latter case it is useful to model the system by a stochastic or noise driven model which leads to the study of stochastic impulsive differential systems. Many of the physical systems may also contain some information about the derivative of the state component and such systems are called neutral systems. Practically, such systems are interesting because they serve as mathematical models in applied problems. Therefore, the investigation of neutral stochastic impulsive differential equations attracts great attention, especially as regards to controllability.

But in many applications such as man-machine systems, biomedical systems, process control, remote control and robotics, population models, delays are inherent in the control due to transportation lags and conduction or communication lines. Due to the transmission of the signal, the mechanical transmission needs a length of time. There are important practical applications for systems such as economic, biological, physiological and spacelight industry systems, having the phenomena of distributed time delays in control. As the demand on the control system increases, it will be more and more important to take the control delays into account in the

analysis and the design of the control system. Controllability is one of the most important properties of control systems. The controllability of nonlinear deterministic systems with delays in control has been extensively studied. However, there seems to have not yet been any study of stochastic behaviour of such models. Therefore the study of controllability of such models is of quite fundamental importance when the parameters are subject to some random disturbances like environmental factors.

In this paper we investigate the complete controllability for nonlinear neutral stochastic impulsive systems with distributed delays in control represented by the Itô equation of the form

$$\begin{aligned} d[x(t) - g(t, x(t))] &= \left[Ax(t) + \int_{-h}^{0} u(t+s)dH(t,s)\right] dt \\ &\quad + \sigma(t, x(t))dw(t),\ t \neq t_k \\ \Delta x(t_k) &= I_k(t_k, x(t_k^-)),\ t = t_k \\ x(t_0) &= x_0,\ t_0 \geq 0, \end{aligned}$$

where $k = 1, 2, \ldots, \rho$, A is a known constant matrix, $x(t) \in \mathbb{R}^n$ is the vector describing the instantaneous state of the stochastic system, $u(t) \in \mathbb{R}^m$ is a control input to the stochastic dynamical system, $H(t,s)$ is $n \times m$ continuous matrix in t for fixed s and is of bounded variation in s on the interval $[-h, 0]$, $\sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$, $g : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, w is an n-dimensional Wiener process, $I_k : \Omega \to \mathbb{R}^n, \Omega \subset [t_0, T] \times \mathbb{R}^n$, $\Delta x(t) = x(t^+) - x(t^-)$, where

$$\lim_{h \to 0^+} x(t+h) = x(t^+),\ \lim_{h \to 0^+} x(t-h) = x(t^-)$$

and $0 = t_0 < t_1 < t_2 < \cdots < t_\rho < t_{\rho+1} = T$, $I_k(x(t_k^-)) = \big(I_{1k}(x(t_k^-)), \ldots, I_{nk}(x(t_k^-))\big)^T$ represents the impulsive perturbation of x at time t_k and $x(t_k^-) = x(t_k), k = 1, 2, \ldots, \rho$, which implies that the solution of the above system is left continuous at t_k. The results are obtained by using the Banach contraction principle. Further we show the complete controllability of nonlinear impulsive stochastic system under the natural assumption that the associated linear impulsive stochastic control system is completely controllable. An example is discussed to illustrate the efficiency of the obtained results.

2000 Mathematics Subject Classification: 93B05, 93C10.

(B-13)

First-Order Nonlinear Evolution Equations and Yosida Approximations Based on Relative Maximal Monotonicity Models

Ram U. Verma
(Florida Institute of Technology, **USA**)
(e-mail: verma99@msn.com)

Abstract

Based on a general model for the Yosida approximation, the solvability of a general class of first-order nonlinear evolution equations is examined. The notion of the relative maximal monotonicity plays a greater role in developing a general model for the existing Yosida approximation and its characerizations in the literature.

(B-14)

On Quadratic Perturbations of Second Order Ordinary Functional Integro-Differential Equations

Bapurao C. Dhage
(Kasubai, Gurukul Colony, Ahmedpur-413 515, Dist: Latur Maharashtra, **India**)
(e-mail: bcdhage@yahoo.co.in)

Abstract

In this paper, an existence theorem for a quadratic perturbations of second order functional integro-differential equations is proved under mixed generalized Lipschitz and Carathéodory conditions. The existence of extremal solutions is also proved under certain monotonicity conditions. Our results of this paper include the existence results of Dhage [Comm. Appl. Anal. 10 (2006), 161-175] and Dhage and Ntouyas [Dynamic Systems & Appl. 16 (2007), 217-232] as special cases under weaker conditions.

(B-15)

Nonhomogeneous Multi-point Boundary Value Problems with P-Laplacian Operator

Shijie Dong
(Mechanical Engineering College, **P. R. China**)
(e-mail: jie_ ds@sina.com)
Haiyun Zhou
(Mechanical Engineering College, **P. R. China**)
(e-mail: witman66@yahoo.com.cn)

Abstract

The nonhomogeneous multi-point boundary value problem (BVP)

$$\begin{cases} (\phi_p(u'))' + a(t)f(u) = 0, \ 0 < t < 1, \\ u'(0) = 0, \ u(1) - \overline{=}b, \end{cases}$$

is studied in this paper, where $\phi_p(s) = |s|^{p-2}s, \ p > 1$. Sufficient conditions are given for the existence of solutions of the BVP by using Krasnoselskii's fixed point theorem.

Keywords and Phrases: p-Laplacian, nonhomogeneous boundary conditions, Krasnoselskii's fixed point theorem, positive solution.

(B-16)

Existence of Solutions of One Kind Nonlinear Elliptic System with (p, q)-Laplacian Operator

Li Wei
(Hebei University of Economics and Business, **P. R. China**)
(e-mail: diandianba@yahoo.com)

Abstract

In this paper, we shall use the perturbation results of the ranges of m-accretive mappings to discuss the existence of solutions of one kind nonlinear elliptic system with (p, q)-Laplacian operator, which can be considered as the extension of discussion?for the case of p-Laplacian operator.

(B-17)

Convergence of Newton's Method for Solving a Quadratic Matrix Equation

Yin-Huan Han
(Pusan National University, **Korea**)
Hyun-Min Kim
(Pusan National University, **Korea**)
(e-mail: hyunmin@pusan.ac.kr)

Abstract

Newton's method is an efficient numerical method for solving nonlinear matrix equation. We specially consider the quadratic matrix equations which is one of the nonlinear matrix equation. In this paper, an iterative method is suggested for solving the generalized Sylvester equation $A_1EB_1 + A_2EB_2 = D$ where A_1, A_2, B_1, B_2 and $D \in \mathbb{C}^{n\times n}$. By Newton's method with our algorithm, we show that when the quadratic matrix equations are consistent, for any symmetric and generalized centro-symmtric (or bisymmetric) starting matrices, a solvent which has the same property with starting matrix can be found. Finally, we give some numerical results.

Keywords and Phrases: Qquadratic matrix equation, solvent, Newton's method, symmetric and generalized centro-symmetric, bisymmetric.

(B-18)

Controllability for Nonlinear Variational Inequalities of Parabolic Type

Jin-Mun Jeong
(Pukyong National University, **Korea**)
(e-mail: jmjeong@pknu.ac.kr)
Eun Young Ju
(Pukyong National University, **Korea**)

Abstract

Consider the following the variational inequality problem with nonlinear term

in Hilbert space:

$$\begin{cases} (x'(t) + Ax(t), x(t) - z) + \phi(x(t)) - \ \phi(z) \\ \quad \leq (f(t, x(t)) + k(t), x(t) - z), \text{ a.e., } 0 < t \leq T, \quad z \in V, \\ x(0) = x_0. \end{cases} \tag{VIP}$$

When the nonlinear mapping f is a Lipschitz continuous from $\mathbb{R} \times V$ into H, we will obtain the existence for solutions of (NDE) by converting the problem into the contraction mapping principle and the norm estimate of a solution of the above nonlinear equation on $L^2(0,T;V) \cap W^{1,2}(0,T;V^*) \cap C([0,T];H)$. Thereafter, we can obtain the approximate controllability for the nonlinear functional differential control problem governed by the variational inequality.

2000 Mathematics Subject Classification: Primary 93C20, Secondary 49J40.

Keywords and Phrases: Approximate controllability, regularity, parabolic variational inequalities, subdifferential operator.

(B-19)

Integral Equation Method for the Schrödinger Equation with a Nonlocal Potential

Sheon-Young Kang
(National Institute for Mathematical Sciences, **Korea**)
(e-mail: skang@nims.re.kr)

Abstract

A new non-iterative integral equation method for an integrodifferential Schrödinger equation based on the Nyström Clenshaw-Curtis quadrature for the equation is described and compared with two other methods: One is iterative method which was adopted for nuclear phys -ics application. The other is non-iterative solution of the equation developed since late 1960s . The method presented here uses the Chebyshev polynomial to approximate the integrand and it turns out to be more accurate than the other two methods by many orders of magnitude for the same number of support points. Accompanying C^{++} code for the numerical method is available upon request.

Keywords and Phrases: Schrödinger equation, integral equation, Nyström type, Clenshaw-Curtis quadrature.

(B-20)

Global Existence and Stabilization for A Nonlinear Dissipative-coriolis Hyperbolic Equation of Kirchoff Type with The Mixed Boundary Conditions

Daewook Kim
(Pusan National University, **Korea**)
(e-mail: kdw@pusan.ac.kr)
Hyun-Wook Nam
(Pusan National University, **Korea**)
Il Hyo Jung
(Pusan National University, **Korea**)

Abstract

In this presentation, we try to prove the global and local existence of the hyperboric equation with the mixed boundary conditions. First, we introduce some assumptions and conditions of the system relating the equation. And we prove the existence of the system using the Faedo-Galerkin approximation and the smallness conditions on the initial data and non-degeneracy condition. Finally, we get some results of the uniform decay of the system with respect to expoential decay. Additionally, we can get the numerical results with respect to the behavior of solutions about the system under restricted conditions.

(B-21)

Controllability of Neutral Stochastic Impulsive Systems with Distributed Control Delays

S. Karthikeyan
(Bharathiar University, **India**)
(e-mail: karthi82india@gmail.com)

Abstract

Many evolution processes, optimal control models in economics, stimulated neural networks, frequency modulated systems and some motions of missiles or aircrafts, automatic control systems, artificial intelligence and robotics are characterized by the fact that at certain moments of time they experience changes of states abruptly. The impulsive differential equations provide a natural description of this type of observed evolutionary processes which are subject to short term perturbations acting instantaneously in the form of impulses. Uncertainty can be incorporated either as an expression of our lack of precise knowledge or as a true driving force. In the latter case it is useful to model the system by a stochastic or noise driven model which leads to the study of stochastic impulsive differential systems. Many of the physical systems may also contain some information about the derivative of the state component and such systems are called neutral systems. Practically, such systems are interesting because they serve as mathematical models in applied problems. Therefore, the investigation of neutral stochastic impulsive differential equations attracts great attention, especially as regards to controllability.

But in many applications such as man-machine systems, biomedical systems, process control, remote control and robotics, population models, delays are inherent in the control due to transportation lags and conduction or communication lines. Due to the transmission of the signal, the mechanical transmission needs a length of time. There are important practical applications for systems such as economic, biological, physiological and spacelight industry systems, having the phenomena of distributed time delays in control. As the demand on the control system increases, it will be more and more important to take the control delays into account in the analysis and the design of the control system. Controllability is one of the most important properties of control systems. The controllability of nonlinear deterministic systems with delays in control has been extensively studied. However, there seems to have not yet been any study of stochastic behaviour of such models. Therefore the study of controllability of such models is of quite fundamental importance when the parameters are subject to some random disturbances like environmental factors.

In this paper we investigate the complete controllability for nonlinear neutral stochastic impulsive systems with distributed delays in control represented by the Itô equation of the form

$$\begin{aligned} d[x(t) - g(t, x(t))] &= \left[Ax(t) + \int_{-h}^{0} u(t+s) dH(t,s) \right] dt \\ &\quad + \sigma(t, x(t)) dw(t), \quad t \neq t_k \\ \Delta x(t_k) &= I_k(t_k, x(t_k^-)), \ t = t_k \\ x(t_0) &= x_0, \ t_0 \geq 0, \end{aligned}$$

where $k = 1, 2, \ldots, \rho$, A is a known constant matrix, $x(t) \in \mathbb{R}^n$ is the vector describing the instantaneous state of the stochastic system, $u(t) \in \mathbb{R}^m$ is a control

input to the stochastic dynamical system, $H(t,s)$ is $n \times m$ continuous matrix in t for fixed s and is of bounded variation in s on the interval $[-h, 0]$, $\sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $g : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, w is an n-dimensional Wiener process, $I_k : \Omega \to \mathbb{R}^n, \Omega \subset [t_0, T] \times \mathbb{R}^n$, $\Delta x(t) = x(t^+) - x(t^-)$, where

$$\lim_{h\to 0^+} x(t+h) = x(t^+), \quad \lim_{h\to 0^+} x(t-h) = x(t^-)$$

and $0 = t_0 < t_1 < t_2 < \cdots < t_\rho < t_{\rho+1} = T$, $I_k(x(t_k^-)) = \big(I_{1k}(x(t_k^-)), \ldots, I_{nk}(x(t_k^-))\big)^T$ represents the impulsive perturbation of x at time t_k and $x(t_k^-) = x(t_k), k = 1, 2, \ldots, \rho$, which implies that the solution of the above system is left continuous at t_k. The results are obtained by using the Banach contraction principle. Further we show the complete controllability of nonlinear impulsive stochastic system under the natural assumption that the associated linear impulsive stochastic control system is completely controllable. An example is discussed to illustrate the efficiency of the obtained results.

2000 Mathematics Subject Classification: 93B05, 93C10.

(B-22)

On Some Results for the Approximation of Generalized Jordan Triple Derivations in Banach Algebras

Ick-Soon Chang
(Mokwon University, **Korea**)
(e-mail: ischang@mokwon.ac.kr)

Abstract

We adopt the idea of Cădariu and Radu to prove the stability of generalized Jordan triple derivations and generalized Jordan triple linear derivations. In addition, we take account of the applications of such derivations.

(B-23)

Integral Inequalities and Applications to the Calculus of Variations

Wing-Sum Cheung
(The University of Hong Kong, **Hong Kong**)
(e-mail: wscheung@hku.hk)

Abstract

Some simple integral inequalities are established and these are applied to solve certain Calculus of Variations problems directly.

(B-24)

Remarks on the Boundedness of the Solutions of Stochastic Functional Differential Equation with Infinite Delay

Young-Ho Kim
(Changwon National University, **Korea**)
(e-mail: yhkim@changwon.ac.kr)

Abstract

In this paper, we obtain some results on the solutions of a stochastic functional differential equations with infinite delay at phase space $BC((-\infty, 0]; R^d)$ which denotes the family of bounded continuous R^d-value functions φ defined on $(-\infty, 0]$ with norm $\|\varphi\| = \sup_{-\infty<\theta\leq 0} |\varphi(\theta)|$ under non-Lipschitz condition with Lipschitz condition being considered as a special case and a weakened linear growth condition. The solution is constructed by the successive approximation.

(B-25)

Inverse Problems for the Phase Field Systems with Single Measurement using Carleman estimate

N. Barani Balan
(Bharathiar University, **India**)
(e-mail: baranibalan1982@gmail.com)

Abstract

In this note, we establish the stability result for the inverse problem consisting of retrieving one smooth coefficient of the following coupled parabolic equations with Dirichlet boundary arising from phase transitions. In the mathematical literature, the free boundary problems arising from phase transitions have been studied for over a century. Most of the work is concerned with the classical Stefan problem which incorporates the physics of latent heat and heat diffusion in a homogeneous medium. It should be recalled that phase field models were first introduced by Caginalp and Fix. In later years a great effort has been devoted by mathematicians to the study of several variants of the model, and interesting results have been obtained in the direction of existence and regularity of solutions, as well as of their dependence on the physical parameters.

The simplest phase field model can be written as a system of parabolic equations describing the phase transitions between two states, for example, solid or liquid, in pure material with (p, q) as the solution

$$\left.\begin{array}{l} p_t + l(x)q_t - \Delta p + a(x)q = f_1(x), \quad \text{in } Q, \\ q_t - \Delta q + b(x,t)q + c(x)p = f_2(x), \quad \text{in } Q, \\ p(x,\theta) = p_\theta(x),\ q(x,\theta) = q_\theta(x), \quad \text{in } \Omega, \\ p(x,t) = h_1(x,t),\ q(x,t) = h_2(x,t), \quad \text{on } \Sigma, \end{array}\right\}$$

where Ω is an open bounded subset of $\mathbb{R}^n$ for the dimension $n \leq 3$ and with boundary $\partial\Omega$ of class C^2. The coefficient $l \in L^\infty(\Omega)$ is the latent heat, $b \in C^1(Q)$ and for some fixed $\theta \in (0, T)$, the semi-initial values $p_\theta, q_\theta : \Omega \to \mathbb{R}$ are sufficiently regular (for instance, $(p_\theta, q_\theta) \in (H^2(\Omega))^2$). The nonzero smooth Dirichlet boundary data $h_1, h_2 : \Sigma \to \mathbb{R}$ are kept fixed and $(f_1, f_2) \in (L^2(\Omega))^2$ are given functions. The solution p denotes the temperature distribution of a material which occupies the region Ω and can be in either of two phases, solid or liquid (if the melting temperature is taken to be zero) and the smooth function q is called as the "phase field function" and it is scaled so that q near +1 corresponds to one phase, for example liquid phase and q near -1 corresponds to the other, solid phase. The phase field function can be related to macroscopic observables and serves to distinguish

the phases. In many areas of statistical mechanics it has also been related to microscopic quantities. In a system with complex dynamics, the order parameter is coupled with other variables, and is constrained to have a fixed value on each side of the equilibrium coexistence curve in the pressure-temperature plane. The unknown coefficient $a(x)$ and the coefficient $c(x)$ are assumed to be sufficiently smooth and shall be kept independent of time t.

The proof of the stability results rely on Carleman estimates and certain energy estimates for parabolic system.

2000 Mathematics Subject Classification: 35K05, 35R30, 80A22.

(B-26)

Implicit Solution Function of P_0 and Z Matrix Linear Complementarity Constraints

Xiaojun Chen
(The Hong Kong Polytechnic University, **Hong Kong**)
(e-mail: maxjchen@polyu.edu.hk)
Shuhuang Xiang
(Central South University, **P. R. China**)
(e-mail: xiangsh@mail.csu.edu.cn)

Abstract

Using the least element solution of the P_0 and Z matrix linear complementarity problem (LCP), we define an implicit solution function for linear complementarity constraints (LCC). We show that the sequence of solution functions defined by the unique solution of the regularized LCP is monotonically increasing and converges to the implicit solution function as the regularization parameter goes down to zero. Moreover, each component of the implicit solution function is convex. We find that the solution set of the irreducible P_0 and Z matrix LCP can be represented by the least element solution and a Perron-Frobenius eigenvector. These results are applied to convex reformulation of mathematical programs with P_0 and Z matrix LCC. Preliminary numerical results show the effectiveness and the efficiency of the reformulation.

Keywords and Phrases: Linear complementarity problem, P_0-mat -rix, Z-matrix, regularization technique, Perron-Frobenius theorem.

(B-27)

On the Existence of Solutions for Nonlinear First-order Implicit Impulsive Integro-differential Equations

H. Y. Lan
(Sichuan University of Science and Engineering, **P. R. China**)
(e-mail: lanhengyou@hotmail.com)

Abstract

By using the Mönch fixed point theorem, the purpose of this paper is to obtain the new existence theorems of solutions for a new class of initial value problems of nonlinear first-order implicit impulsive integro-differential equations in Banach spaces under some weaker conditions.

2000 Mathematics Subject Classification:

(B-28)

Asymptotic Behavior of Bounded Solutions to Some Second Order Evolution Equations and Difference Equations

Behzad Djafari Rouhani
(University of Texas at El Paso, **USA**)
(e-mail: behzad@math.utep.edu)

Abstract

We consider the following second order nonhomogeneous evolution equation

$$\begin{cases} u''(t) - cu'(t) \in Au(t) + f(t) \quad \text{a.e.} \quad t \in (0, +\infty) \\ u(0) = u_0, \quad \sup\limits_{t \geq 0} |u(t)| < +\infty \end{cases}$$

where A is a monotone operator in a real Hilbert space H, $c \geq 0$, and $f : \mathbb{R}^+ \to H$ is a given function, as well as its discrete analogue corresponding to the following

second order difference equation

$$\begin{cases} u_{n+1} - 2u_n + u_{n-1} \in c_n A u_n; \ \ n \geq 1 \\ u_0 \in H, \quad \sup\limits_{n \geq 0} |u_n| < +\infty \end{cases}$$

where $\{c_n\}$ is a positive real sequence. We prove ergodic theorems, as well as weak and strong convergence theorems for solutions to these equations, converging to an element of $A^{-1}(0)$, implying in particular that solutions exist if and only if $A^{-1}(0) \neq \emptyset$. Our results extend and give simpler proofs to previous results by several authors who studied special cases of similar problems by assuming that $A^{-1}(0) \neq \emptyset$, and have many applications in approximation and optimization theory.

III. SESSION C

(C-01)

An Extragradient-viscosity Approximation Method for General Euilibrium Problems, Variational Inequalities and Fixed Point of an Infinite Family of Nonexpansive Mappings*

Phayap Katchang
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: phayapk@hotmail.com)
Tareerat Tanutpanit
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: tareerat@yahoo.com)
Poom Kumam
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: poom.kum@kmutt.ac.th)

Abstract

The purpose of this paper is to investigate the problem of finding a common element of the set of solutions for mixed equilibrium problems, the set of solutions of the variational inequalities and the set of fixed points of a family of infinitely nonexpansive mappings in the setting of Hilbert spaces. We propose an iterative scheme by the extragradient viscosity approximation method for finding the common element of the above three sets. Moreover, it is proven that the sequences generated by this iterative scheme converge strongly to a common element of the set of solutions of mixed equilibrium problems, the set solutions of the variational inequalities and the set of fixed points of an infinite family of nonexpansive mapping. Our results improve and extend the corresponding results of Peng and Yao [Strong convergence theorems of iterative scheme based on the extragradient method for mixed equilibrium problems and fixed point problems, *Math. and Comp. Model.*, doi:10.1016/j.mcm.2008.11.014], Kumam and Katchang [A viscosity of extragradient approximation method for finding equilibrium problems, variational inequalities and fixed point problems for nonexpansive mapping, *Nonlinear Anal: Hybrid Systems*, doi:10.1016/j.nahs.2009.03.006], Kumam [Strong Convergence Theorems by an Extragradient Method for Solving Variational Inequalities and Equilibrium Problems in a Hilbert space, *Turk. J. Math.*, 33 (2009), 85-98] and some well-known results in the literature.

2000 Mathematics Subject Classification: 46C05, 47D03, 47H09, 47H10, 47H20.

Keywords and Phrases: Nonexpansive mapping, monotone mapping, equilibrium problem, variational inequality.

(C-02)

Mixed Vector FQ-implicit Variational Inequality with Local Non-Positivity

Byung-Soo Lee
(Kyungsung University, **Korea**)
(e-mail: bslee@ks.ac.kr)

Abstract

This paper introduces a local non-positivity of two set-valued mappings (F, Q) and considers the existences and properties of solutions for set-valued mixed vector FQ-implicit variational inequality problems and set-valued mixed vector FQ-complemen-tarity problems in the neighborhood of a point belonging to an underlined domain K of the set-valued mappings, where the neighborhood is contained in K.

This paper generalizes and extends many results in [1-6].

(C-03)

An Iterative Algorithm of Solution for Quadratic Minimization Problem in Hilbert Spaces

Guanghui Gu
(Cangzhou Normal University, **P. R. China**)
(e-mail: hbczguguanghui@yahoo.com.cn)

Abstract

The purpose of this talk is to introduce an iterative algorithm for finding a solution of quadratic minimization problem in the set of fixed points of a nonexpansive

mapping and to prove a strong convergence theorem of the solution for quadratic minimization problem. The result of this article improved and extended the result of G. Marino and H. K. Xu and some others.

Keywords and Phrases: Iterative method, nonexpansive mapping, fixed point, minimization problem.

(C-04)

Approximation of A Common Zero Point of A Finite Family of m-accretive Mappings in Banach Spaces

Rudong Chen
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: tjcrd@yahoo.com.cn)
Qinwei Fan
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: fanqinwei2008@yahoo.com.cn)

Abstract

In this paper we introduce a composite iteration scheme for approximating a zero point of finite family of m-accretive mappings in the framework of strictly convex real Banach spaces which has a uniformly Gâteaux differentiable norm. Strong convergence of the composite iteration scheme $\{x_n\}$ defined by

$$\begin{cases} y_n = \beta_n x_n + (1-\beta_n) S_r x_n \\ x_{n+1} = \alpha_n u + (1-\alpha_n) y_n \end{cases}$$

where $A_i : K \to E$, $i = 1, 2, 3, \cdots, r$ is a family of m-accretive mappings with $\cap_{i=1}^{r} N(A_i) \neq \emptyset$ and $S_r := a_0 I + a_1 J_{A_1} + a_2 J_{A_2} + \cdots + a_r J_{A_r}$, with $J_{A_i} =: (I + A_i)^{-1}$ for $0 < a_i < 1$, $i = 0, 1, 2, \cdots, r$, $\sigma_{i=0}^{r} a_i = 1$, and $u \in K$ is an arbitrary (but fixed) element in K and $\{\alpha_n\}$ in $(0, 1)$, and $\{\beta_n\}$ in $[0, 1]$ is established. Under certain appropriate assumptions on the sequences $\{\alpha_n\}$ and $\{\beta_n\}$, that $\{x_n\}$ defined by the above iteration scheme converges strongly to a common zero point of $\{A_i\}_{i=1}^{r}$ is proved. The results present in this paper improve and extend results of T. H. Kim, H. K. Xu [Strong convergence of modified Mann iterations, Nonlinear Anal. 61(2005)51-60] and H.Zegeye, N.Shahzad [Strong convergence theorems for a common zero of a finite family of m-accretive mappings, Nonlinear Anal. 66(2007)1161-1169] and some others.

(C-05)

Strong Convergence Theorems for Monotone Mappings and Weak Relatively Nonexpansive Mappings and Equilibrium Problems

Yongfu Su
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: suyongfu@gmail.com)
Jinlong Kang
(Xi'an Communication of Institute, **P. R. China**)
(e-mail: kangjinlong@yahoo.com)
Hong Zhang
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: zhanghongmath@gmail.com)

Abstract

In this talk, we introduce an iterative scheme for finding a common element of the set of fixed points of a weak relatively nonexpansive mapping, the set of solutions of the variational inequality for the monotone mapping and the set of solutions of an equilibrium problem in a 2-uniformly convex and uniformly smooth Banach space. Then we show that the iterative sequence converges strongly to a common element of the three sets.

Keywords and Phrases: Relatively weak nonexpansive mapping, monotone mapping, variational inequality, equilibrium problem, fixed point.

(C-06)

A Projection-Proximal Point Algorithm for Solving Generalized Variational Inequalities

Fu Quan Xia
(Sichuan Normal University, **P. R. China**)
(e-mail: fuquanxia@sina.com)

Abstract

In this paper, a projection-proximal point method for solving a class of general-

ized variational inequalities is considered in Hilbert spaces. We investigate a general iterative algorithm, which consists of an inexact proximal point step followed by a suitable orthogonal projection onto a hyperplane. We prove the convergence of the algorithm for a maximal monotone mapping without upper semi-continuity and a pseudo-monotone mapping with upper semi-continuity, respectively. We also analyze the convergence rate of the iterative sequence under some suitable conditions.

(C-07)

A New Iterative Method for Solving a System of General Variational Inequalities

Min Liu
(Yibin University, **P. R. China**)
(e-mail: liuminybsc@yahoo.com.cn)
Shih-sen Chang
(Yibin University, **P. R. China**)
(e-mail: changss@yahoo.cn)

Abstract

In this paper, we introduce and consider a new system of general variational inequalities involving six different operators. Using the projection operator technique, we introduce a new iterative scheme for this system of general variational inequalities. Under suitable conditions, some strong convergence theorems are proved. Our results improve the main results in S. S. Chang, H. W. J. Lee, C. K. Chan [2] and M. Aslam Noor, K. Inayat Noor [5]. Moreover, the iterative scheme and the method of proof adopted in the paper is differen from that of [5].

(C-08)

General Wiener-Hopf Equations for Nonexpansive Mappings and General Variational Inequalities

Meijuan Shang
(Shijiazhuang University, **P. R. China**)
(e-mail: meijuanshang@yahoo.com.cn)

Yongfu Su
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: suyongfu@gmail.com)
Xiaolong Qin
(Gyeongsang National University, **Korea**)
(e-mail: qxlxajh@163.com)

Abstract

In this paper, we show that the general variational inequalities are equivalent to the general Wiener-Hopf equations and use this alterative equivalence to suggest and analyze a new iterative method for finding the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the general variational inequality. Our results include the previous results [M.Aslam Noor, Wiener-Hopf equations and variational inequalities, J. Optim. Theory Appl. 79 (1993) 197-206], [M.Aslam Noor, General variational inequalities and nonexpansive mappings, J. Math. Anal. Appl. 331 (2007) 810-822] and many others as special cases and may be considered as an improvement and refinement of the previously known results.

(C-09)

Browder-Tikhonov Regularization for a Class of Evolution Second Order Hemivariational Inequalities

Yi-bin Xiao
(University of Electronic Science and Technology of China, **P. R. China**)
(e-mail: xiaoyb9999@hotmail.com)
Nan-jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)

Abstract

In this paper, we consider a class of evolution second order hemivariational inequalities with non-coercive operators which are assumed to be known approximately. Using the so-called Browder-Tikhonov regularization method, we prove that the regularized evolution hemivariational inequality problem is solvable. We construct a sequence based on the solvability of the regularized evolution hemivariational in- equality problem and show that every weak cluster of this sequence is a solution for the evolution second order hemivariational inequality.

(C-10)

An Iterative Algorithm Based on M-Proximal Mappings for a System of Generalized Implicit Variational Inclusions in Banach Spaces

Kaleem Raza Kazmi
(Aligarh Muslim University, **India**)
(e-mail: krkazmi@gmail.com)

Abstract

In this paper, we give the notion of M-proximal mapping, an extension of P-proximal mapping given by Ding and Xia [A new class of completely generalized quasi-variational inclusions in Banach spaces, J. Comput. Appl. Math. 147 (2002) 369-383], for a nonconvex, proper, lower semicontinous and subdifferentiable functional on Banach space and prove its existence and Lipschitz continuity. Further, we consider the following new system of generalized implicit variational inclusions in Banach spaces:

For each $i = 1, 2, 3$, let E_i be real Banach space with norm $||\cdot||_i$ and let E_i^* be its dual space with norm $||\cdot||_{*i}$; let A_{1i}, A_{2i}, $A_{3i} : E_i \to CB(E_i^*)$, $D_i : E_i \to CB(E_i)$ be set-valued mappings; let $N_i : E_1^* \times E_2^* \times E_3^* \to E_i^*$ and $g_i : E_i \to E_i$ be single-valued mappings; let $\phi_i : E_i \times E_i \to R \cup \{+\infty\}$ be such that for each fixed $z_i \in E_i$, $\phi_i(\cdot, z_i) : E_i \to R \cup \{+\infty\}$ is a proper, lower semicontinuous and subdifferentiable and $g_i(E_i) \cap dom\partial\phi_i(\cdot, z_i) \neq \emptyset$.

Find $(x_1, x_2, x_3,\ u_{11}, u_{12}, u_{13}, u_{21}, u_{22}, u_{23}, u_{31}, u_{32}, u_{33},\ z_1, z_2, z_3)$ such that for each $i = 1, 2, 3$, $x_i \in E_i$, $u_{i1} \in A_{i1}(x_1)$, $u_{i2} \in A_{i2}(x_2)$, $u_{i3} \in A_{i3}(x_3)$, $z_i \in D_i(x_i)$, and

$$\langle N_i(u_{i1}, u_{i2}, u_{i3}), y_i - g_i(x_i)\rangle \geq \phi_i(g_i(x_i), z_i) - \phi_i(y_i, z_i), \quad \forall y_i \in E_i,$$

and show its equivalence with a system of implicit Wiener-Hopf equations using the concept of M-proximal mappings. Using this equivalence, we propose a new iterative algorithm for the system of generalized implicit variational inclusions. Furthermore, we prove the existence of solution of the system of generalized implicit variational inclusions and discuss the convergence and stability analysis of the iterative algorithm. We also discuss some further extensions of M-proximal mappings and the theorems presented in this work. The theorems presented in this paper can be viewed as significant generalizations of many known and important results under Hilbert spaces as well as reflexive Banach spaces setting in recent literature.

2000 Mathematics Subject Classification: 47H04, 49J40.

Keywords and Phrases: System of generalized implicit variational inclusions, M-proximal mapping, mixed lipschitz mapping, system of implicit Wiener-Hopf equations, iterative algorithm, convergence and stability.

(C-11)

Relaxed Extragradient Viscosity Approximation Method for Equilibrium Problems and Variational Inequalities for Relaxed Cocoercive Mappings*

Chaichana Jaiboon
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: s0501403@st.kmutt.ac.th, chaichana111@hotmail.com)
Poom Kumam
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: poom.kum@kmutt.ac.th)

Abstract

In this paper, we introduce a new iterative scheme for nding the common element of the set of: xed points of countable family for nonexpansive mappings; equilibrium; and the variational inequality problems for a relaxed cocoercive, Lipschitz continuous mappings by relaxed extragradient methods. We show that the sequence converges strongly to a common element of the above three sets under some parameter controlling conditions. Our results improve and extend the recent ones announced by many other.

2000 Mathematics Subject Classification: 46C05, 47D03, 47H09, 47H10, 47H20.

Keywords and Phrases: Nonexpansive mapping, monotone mapping, equilibrium problem, variational inequality.

(C-12)

From an Abstract Maximal Element Principle to Optimization Problems, Stationary Point Theorems and Common Fixed Point Theorems

Lai-Jiu Lin
(National Changhua University of Education, **Taiwan**)
(e-mail: maljlin@cc.ncue.edu.tw)
Wei-Shih Du
(National Kaohsiung Normal University, **Taiwan**)
(e-mail: wsdu@nknucc.nknu.edu.tw)

Abstract

In this paper, we first establish an existence theorem related with intersection theorem, maximal element theorem and common fixed point theorem for multivalued maps by applying an abstract maximal element principle proved by Lin and Du. Some new stationary point Theorems, minimization problems, new fixed point theorems and a system of nonconvex equilibrium theorem are also given.

(C-13)

Approximation of Solutions for Generalized Wiener-Hopf Equations and Generalized Variational Inequalities

Guanghui Gu
(Cangzhou Normal University, **P. R. China**)
(e-mail: hbczguguanghui@yahoo.com.cn)
Yongfu Su
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: suyongfu@gmail.com)

Abstract

The purpose of this talk is to introduce a new generalized class of the Wiener-Hopf equations and a new generalized class of the variational inequalities. Using the projection technique, we show that the generalized Wiener-Hopf equations are

equivalent to the generalized variational inequali- ties. We use this alternative equivalence to suggest and analyze an iterative scheme for finding the solution of the generalized Wiener-Hopf equations and the solution of the generalized variational inequalities. The results presented in this paper may be viewed as significant and improvement of the previously known results. In special, our results improve and extend the resent results of M. A. Noor and Z. Y. Huang [M. A. Noor and Z. Y. Huang, Wiener-Hopf equation technique for variational inequalities and nonexpansive mappings, Appl. Math. Comput.(2007), doi:10.1016/j.amc.2007.02.117].

Keywords and Phrases: Wiener-Hopf equations, variational inequalities, iterative algorithm, nonexpansive mappings.

(C-14)

Shrinking Projection Methods for Firmly Nonexpansive Mappings and Its Applications

Yongfu Su
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: suyongfu@gmail.com)

Abstract

The purpose of this talk is to study the shrinking projection method for finding common fixed points of firmly nonexpansive mappings. Some strong convergence theorems are proved. The main convergence the- orem is also applied to the equilibrium and optimization problems. The results of this paper improve and extend the results of Koji Aoyama, Fumiaki Kohsaka, Wataru Takahashi [Koji Aoyama, Fumiaki Kohsaka, Wataru Taka- hashi, Shrinking projection methods for firmly nonexpansive mappings, Nonlinear Analysis (2009), doi:10.1016/j.na. 2009.02.001] in the following respects: (1) the main convergence theorem has been proved by using the new method; (2) the condition of family of firmly nonexpansive mappings $\{T_n\}_{n=1}^{i} nfty$ has been relaxed from the condition (Z) to uniformly closed; (3) the application has been given to find the solution of equilibrium and optimization problems.

Keywords and Phrases: Shrinking projection method, firmly nonexpansive mapping, uniformly closed, equilibrium problem.

(C15)

Generalized Equilibrium Problem and k-strict Pseudocontraction

Li Yang
(Southwest University of Science and Technology, **P. R. China**)
(e-mail: yanglizxs@yahoo.com.cn)

Abstract

We introduce and study a new iterative method for finding a common element of the set of solutions for a generalized equilibrium problem and the set of fixed points for a k-strict pseudocontractive mapping in Hilbert space. The strong convergence theorem obtained in this paper extends and improves some recent results.

(C-16)

An Iterative Method for Equilibrium Problems and Quasi-variational Inclusion Problems

Min Liu
(Yibin University, **P. R. China**)
(e-mail: liuminybsc@yahoo.com.cn)
Shih-sen Chang
(Yibin University, **P. R. China**)
(e-mail: changss@yahoo.cn)

Abstract

In this paper, we introduce a hybrid iterative scheme for finding a common element of the set of solutions for an equilibrium problems, the set of common fixed point for a family of infinite nonexpansive mappings and the set of solutions of the variational inclusion problem with multi-valued maximal monotone mappings and inverse-strongly monotone mappings in Hilbert space. Under suitable conditions, some strong convergence theorems are proved. Our results extend the recent results in Takahashi and Toyoda [19], Takahashi and Takahashi [18], Chang, Lee and Chan [8, 9].

(C-17)

The Approximate Solvability of a System of Nonlinear Variational Inequalities

Meijuan Shang
(Shijiazhuang University, **P. R. China**)
(e-mail: meijuanshang@yahoo.com.cn)
Yongfu Su
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: suyongfu@gmail.com)
Xiaolong Qin
(Gyeongsang National University, **Korea**)
(e-mail: qxlxajh@163.com)

Abstract

In this paper, we consider the approximate solvability of a system of nonlinear variational inequalities involving two different relaxed cocoercive mappings in the framework of Hilbert spaces. From the numerical and approximation point of views, we suggest and analyze a new iterative scheme for finding the common element of the set of fixed points of nonexpansive mappings and the set of solutions of a system of nonlinear variational inequalities. Our results extend and improve the corresponding results announced by many others.

(C-18)

Approximate Solvability of a System of Nonlinear Relaxed Cocoercive Variational Inequalities and Lipschitz Continuous Mapping in Hilbert Spaces

Yeol Je Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gsnu.ac.kr)
Narin Petrot
(Naresuan University, **Thailand**)
(e-mail: narinp@nu.ac.th)

Abstract

In this work, we study the approximate solvability of a system of nonlinear relaxed cocoercive variational inequalities and a Lipschitz continuous mapping in the framework of Hilbert spaces. Our results represent significant and important refinements of the recently known results of [S. S. Chang, H. W. Joseph Lee, C. K. Chan, Generalized system for relaxed cocoercive variational inequalities in Hilbert spaces, Appl. Math. Lett. 20(2007), 329-334.], [R.U. Verma, A new class of iterative algorithms for approximation-solvability of nonlinear variational inequalities, Comput. Math. Appl. 41 (2001) 505-512.], [R.U. Verma, Projection methods, algorithms, and a new system of nonlinear variational inequalities, Comput. Math. Appl. 41 (2001) 1025-1031], [R. U. Verma, General convergence analysis for two-step projection methods and application to variational problems, Appl. Math. Lett. 18 (2005), 1286-1292] and many others.

Keywords and Phrases: System of nonlinear relaxed cocoercive variational inequalities, relaxed cocoercive mapping, Lipschitz continuous mapping.

(C-19)

An Explicitly Iterative Algorithm for Zeros of Accretive Operators

Yisheng Song
(Henan Normal University, **P. R. China**)
(e-mail: songyisheng123@yahoo.com.cn)

Abstract

In this paper, for general Lipschitz accretive operator A,an iteration scheme is defined as follows:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n(u - \beta_n Ax_n).$$

The strong convergence of the given interative sequence is established for finding some zero of Awhenever $\alpha_n, \beta_n \in (0, 1)$ satisfying conditions:

$$\lim_{n\to\infty} \alpha_n = 0, \sum_{n=1}^{+\infty} \alpha_n = +\infty, \lim_{n\to\infty} \beta_n = 0.$$

Furthermore, some applications for equilibrium problems are given also. In particular, the iteration coefficient are simpler and more general (for example, $\alpha_n = \beta_n = \frac{1}{n+1}$).

(C-20)

Weak and Strong Convergence Theorems for Equilibrium Problems and Countable Strict Pseudo-contractions in Hilbert Space

Rudong Chen
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: tjcrd@yahoo.com.cn)
Shujun Cui
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: cuishujun@yahoo.cn)

Abstract

In this paper, we introduce two iterative sequence for finding a common element of the set of solutions of an equilibrium problem and the set of fixed points of a countable family of strict pseudo-contractions in Hilbert Space. Then we study the weak and strong convergence of the sequences.

(C-21)

Strong Convergence Theorems by Using The Hybrid Projection Method in Mathematical Programming for Variational Inequality, Fixed Point and Equilibrium Problems

Ornrudee Suttisri
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: ornrudee.sut@kmutt.ac.th)
Tareerat Tanutpanit
(King Mongkut's University of Technology Thonburi, **Thailand**)

(e-mail: tareerat@yahoo.com)
Poom Kumam
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: poom.kum@kmutt.ac.th)

Abstract

In this talk, we introduce and study an iterative scheme by a new hybrid method for finding a common element of the set of fixed points of a countable family of nonexpansive mappings, the set of solutions of an general equilibrium problem and the set of solutions of the general system of variational inequality for for cocoercive mapping mappings in a real Hilbert space. We show that the iterative sequence converges strongly to a common element of the above three sets, which solves some fixed point problems, a system of variational inequality problems and general equilibrium problems by using the hybrid method in mathematical programming which connected with optimization problems. The results are connected with Kumam's result [1, 2], Shinzato and Takahashi's result [3], Tada and Takahashi's result [4] and Takahashi's et. al. result [5] and many others.

2000 Mathematics Subject Classification: 46C05, 47D03, 47H09, 47H10, 47H20.

Keywords and Phrases: Nonexpansive mapping, monotone mapping, equilibrium problem, variational inequality.

(C-22)

Generalized Bi-quasi-variational Inequalities for Quasi-pseudo-monotone Type II Operators on Compact Sets

Mohammad Chowdhury
(Lahore University of Management Sciences,**Pakistan**)
(e-mail: msrchowdhury@yahoo.com.au)

Abstract

In this paper, the authors prove some existence results of solutions for a new class of generalized bi-quasi-variational inequalities (GBQVI) for quasi-pseudo-monotone type II operators defined on compact sets in locally convex Hausdorff topological¿

vector spaces. In obtaining these results on GBQVI for quasi-pseudo-monotone type II operators, Tan's generalized version¿ Ky Fan's minimax inequality tool.

(C-23)

An Iterative Method for Variational Inequality, Mixed Equilibrium and Fixed Point Problems of Strictly Pseudocontractive Mappings

Rattanaporn Wangkeeree
(Naresuan University, **Thailand**)
(e-mail: rattanapornw@nu.ac.th)
Rabian Wangkeeree
(Naresuan University, **Thailand**)
(e-mail: rabianw@nu.ac.th)

Abstract

In this paper, we introduce an iterative scheme for finding a common element of the set of fixed points of a k-strictly pseudocontractive mapping, the set of solutions of the variational inequality for an inverse-strongly monotone mapping and the set of solutions of the mixed equilibrium problem in a real Hilbert space. Under suitable conditions, some strong convergence theorems for approximating a common element of the above three sets are obtained. As applications, at the end of the paper we first apply our results to study the optimization problem and we next utilize our results to study the problem of finding a common element of the set of fixed points of two families of finitely k-strictly pseudocontractive mapping, the set of solutions of the variational inequality, and the set of solutions of the mixed equilibrium problem. The results presented in the paper improve some recent results announced by Kim and Xu [1], Yao, Chen, and Yao [5], Marino, Colao, Qin, and Kang [2], Liu [3], Plubtieng and Punpaeng [4].

2000 Mathematics Subject Classification: 47H09, 47H10, 47H17.

Keywords and Phrases: Strictly pseudocontractive mapping, inverse strongly monotone mapping, variational inequality, mixed equilibrium problem, strong convergence, fixed point, metric projection.

(C-24)

Systems of Relaxed Cocoercive Generalized Variational Inequalities via Nonexpansive Mappings

Yan Hao
(Zhejiang Ocean University, **P. R. China**)
(e-mail: hymm6512@163.com)

Abstract

In this paper, we consider a system of general variational inequalities. We introduce an iterative algorithm for the system of general variational inequalities and study the algorithmic convergence analysis. Strong convergence theorems are established. The results presented in this paper mainly improve and extend the corresponding results announced by many others.

(C-25)

New Iterative Scheme for Equilibrium Problem, Relatively Nonexpansive Mappings and Maximal Monotone Operators in Banach Spaces

Li Wei
(Hebei University of Economics and Business, **P. R. China**)
(e-mail: diandianba@yahoo.com)
Haiyun Zhou
(Ordnance Engineering College, **P. R. China**)
(e-mail: witman66@yahoo.com.cn)

Abstract

In this paper, we shall introduce a new hybrid iterative scheme to approximate the common set of the solution of the equilibrium problems, the set of fixed points of relatively nonexpansive mappings, the set of zeroes of maximal monotone operators in uniformly smooth and uniformly convex Banach spaces. Some strong convergence theorems of the iterative schemes are obtained which extend and complement some previous work. Moreover, the applications of the iterative schemes on optimization problem and variational inequalities problems are demonstrated.

IV. SESSION D

(D-01)

Scalarization Approaches for Set-valued Vector Optimization Problems and Vector Variational Inequalities

Jun Li
(China West Normal University, **P. R. China**)
(e-mail: junli1026@163.com)
Sy-Ming Guu
(Yuan Ze University, **Taiwan**)
Nan-Jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)

Abstract

The scalarization approaches of Giannessi, Mastroeni and Pellegrini are extended to the study of set-valued vector optimization problems and set-valued weak vector optimization problems. Some equivalence results among set-valued (scalar) optimization problems, set-valued (scalar) quasi-optimization problems, set-valued vector optimization problems and set-valued weak vector optimization problems are established under convexity assumption of the objective functions. Some examples are provided to illustrate these results. The approaches are furthermore exploited to investigate set-valued vector variational inequalities and set-valued weak vector variational inequalities, which are different from that suggested by Konnov. Some equivalence relations among set-valued (scalar) variational inequalities, set-valued (scalar) quasi-variational inequalities, set-valued vector variational inequalities and set-valued weak vector variational inequalities are also derived under suitable conditions.

(D-02)

Weak Stability and Strong Duality of a Class of Nonconvex Infinite Programs via Augmented Lagrangian

Ta Quang Son
(Nhatrang College of Education, **Vietnam**)
(e-mail: taquangson@gmail.com)
Do Sang Kim
(Pukyong National University, **Korea**)
(e-mail: dskim@pknu.ac.kr)

Abstract

In this talk, we present duality of a class of nonconvex infinite programs via augmented Lagrangian. Based on a concept of weak-subgradient applied to the marginal functions of the problems, we investigate relations between their weak-stability and strong duality. Saddle point theorems of augmented Lagrange functions are also established. Applications to semi-infinite programs and convex infinite programs are discussed. Example is given.

2000 Mathematics Subject Classification: 49N15, 49K40, 90C26.

Keywords and Phrases: Weak-subgradient, weak-stability, strong duality, saddle point theorems.

(D-03)

Optimality and Mixed Duality for Nondifferentiable G-Invex Multiobjective Programs

Ho Jung Kim
(Pukyong National University, **Korea**)
Do Sang Kim
(Pukyong National University, **Korea**)
(e-mail: dskim@pknu.ac.kr)

Abstract

We consider a class of nondifferentiable multiobjective programs with inequality and equality constraints in which each component of the objective function contains a term involving the support function of a compact convex set. G-Karush-Kuhn-Tucker optimality conditions of our nondifferentiable multiobjective programs are presented by using suitable G-invex functions. We formulate G-mixed dual problems and establish weak, strong and converse duality theorems for a weak Pareto optimal solution under G-invexity assumptions. As special cases of our mixed duality results, we give G-Mond-Weir type and G-Wolfe type duality relations.

2000 Mathematics Subject Classification: 90C29, 90C46, 90C26.

Keywords and Phrases: Nondifferentiable Multiobjective programming, *G*-Karush-Kuhn-Tucker optimality conditions, duality theorems, *G*-invex functions.

(D-04)

Efficiency and Generalized Convex Duality for Nondifferentiable Multiobjective Programs

Kwan Deok Bae
(Pukyong National University, **Korea**)
Yu Jung Lee
(Pukyong National University, **Korea**)
Do Sang Kim
(Pukyong National University, **Korea**)
(e-mail: dskim@pknu.ac.kr)

Abstract

In this talk, we introduce nondifferentiable multiobjective programming problems involving the support function of a compact convex set and linear functions. Characterizations of (properly) efficient solutions due to Geoffrion, Chankong and Haimes are presented. We formulate Mond-Weir type and Wolfe type dual problems and establish weak and strong duality theorems for efficient solutions by using suitable generalized convexity conditions. Some special cases of our duality results are given.

2000 Mathematics Subject Classification: 90C30, 90C29, 90C46.

(D-05)

On The Transferred Superstability of The Sine Functional Equation

Gwang Hui Kim
(Kangnam University, **Korea**)
(e-mail: ghkim@kangnam.ac.kr)

Abstract

In this paper, we will investigate the transferred superstability for the following functional equations :

$$f(x+y)+g(x-y)=2f(x)g(y),$$
$$f(x+y)+g(x-y)=2g(x)f(y),$$
$$f(x+y)-g(x-y)=2f(x)g(y),$$
$$f(x+y)-g(x-y)=2g(x)f(y),$$

which can be considered the mixed sine functional equation and cosine functional equation, the mixed hyperbolic sine functional equation and hyperbolic cosine functional equation, and also the exponential functional equation.

2000 Mathematics Subject Classification: 39B82, 39B52.

Keywords and Phrases: Stability, superstability, functional equation, d'Alembert equation, sine functional equation.

(D-06)

One Generalization of the Hyers-Ulam Stability

Kyoo-Hong Park
(Seowon University, **Korea**)
(e-mail: parkkh@seowon.ac.kr)

Abstract

In this paper we study on the generalization of the Hyers-Ulam stability for the quadratic derivation. To see this we let $\mathcal{X}$ be a unital Banach algebra. If $f : \mathcal{X} \to \mathcal{X}$

is an approximately quadratic derivation in the sense of Hyers-Ulam-J.M. Rassias, then we show that $f : \mathcal{X} \to \mathcal{X}$ is an exactly quadratic derivation.

2000 Mathematics Subject Classification: 39B52, 46H99, 39B72, 39B82.

Keywords and Phrases: Quadratic derivation, approximate quadratic derivation, stability.

(D-07)

Generalized Hyers–Ulam Stability of Additive Functional Equations

Hark-Mahn Kim
(Chungnam National Uinversity, **Korea**)
(e-mail: hmkim@cnu.ac.kr)
EunYonug Son
(Chungnam National Uinversity, **Korea**)

Abstract

In this talk, we obtain the general solution and the generalized Hyers–Ulam stability theorem for an additive functional equation

$$af(x+y) + 2f\Big(\frac{x}{2} + y\Big) + 2f\Big(x + \frac{y}{2}\Big) = (a+3)[f(x) + f(y)]$$

for any fixed integer a. Given $f : X \to Y$, we set

$$Df_a(x,y) := af(x+y) + 2f\Big(\frac{x}{2} + y\Big) + 2f\Big(x + \frac{y}{2}\Big) - (a+3)[f(x) + f(y)]$$

for all $x, y \in X$ and for any fixed integer a. Assume that a function $f : X \to Y$ satisfies the functional inequality

$$\|Df_a(x,y)\| \leq \varphi(x,y)$$

for all $x, y \in X$. If φ satisfies the condition $\Phi(x,y) = \sum_{k=0}^{\infty} \frac{1}{2^k}\varphi(2^k x,\, 2^k y) < \infty$, then there exists a unique additive mapping $T : X \to Y$ such that T satisfies the inequality

$$\|f(x) - (a+3)f(0) - T(x)\| \leq \frac{1}{2}\Phi(2x, 0)$$

for all $x \in X$, where $\|(a+2)f(0)\| \leq \varphi(0,0)$. The mapping T is defined by

$$T(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

for all $x \in X$.

(D-08)

Stable Approximation of Approximate Quadratic Mappings by Quadratic Mappings

Kil-Woung Jun
(Chungnam National Uinversity, **Korea**)
(e-mail: kwjun@cnu.ac.kr)
Juri Lee
(Daejin University, **Korea**)
(e-mail: jrlee@daejin.ac.kr)

Abstract

In this talk, we investigate the general solution of the quadratic functional equation

$$f(2x+y) + 3f(2x-y) = 4f(x-y) + 12f(x),$$

and then we are to prove the generalized Hyers–Ulam stability of the equation by using direct method and fixed point method. As corollaries we have very similar results for special control functions by using direct method and fixed point method.

(D-09)

Stability for Quadratic Functional Equations via Fixed Point and Direct Method

Hark-Mahn Kim
(Chungnam National Uinversity, **Korea**)
(e-mail: hmkim@cnu.ac.kr)

Abstract

In 1940 S.M. Ulam proposed the famous Ulam stability problem. Cădariu and Radu applied the fixed point theorem of alternative to the investigation of Cauchy and Jensen functional equations. In this talk, we prove the generalized Hyers–Ulam stability of quadratic functional equations via the fixed point method and investigate new theorems via direct method concerning the stability of quadratic functional equations. As a result, we have the following theorem:

Let $f : X \to Y$ be a function with $f(0) = 0$ for which there exists a function $\varphi : X^n \to [0, \infty)$ such that there exists a constant $L, 0 < L < 1$, satisfying the inequalities

$$\|Df(x_1, \cdots, x_n)\|_\beta \leq \varphi(x_1, \cdots, x_n),$$
$$\varphi(nx_1, \cdots, nx_n) \leq n^{2\beta} L\varphi(x_1, \cdots, x_n)$$

for all $x_1, \cdots, x_n \in X$. Then there exists a unique quadratic function $Q : X \to Y$ defined by $\lim_{k\to\infty} \frac{f(n^k x)}{n^{2k}} = Q(x)$ such that

$$\|f(x) - Q(x)\|_\beta \leq \frac{1}{2^\beta n^{2\beta}(1 - L)} \varphi(x, \cdots, x)$$

for all $x \in X$.

(D-10)

A Fixed Point Approach to the Fuzzy Stability of a Quadratic-quartic Functional Equation

Choonkil Park
(Hanyang University, **Korea**)
(e-mail: baak@hanyang.ac.kr)

Abstract

Using the fixed point method, we prove the generalized Hyers-Ulam stability of the following quadratic-quartic functional equation

$$f(x + 2y) + f(x - 2y) = 4f(x + y) + 4f(x - y) - 6f(x) + 2f(2y) - 8f(y)$$

in fuzzy Banach spaces.

2000 Mathematics Subject Classification: Primary 46S40, 39B 72, Secondary 39B52, 46S50, 26E50, 47H10.

Keywords and Phrases: Fuzzy Banach space, fixed point, generalized Hyers-Ulam stability, quadratic-quartic functional equation.

(D-11)

Superstability And Stability of The Pexiderized Multiplicative Functional Equation

Young Whan Lee
(Daejeon University, **Korea**)
(e-mail: ywlee@dju.ac.kr)

Abstract

We obtain the superstability of the Pexiderized multiplicative functional equation

$$f(xy) = g(x)h(y)$$

and investigate the stability of this equation in the following form:

$$\frac{1}{1+\psi(x,y)} \leq \frac{f(xy)}{g(x)h(y)} \leq \psi(x,y).$$

2000 Mathematics Subject Classification: 39B72, 39B22.

Keywords and Phrases: Multiplicative functional equation, stability of functional equations, superstability of functional equations, Cauchy functional equation.

(D-12)

Hyers-Ulam Stability of the Functional Equation $f(3x) = 4f(3x-3) + f(3x-6)$

Soon-Mo Jung
(Hong-Ik University, **Korea**)
(e-mail: smjung@hongik.ac.kr)

Abstract

In this paper, a functional equation $f(3x) = 4f(3x-3) + f(3x-6)$, which is regarded as a generalized Fibonacci functional equation, will be solved and its Hyers-Ulam stability will be also investigated in the class of functions $f : \mathbb{R} \to X$, where X is a real Banach space.

(D-13)

Stability Results for Quadratic Program with Conic Constraint

Gue Myung Lee
(Pukyong National University, **Korea**)
(e-mail: gmlee@pknu.ac.kr)
Nguyen Nang Tam
(Hanoi Pedagogical Institute No. 2, **Vietnam**)
Nguyen Dong Yen
(Institute of Mathematics, Vietnamese Academy of Science and Technology, **Vietnam**)

Abstract

Stability of a general indefinite quadratic program whose constraint set is the intersection of an affine subspace and a closed convex cone is investigated. We present a systematical study of several stability properties of the Karush-Kuhn-Tucker point map, the global solution map, and the optimal value function, assuming that the problem data undergoes small perturbations. Some techniques from our preceeding works on stability of indefinite quadratic programs under linear constraints have found further applications and extensions in this talk.

(D-14)

On Følner conditions

Jung Im Kang
(National Institute for Mathematical Sciences, **Korea**)
(e-mail: jikang@nims.re.kr)

Abstract

Følner gave a necessary and sufficient conditions for a group to be amenable. And then, Namioka extended this research, particularly to semigroups, and investigated numerous Følner type conditions.

In this talk, we will give an overview of various Følner type conditions and related definitions. In particular, we will study Følner nets(or sequences) which play an important role in the approximation of fixed points for semigroups of nonexpansive mappings, where they are referred to as strongly left-regular nets. What a Følner sequence means that, roughly, given a (semi-)group G that acts on a set X, a Følner sequence for the action is a sequence of finite subsets of X which exhaust X and which " do not move too much " when acted on by any group element. And, we will discuss fixed point properties of various amenable semi-groups in terms of Følner sequences.

2000 Mathematics Subject Classification: 43A07, 46T99, 47H10.

Keywords and Phrases: Amenable, Følner condition, Følner net, fixed point, semigroup of mappings.

(D-15)

Weak Sharp Minima for Set-valued Vector Variational Inequalities with an Application

Jun Li
(China West Normal University, **P. R. China**)
(e-mail: junli1026@163.com)
Nan-Jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)
Xiao-Qi Yang
(The Hong Kong Polytechnic University, **Hong Kong**)

Abstract

In this paper, the notion of weak sharp minima is employed to the investigation of set-valued vector variational inequalities. The gap function φ_T for set-valued strong vector variational inequalities (for short, SVVI) is proved to be less than the gap function ϕ_T for set-valued weak vector variational inequalities (for short,

WVVI) under certain conditions, which implies that the solution set of SVVI is equivalent to the solution set of WVVI. Moreover, it is shown that weak sharp minima for the solution sets of SVVI and WVVI hold for $\sqrt{\min_{1\leq i\leq n} p_{T_i}}$ and for gap functions $\sqrt{\varphi_T}$ and $\sqrt{\phi_T}$ under the assumption of strong pseudomonotonicity, where p_{T_i} is a gap function for i-th component of SVVI and WVVI. As an application, the weak Pareto solution set of vector optimization problems (for short, VOP) is proved to be weak sharp minimum for $\sqrt{\min_{1\leq i\leq n} p_{\nabla g_i}}$ when each component g_i of objective function is strongly convex.

(D-16)

Values of the Polygamma Functions at Rational Arguments

Junesang Choi
(Dongguk University, **Korea**)
(e-mail: junesang@mail.dongguk.ac.kr)

Abstract

Gauss in 1812, in his celebrated memoir on the hypergeometric series, presented a remarkable formula for the Psi (or Digamma) function, $\psi(z)$, at rational arguments z, which can be expressed in terms of elementary functions. Davis in 1935 extended Gauss's result to the Polygamma functions $\psi^{(n)}(z)$ $(n \in \mathbb{N})$ by using a known series representation of $\psi^{(n)}(z)$ in an elementary yet technical way. Kölbig in 1996, in his CERN technical report, also gave two extensions to $\psi^{(n)}(z)$ by using the series definition of Polylogarithm function and the above-known series representation. Here we show how to derive general formulae expressing $\psi^{(n)}(z)$ $(n \in \mathbb{N}_0)$ at rational arguments in terms of other functions, in two ways. In addition, several special cases are also considered and, as a by-product of our main results, we derive, in a simple and unified manner, all formulae given by Gauss, Davis and Kölbig. Finally, it should be noted that all our results, in view of the relationship between $\psi^{(n)}(z)$ and the Hurwitz zeta function, $\zeta(s,a)$, could be rewritten in the representation of $\zeta(s,a)$.

2000 Mathematics Subject Classification: Primary 11M06, 11M 35, 33B15, Secondary 11B68, 11B73, 11M36, 33B30.

Keywords and Phrases: Polygamma functions, Psi (or Digamma) function, Hurwitz (or generalized) Zeta function, Bernoulli numbers and polynomials, Poly-

logarithms, generalized and associated Clausen functions, Riemann Zeta function, Gamma function.

(D-17)

Inequalities in Terms of the Gâteaux Derivatives for Convex Functions in Linear Spaces with Applications

S.S. Dragomir
(Victoria University, **Australia**)
(e-mail: sever.dragomir@vu.edu.au)
(URL: http://www.staff.vu.edu.au/rgmia/dragomir)

Abstract

For a convex function $f : X \to \mathbb{R}$ defined on a linear space X, perhaps one of the most important result is the well known Jensen's inequality

$$f\left(\sum_{i=1}^{n} p_i x_i\right) \leq \sum_{i=1}^{n} p_i f\left(x_i\right), \tag{1.1}$$

which holds for any n-tuple of vectors $\mathbf{x} = (x_1, \cdots, x_n) \in X^n$ and any probability distribution $\mathbf{p} = (p_1, \cdots, p_n) \in \mathbb{P}^n$.

The following refinement of Jensen's inequality in terms of the Gate -aux lateral derivatives $\nabla_{+(-)}$ holds:

Theorem 1. *Let $f : X \to \mathbb{R}$ be a convex function defined on a linear space X. Then for any n-tuple of vectors $\mathbf{x} = (x_1, \cdots, x_n) \in X^n$ and any probability distribution $\mathbf{p} = (p_1, \cdots, p_n) \in \mathbb{P}^n$ we have the inequality*

$$\begin{aligned} &\sum_{i=1}^{n} p_i f\left(x_i\right) - f\left(\sum_{i=1}^{n} p_i x_i\right) \\ &\geq \sum_{k=1}^{n} p_k \nabla_+ f\left(\sum_{i=1}^{n} p_i x_i\right)\left(x_k\right) - \nabla_+ f\left(\sum_{i=1}^{n} p_i x_i\right)\left(\sum_{i=1}^{n} p_i x_i\right) \\ &\geq 0. \end{aligned} \tag{1.2}$$

The following particular case that provides a refinement for the generalised triangle inequality in normed linear spaces is of interest

Corollary 1. *Let $(X, \|\cdot\|)$ be a normed linear space. Then for any $p \geq 1$, for any n-tuple of vectors $\mathbf{x} = (x_1, \cdots, x_n) \in X^n$ and any probability distribution $\mathbf{p} = (p_1, \cdots, p_n) \in \mathbb{P}^n$ with $\sum_{i=1}^n p_i x_i \neq 0$ we have the inequality*

$$\begin{aligned} &\sum_{i=1}^n p_i \|x_i\|^p - \left\| \sum_{i=1}^n p_i x_i \right\|^p \\ &\geq p \left\| \sum_{i=1}^n p_i x_i \right\|^{p-2} \left[\sum_{k=1}^n p_k \left\langle x_k, \sum_{j=1}^n p_j x_j \right\rangle_s - \left\| \sum_{i=1}^n p_i x_i \right\|^2 \right] \\ &\geq 0. \end{aligned} \tag{1.3}$$

If $p \geq 2$ the inequality holds for any n-tuple of vectors and probability distribution.

The following result is of interest as well:

Theorem 2. *Let $f : X \to \mathbb{R}$ be a convex function defined on a linear space X. Then for any n-tuple of vectors $\mathbf{x} = (x_1, \cdots, x_n) \in X^n$ and any probability distribution $\mathbf{p} = (p_1, \cdots, p_n) \in \mathbb{P}^n$ we have the inequality*

$$\begin{aligned} &\sum_{k=1}^n p_k \nabla_- f(x_k)(x_k) - \sum_{k=1}^n p_k \nabla_- f(x_k) \left(\sum_{i=1}^n p_i x_i \right) \\ &\geq \sum_{i=1}^n p_i f(x_i) - f\left(\sum_{i=1}^n p_i x_i \right). \end{aligned} \tag{1.4}$$

The following reverse of the generalised triangle inequality holds:

Corollary 2. *Let $(X, \|\cdot\|)$ be a normed linear space. Then for any $p \geq 1$, for any n-tuple of vectors $\mathbf{x} = (x_1, \cdots, x_n) \in X^n \setminus \{(0, \cdots, 0)\}$ and any probability distribution $\mathbf{p} = (p_1, \cdots, p_n) \in \mathbb{P}^n$ we have the inequality*

$$\begin{aligned} &p \left[\sum_{k=1}^n p_k \|x_k\|^p - \sum_{k=1}^n p_k \|x_k\|^{p-2} \left\langle \sum_{i=1}^n p_i x_i, x_k \right\rangle_i \right] \\ &\geq \sum_{i=1}^n p_i \|x_i\|^p - \left\| \sum_{i=1}^n p_i x_i \right\|^p. \end{aligned} \tag{1.5}$$

Bounds for mean f-deviations and for f-divergence measures with applications in Information Theory are also provided. For other results related to the Jensen inequality see the references below.

2000 Mathematics Subject Classification: 26D15; 94.

Keywords and Phrases: Convex functions, Gâteaux derivatives, Jensen's inequality, norms, mean f-deviations, f-Divergence measures.

(D-18)

Some Numerical Methods for Solving a Matrix Polynomial

Young Jin Kim
(Pusan National University, **Korea**)
Hyun-Min Kim
(Pusan National University, **Korea**)
(e-mail: hyunmin@pusan.ac.kr)

Abstract

We can consider some numerical methods for solving a matrix polynomial which has the form

$$P(X) = A_0X^m + A_1X^{m-1} + \cdots + A_{m-1}X + A_m = 0 \tag{1}$$

where A_i are complex square matrices for $0 \leq i \leq m$. For solving a matrix polynomial (1), Newton's method was studied by Kratz and Stickel [3]. Seo and Kim [4] improved Newton's method with Schur Algorithm and induced the exact line search. The conjugate gradient method [2] and Bernoulli's iteration [1] were also considered. Finally, we show some numerical experiments.

Keywords and Phrases: Matrix polynomial, Newton's method, conjugate Gradient method, Bernoulli's iteration.

(D-19)

On a Family of Means Generated by The Hardy-Littlewood Maximal Inequality

Josip Pecaric
(Aleksandra Cizmesija, Iva Pranjic and Pokaz Dora Pokaz)
(e-mail: pecaric@element.hr)

Abstract

In this paper, a functional defined as the difference between the right-hand and the left-hand side of the Hardy-Littlewood maximal inequality is studied and its properties, such as exponential and logarithmic convexity, are explored. Furthermore, related analogues of the Lagrange and Cauchy mean value theorems are derived. Finally, using this functional, a new family of the Cauchy-type means is generated and their monotonicity property is proven.

(D-20)

Nonlinear Variational Inclusion Problems Based on the Yosida Approximation

Ram U. Verma
(Florida Institute of Technology, **USA**)
(e-mail: verma99@msn.com)

Abstract

In this communication, the approximation solvability of a general class of nonlinear variational inclusion problems using the relaxed proximal point algorithm based on the Yosida approximation is explored, while the weak convergence is achieved.

(D-21)

Optimality Conditions, Duality and Saddle Points for Nondifferentiable Multiobjective Fractional Programs

Xian-jun Long
(Sichuan University, **P. R. China**)
Nan-jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)

Abstract

In this paper, a new class of nondifferentiable multiobjective fractional programs

is introduced and studied, in which every component of the objective function contains a term involving the support function of a compact convex set. Kuhn-Tucker necessary and sufficient optimality conditions, duality and saddle point results for weakly efficient solutions of the nondifferentiable multiobjective fractional programming problems are given. The results presented in this paper improve and extend some the corresponding results in the literature.

2000 Mathematics Subject Classification: 90C26, 90C29, 90C46.

Keywords and Phrases: Nondifferentiable multiobjective fractional programming, Kuhn-Tucker optimality condition, duality, saddle poin -t, weakly efficient solution, (F, α, ρ, d)-convex function.

(D-22)

Existence of Solutions for Variational Inequalities on Riemannian Manifolds

Shu-Long Li
(Southern Medical University, **P. R. China**)
(e-mail: shulong2004@163.com)
Chong Li
(Zhejiang University, **P. R. China**)
(e-mail: cli@zju.edu.cn)
Yeong-Cheng Liou
(Cheng Shiu University, **Taiwan**)
(e-mail: simplex_liou@hotmail.com)
Jen-Chih Yao
(National Sun Yat-sen University, **Taiwan**)
(e-mail: yaojc@math.nsysu.edu.cn)

Abstract

We establish the existence and uniqueness results for variational inequality problems on Riemannian manifolds and solve completely the open problem proposed Németh. Also the relationships between the constrained optimization problem and the variational inequality problems as well as the projections on Riemannian manifolds are studied.

2000 Mathematics Subject Classification: Primary 34A55, Secondary 34B24.

Keywords and Phrases: Variational inequalities, Riemannian manifold, Monotone vector fields.

(D-23)

An Iterative Method for a System of Linear Complementarity Problems with Perturbations and Interval Data

Hui-qiang Ma
(Sichuan University, **P. R. China**)
Jiu-ping Xu
(Sichuan University, **P. R. China**)
Nan-jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)

Abstract

In this paper, we introduce a total step method for solving a system of linear complementarity problems with perturbations and interval data. It is applied to two interval matrices $[A]$ and $[B]$ and two interval vectors $[b]$ and $[c]$. We prove that the sequence generated by the total step method converges to $([x^*], [y^*])$ which includes the solution set for the system of linear complementarity problems defined by any fixed $A \in [A]$, $B \in [B]$, $b \in [b]$ and $c \in [c]$. We also consider a modification of the method and show that, if we start with two interval vectors containing the limits, then the iterates contain the limits. We close our paper with two examples which illustrate our theoretical results.

2000 Mathematics Subject Classification: 90C33, 49J40.

Keywords and Phrases: A system of Linear complementarity problems, perturbation, iterative method, total step method, interval computation.

(D-24)

On the Convergence of An Inexact Newton-like Method Containing Outer Inverses

Ioannis K. Argyros
(Cameron university, **USA**)
(e-mail: iargyros@cameron.edu)

Abstract

We introduce the new idea of recurrent functions to provide a semilocal convergence analysis for an inexact Newton–like method, using outer inverses. It turns out that our sufficient convergence conditions are weaker than in earlier studies in many interesting cases. Applications and numerical examples, involving a nonlinear integral equation of Chandrasekhar-type, and a differential equation are also provided.

Keywords and Phrases: Inexact Newton-like method, Newton's method, recurrent functions, Banach space, semilocal convergence.

(D-25)

Nonlinear Separation and Extension Properties for Positive Definite Functions on Groups

Anthony To-Ming Lau
(University of Alberta, **Canada**)
(e-mail: tlau@math.ualberta.ca)

Abstract

A well known theorem of Hahn Banach asserts that if E is a closed subspace of a Banach space and F is a closed subspace of E, then each continuous linear functional on F can be extended to a continuous linear functional on E. As a consequence, any element in E but not in F can be separated by continuous linear functional on E. In my talk, I shall discuss similar properties for closed subgroups of a locally compact group and their relations with the invariant complementation problem for the group von Neumann algebra of a locally compact group.

(D-26)

Stability of a 2-dimensional Functional Equation in a Class of Vector Variable Functions

Won-Gil Park
(NIMS, **Korea**)
(e-mail: wgpark@nims.re.kr)

Jae-Hyeong Bae
(Kyung Hee University, **Korea**)
(e-mail: jhbae@khu.ac.kr)

Abstract

We prove the Hyers-Ulam stability of a 2-dimensional quadratic functional equation in a class of vector variable functions in Banach modules over a unital $C^\star$-algebra.

(D-27)

Optimal Control Applied to Mathematical Model of Infectious Diseases

Gul Zaman
(National University of Sciences and technology, **Pakistan**)
(e-mail: gzaman@camp.edu.pk)

Abstract

In this work we introduce a optimal control theory to control the infectious diseases in a community. We rst established the existence of an optimal control pair for the optimal control problem. Then, we use control variable and present how to derive the optimality system from the optimal control problem.The validation results of these control strategies are presented and discussed.

V. SESSION E

(E-01)

Convergence of Modified Mann's Iteration Methods for Discrete Asymptotically Strict Pseudo-contractive Semigroups

Tae-Hwa Kim
(Pukyong National University, **Korea**)
(e-mail: taehwa@pknu.ac.kr)

Abstract

In this paper, we first propose some modifications of Mann's iteration method and hybrid method for a discrete asymptotically strict pseudo-contractive semigroup $\mathcal{S} = \{T_n : n \geq 0\}$ defined on a nonempty closed convex subset in a Hilbert space, and next prove weak convergence of modified Mann's iteration method, and strong convergence of modified hybrid method, respectively, for such a semigroup $\mathcal{S}$. Also, corresponding convergence analogues of mean type algorithms for such a semigroup $\mathcal{S} = \{T_n : n \geq 0\}$ are also investigated, and some applications for a single asymptotically strict pseudo-contraction are added, which improve the corresponding ones due to Kim and Xu [*Nonlinear Anal.* 68 (2008), 2828–2836].

2000 Mathematics Subject Classification: Primary 47H09, Secondary 65J15.

Keywords and Phrases: Discrete semigroup, asymptotically strict pseudo-contraction, modified Mann's iteration method, weak (strong) convergence, fixed point, projection.

(E-02)

Viscosity Approximation for Semigroup of Asymptotically Nonexpansive Mappings

Balwant Singh Thakur

(Pt. Ravishankar Shukla University, **India**)
(e-mail: balwant_rsu@yahoo.com, balwant.rsu@gmail.com)

Abstract

Viscosity method also called elliptic regularization, provide an efficient approach to a large number of problems coming from different branches of mathematical and physical sciences. One of the recent trend in the iterative construction of fixed point of nonlinear mappings is to use viscosity approximation method. In this paper we study both implicit and explicit viscosity method to approximate common fixed point of semigroup of asymptotically nonexpansive mappings which converges strongly to a solution of a variational inequality.

(E-03)

A Note on CQ Method for Nonexpansive Nonself Mappings in Hilbert Spaces

Yeol Je Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)
Nguyen Buong
(Vietnamse Academy of Science and Technology, Institute of Information Technology, **Vietnam**)
(e-mail: nbuong@ioit.ac.vn)
Jong Kyu Kim
(Kyungnam University, **Korea**)
(e-mail: jongkyuk@kyungnam.ac.kr)

Abstract

The purpose of this note is to present another approach to construct the CQ method for finding a fixed point of a family of nonexpansive nonself mappings in Hilbert spaces. Applications to solve the problem of common solution of a finite family of equilibrium problems and common fixed point of a finite family of strictly pseudocontractive nonself mappings are also showed.

2000 Mathematics Subject Classification: 41A65, 47H17, 47H20.

Keywords and Phrases: Regularization, common fixed points, nonexpansive

mappings.

(E-04)

Hybrid Iterative Scheme with Parallel Method for Generalized Equilibrium Problems and Fixed Point Problems of a Family of Strictly Pseudo-Contraction Mappings

Narin Petrot
(Naresuan University, **Thailand**)
(e-mail: narinp@nu.ac.th)
Issara Inchan
(Uttaradit Rajabhat University, **Thailand**)
(e-mail: peissara@uru.ac.th)
Rabian Wangkeeree
(Naresuan University, **Thailand**)
(e-mail: rabianw@nu.ac.th)
Poom Kumam
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: poom.kum@kmutt.ac.th)

Abstract

The purpose of this work is to introduce a new hybrid iterative scheme with parallel method for finding a common element of the set of a generalized equilibrium problem, the set of solutions to a variational inequality and the set of fixed points of a family of finitely strict pseudo-contraction mappings in a real Hilbert space. Then, we prove convergence theorems by the new iterative method introduced by Takahashi et al. [W. Takahashi, Y. Takeuchi, R. Kubota, Strong Convergence Theorems by Hybrid Methods for Families of Nonexpansive Mappings in Hilbert Spaces, J. Math. Anal. Appl. 341 (1) (2008), 276286.] of the proposed iterative algorithm to a fixed point of a family of finitely strict pseudo-contraction mappings which is a solution of the equilibrium problem and the variational inequality. The results obtained in this paper extend and improve some well-known results in the literature.

2000 Mathematics Subject Classification: 47H09, 47H10.

Keywords and Phrases: Strict pseudo-contraction mappings, equilibrium prob-

lems, minimization problem, fixed points.

(E-05)

An Implicit Iterative Algorithm for Common Fixed Points of Two Nonexpansive Mappings

Safeer Hussain Khan
(Qatar University, **Qatar**)
(e-mail: safeer@qu.edu.qa)

Abstract

In this talk, we introduce an implicit iterative algorithm for approximating common fixed points of two nonexpansive mappings in Banach spaces. We prove some weak and strong convergence theorems for this iterative algorithm. Our results generalize corresponding results in contemporary literature.

2000 Mathematics Subject Classification: 47H09.

Keywords and Phrases: Implicit iterative algorithm, nonexpansive mapping condition (A').

(E-06)

Coincidence Point Theorems and Stability of Iterative Procedures for Mappings in Cone metric Spaces without Continuity

Hemant K. Pathak
(Pt. Ravishankar Shukla University, **India**)
(e-mail: hkpathak05@gmail.com)
Y. J. Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)
S. M. Kang

(Gyeongsang National University, **Korea**)
(e-mail: smkang@gnu.ac.kr)

Abstract

In this note, the existence of coincidence points and common fixed points for contractive mappings in cone metric spaces are obtained. Indeed, we prove: Let (X, ρ) be a normal cone metric space with normality constant K and let $f, g : X \to X$ be mappings such that $f(X) \subset g(X)$ and $g(X)$ is a complete subspace of X. If there exists $\lambda \in [0, 1)$ such that $K\lambda < 1$ and $\rho(fx, fy) \preceq \lambda u$ where $u \in \{\rho(gx, gy), \rho(gx, fx), \rho(gy, fy), [\rho(gx, fy) + \rho(gy, fx)]/2\}$ for all $x, y \in X$, then f and g have a unique point of coincidence in X. Further, if f and g are $\mathcal{P}$-operator pair, then f and g have a unique common fixed point. In the sequel, the stability of iterative procedures for mappings having coincidence point in cone metric spaces is introduced and discussed. Let (X, d) be a cone metric space, P a normal cone with normal constant K and let $f, g : X \to X$ be mappings such that $f(X) \subset g(X)$, $g(X)$ is complete subset of X and such that $\rho(fx, fy) \preceq u$, where $u \in \{k_1\rho(gx, gy), k_2[\rho(gx, fx) + \rho(gy, fy)], k_3[\rho(gx, fy) + \rho(gy, fx)]\}$ for all $x, y \in X$ and for some $k_1 \in [0, 1)$, k_2, $k_3 \in [0, \frac{1}{2})$. Let z be a coincidence point of f and g, that is, there exists $w \in X$ such that $fz = gz = w$. Let $x_0 \in X$ and let the sequence $\{gx_n\}$, generated by $gx_{n+1} = fx_n$, $n = 0, 1, \cdots$, converge to w. Let $\{gy_n\} \subset X$ and defined $\epsilon_n = \rho(fyn, gy_{n+1})$, $n = 0, 1, \cdots$. Then

(1^0) $\rho(w, gy_{n+1}) \leq \rho(w, gx_{n+1}) + \sigma_{i=0}^{n} 2k^{n+1-i}\rho(gx_i, gx_{i+1}) + k^{n+1}\,\rho(gx_0, gy_0) + \sigma_{i=0}^{n} k^{n-i}\rho(gx_0, gy_0) + \sigma_{i=0}^{n} k^{n-i}\epsilon_i$, where $k = \max\{k_1, \frac{k_2}{1-k_2}, \frac{k_3}{1-k_3}\} < 1$.

(2^0) $\lim_{n\to\infty} gy_n = w$, if and only if $\lim_{n\to\infty} \epsilon_n = 0$.

2000 Mathematics Subject Classification: Primary 54H25, 65D 15, Secondary 65J15, 41A25.

Keywords and Phrases: Coincidence point, fixed point, stable iterative process, Jungck-type iterative procedure, ordered Banach space, cone metric space.

(E-07)

Iterative Approximation to Common Fixed Points of Two Asymptotically Quasi-nonexpansive Mappings in the Intermediate Sense

Yonghong Yao

(Tianjin Polytechnic University, **P. R. China**)
(e-mail: yaoyonghong@yahoo.com)
Yeong-Cheng Liou
(Cheng Shiu University, **Taiwan**)

Abstract

In this paper, we consider an iterative method for approximating the common fixed points of two asymptotically quasi-nonexpansive mappings in the intermediate sense. The present results improve, extend some recent corresponding results of Lan [Common fixed point iterative processes with errors for generalized asymptotically quasi-nonexpansive mappings, Comput. Math. Appl., 52(2006), 1403-1412] and many others.

2000 Mathematics Subject Classification: 47H05, 47H10, 47H17.

Keywords and Phrases: Asymptotically quasi-nonexpansive mappings in the intermediate sense, general modified Ishikawa iterative sequence with errors, common fixed point, strong convergence.

(E-08)

Some Results on Asymptotically Hemi-pseudocontractive Mappings in Hilbert Spaces

Xiaolong Qin
(Gyeongsang National University, **Korea**)
(e-mail: qxlxajh@163.com)
Sun Young Cho
(Gyeongsang National University, **Korea**)
(e-mail: ooly61@yahoo.co.kr)
Shin Min Kang
(Gyeongsang National University, **Korea**)
(e-mail: smkang@gnu.ac.kr)

Abstract

The purpose of this paper is to modify Ishikawa's iterative process to have strong convergence without any compact assumptions for asymptotically hemi-pseudocontractive mappings in the framework of real Hilbert spaces.

2000 Mathematics Subject Classification: 47H09, 47J25.

Keywords and Phrases: Hybrid projection algorithm, asymptotically pseudo-contractive mapping, strong convergence, fixed point.

(E-09)

Characterizations of Strict Convexity of G.S.I.P Spaces

Dr. V. Lokesha
(Acharya Institute of technology, **India**)
(e-mail: lokiv@yahoo.com)

Abstract

In this talk, I would like to present some results related to strict convexity on g.s.i.p space.

2000 Mathematics Subject Classification: 46C15.

Keywords and Phrases: G.S.I.P space, strict convexity.

(E-10)

Strong Convergence of an Implicit Iteration Process for a Finite Family of Uniformly L-Lipschitzian Mappings in Banach Spaces

Feng Gu
(Hangzhou Normal University, **P. R. China**)
(e-mail: gufeng99@sohu.com)

Abstract

The purpose of this paper is to prove a strong convergence theorem for a finite family of uniformly L-Lipschitzian mappings in Banach spaces. The results pre-

sented in the paper improve and extend some recent results in Chang [1], Cho et al. [3] Ofoedu [7], Schu [9] and Zeng [11, 12].

(E-11)

A Modified Ishikawa Iterative Algorithm for Fixed Points of Nonexpansive Semigroups*

Kriengsak Wattanawitoon
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: tayao@hotmail.com)
Anirut Luadsong
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: anirut.lua@kmutt.ac.th)
Poom Kumam
(King Mongkut's University of Technology Thonburi, **Thailand**)
(e-mail: poom.kum@kmutt.ac.th)

Abstract

In this talk, we prove a convergence theorem of modified Ishikawa iterative scheme for two nonexpansive semigroups in Hilbert spaces by the two hybrid methods. Our results improve and extend the corresponding results announced by Saejung [Strong convergence theorems for nonexpansive semigroups without Bochner integrals, Fixed Points Theory Appl. (2008), doi:10.1155/2008/745010.] and some others.

2000 Mathematics Subject Classification: 47H09, 47H10.

Keywords and Phrases: Nonexpansive semigroup, modified Ishika- wa iterative scheme, fixed points.

(E-12)

Iterative Regularization for a Common Solution of a Finite Family of Accretive Mappings

Jong Kyu Kim
(Kyungnam University, **Korea**)
(e-mail: jongkyuk@kyungnam.ac.kr)
Nguyen Buong
(Vietnamse Academy of Science and Technology, Institute of Information Technology, **Vietnam**)
(e-mail: nbuong@ioit.ac.vn)
Yeol Je Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)

Abstract

In this paper, a new iterative regularization method is proposed for finding a common solution of a finite family of the following variational inequality problems: find an element $p \in C$, a closed convex subset of a q-uniformly smooth and uniformly convex Banach space X with a normalized duality mapping J, such that $\langle A_i(p), J(y-p)\rangle \geq 0$ for all $y \in C$ and $i = 1, \cdots, N$, where each A_i is a λ_i-inverse strongly accretive mapping from C into X. An application for finding a common fixed point of a finite family of N nonself k_i-strictly pseudocontractive mappings is showed.

(E-13)

A Hybrid Algorithm for Generalized Equilibrium Problem and Strict Pseudocontraction Mappings

Shih-sen Chang
(Yibin University, **P. R. China**)
(Sichuan University, **P. R. China**)
(e-mail: changss@yahoo.cn)

Abstract

The purpose of this paper is to introduce an iterative scheme for finding a

common element of the set of solutions of a generalized equilibrium problem (GEP) and the set of common fixed points of a $k-$strict pseudo-contraction mapping in Hilbert space. Under suitable conditions, some strong convergence theorems for approximating to this common elements are proved by the viscosity approximation method. The results presented in the paper extend and improve some recent results.

(E-14)

A Weak Convergence Theorem Related to Common Fixed Points of Two Families of Nonexpansive Mappings in a Banach Space

Wataru Takahashi
(Tokyo Institute of Technology, **Japan**)
(e-mail: wataru@is.titech.ac.jp)
Yukio Takeuchi
(Takahashi Institute for Nonlinear Analysis, **Japan**)
(e-mail: aho31415@yahoo.co.jp)

Abstract

The authors [8] proved an extension of the Rich's theorem [5] for common fixed points of nonexpansive mappings; also see Kim and Takahashi [7]. This theorem is usefull whenever we consider weak convergence of Mann type iterative sequences with errors. On the other hand, under some conditions, Aoyama, Kimura, Takahashi and Toyoda [1] introduced a useful sequence $\{B_n\}$ and a mapping B generated by given sequence $\{A_n\}$ of nonexpansive mappings with $\cap_n F(A_n) \neq \leq$. This sequence $\{B_n\}$ and mapping B are deeply connected with Bruck's theorem [2]. This sequence and mapping are similar in roll to W-mapping introduced by Takahashi [6] for weak and strong convergence theorems of nonexpansive mappings. We call this sequence $\{B_n\}$ and mapping B an (ABT)-sequence and an (ABT)-mapping, respectively. In this talk, using this sequence and our theorem, we prove a weak convergence theorem related to common fixed points of two families of nonexpansive mappings.

2000 Mathematics Subject Classification: Primary 47H09, Secondary 47H10.

(E-15)

Monotone Generalized Contractions in Partially Ordered Probabilistic D–metric Spaces

Yeol Je Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)
R. Saadati
(Amirkabir University of Technology, **Iran**)
(e-mail: rsaadati@eml.cc)
S. M. Vaezpour
(Amirkabir University of Technology, **Iran**)

Abstract

In this paper, a concept of monotone generalized contraction in partially ordered probabilistic D–metric spaces is introduced and some fixed point theorems are proved. Presented theorems are an extension and improvement of the results in partially ordered metric spaces of Nieto and Rodriguez-Lopez [Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Order 22 (2005), 223–239; Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations, Acta Math. Sinica (in press)] and Ran and Reurings [A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc. Amer. Math. Soc. 132 (2004), 1435–1443] to more general class of contractive type mappings in partially ordered probabilistic metric spaces and include several recent developments.

2000 Mathematics Subject Classification: Primary 54H25, Secondary 47H10.

Keywords and Phrases: Non-decreasing mapping, coincidence, fix -ed point, complete metric space, probabilistic D-metric spaces.

(E-16)

Some Notes on $H(\cdot,\cdot)$-accretive Operators in Banach Spaces

Mohsen Alimohammady

(University of Mazandaran, **Iran**)
(e-mail: m.alimohammady@gmail.com)
Mehdi Roohi
(University of Mazandaran, **Iran**)
(e-mail: mehdi.roohi@gmail.com)

Abstract

In this paper, we consider the new concept of $H(\cdot,\cdot)$-monotone operator and also the generalized resolvent operator associated with this type of monotone operator is considered. Further, some notes about results of [1] and [2] are given.

2000 Mathematics Subject Classification: Primary 47H05, Secondary 47J20, 49J40.

Keywords and Phrases: Monotone operator, resolvent operator, variational inclusion.

(E-17)

Zero Theorems of Accretive Operators in Banach Spaces

Xiaolong Qin
(Gyeongsang National University, **Korea**)
(e-mail: qxlxajh@163.com)
Zeqing Liu
(Liaoning Normal University, **P. R. China**)
(e-mail: zeqingliu@dl.cn)
Shin Min Kang
(Gyeongsang National University, **Korea**)
(e-mail: smkang@gnu.ac.kr)

Abstract

In this paper, we consider the problem of approximating zeros of accretive operators in a real Banach space. We introduce and analysis the Mann-type iterative algorithm with errors and Halpern-type iterative algorithms with errors. Weak and strong convergence theorems are established in a real Banach space. As applications, The problem of finding a minimizer of a proper lower semicontinuous convex function in a real Hilbert space is considered.

2000 Mathematics Subject Classification: 47H05, 47H09, 47J25.

Keywords and Phrases: Nonexpansive mapping, fixed point, accretive operator, zero, range condition.

(E-18)

On the New Composite Implicit Iterative Process for a Finite Family of Asymptotically Nonexpansive Mappings

Feng Gu
(Hangzhou Normal University, **P. R. China**)
(e-mail: gufeng99@sohu.com)
Y. J. Cho
(Gyeongsang National University, **Korea**)
(e-mail: yjcho@gnu.ac.kr)

Abstract

In this paper, we first introduce a new implicit iterative processes with errors for a Finite Family of Asymptotically Nonexpansive Mappings and nonexpansive mappings in Banach spaces, and then we discuss the strong and weak convergence for the iterative processes. The results presented in this paper extend and improve the corresponding results of [1, 3-7, 9-13]. 1

(E-19)

Well-posedness of Hemivariational Inequalities and Inclusion Problems

Yi-bin Xiao
(University of Electronic Science and Technology of China, **P. R. China**)
(e-mail: xiaoyb9999@hotmail.com)
Nan-jing Huang
(Sichuan University, **P. R. China**)
(e-mail: nanjinghuang@hotmail.com)

Abstract

In the present paper, we generalize the concept of well-posedness to a hemivariational inequality, give some metric characterizations of the well-posed hemivariational inequality, and show that the equivalence between the well-posedness of the hemivariational inequality and the well-posedness of the corresponding inclusion problem. Under suitable conditions we also prove that the strong well-posedness of the hemivariational inequality is equivalent to the existence and uniqueness of its solution. Finally, we derive some conditions under which the hemivariational inequality is strongly well-posed in the generalized sense.

(E-20)

A Projected Subgradient Method for Solving Generalized Mixed Variational Inequalities

Fu Quan Xia
(Sichuan Normal University, **P. R. China**)
(e-mail: fuquanxia@sina.com)

Abstract

In this paper, we consider the projected subgradient method for solving generalized mixed variational inequalities. In each step, we choose an ε_k-subgradient u^k of the function f and w^k in a set-valued mapping T, followed by an orthogonal projection onto the feasible set. We prove that the sequence is weakly convergent.

(E-21)

Strong Convergence Theorems for Fixed Point Problems, Variational Inequality Problems and Equilibrium Problems

Yonghong Yao
(Tianjin Polytechnic University, **P. R. China**)
(e-mail: yaoyonghong@yahoo.com)
Yeong-Cheng Liou
(Cheng Shiu University, **Taiwan**)

Abstract

In this paper, we first introduce an iterative algorithm for finding a common element of the set of fixed points of a nonexpansive mapping, the set of solutions of an equilibrium problem, and the solution set of the variational inequality problem for a monotone mapping in a real Hilbert space. Further- more, we prove that the proposed iterative algorithm converges strongly to a common element of the above three sets under some mild conditions. Our result includes the result of Zeng and Yao [L.C. Zeng and J.C. Yao, Strong convergence theorem by an extragradient method for fixed point problems and variational inequality problems, Taiwanese J. Math., 10(2006): 1293-1303] as a special case.

2000 Mathematics Subject Classification: 49J30, 47H10, 47H17, 49M05, 90C25, 90C99.

Keywords and Phrases: Nonexpansive mapping, equilibrium problem, fixed point, variational inequality.

(E-22)

Upper Semicontinuity in a Parametric General Variational Problem and Application

Le Anh Tuan
(Ninh Thuan College of Pedagogy, **Vietnam**)
(e-mail: latuan02@yahoo.com)
Gue Myung Lee
(Pukyong National University, **Korea**)
(e-mail: gmlee@pknu.ac.kr)
Pham Huu Sach
(Hanoi Institute of Mathematics, **Vietnam**)
(e-mail: phsach@math.ac.vn)

Abstract

In this paper, we give sufficient conditions for the upper semicontinuity property of the solution mapping of a general model which includes as special cases many generalized vector quasi-equilibrium problems with set-valued maps. The main result generalizes and improves several recent results. An example is given to illustrate such generalization and improvement. The main result is also applied to

a model which can be interpreted as a system of generalized vector quasi-equilibrium problems with moving cones. The main tools of the paper are some new notions of cone-semicontinuity properties and openness closedness properties of families of set-valued maps.