Nonlinear Functional Analysis and Applications
Vol. 31, No. 1 (2026), pp. 193-210
ISSN: 1229-1595(print), 2466-0973(online)

https://doi.org/10.22771/nfaa.2026.31.01.12
http://nfaa.kyungnam.ac.kr/journal-nfaa

ON COVID-19 MATHEMATICAL MODEL: THEORETICAL AND COMPUTATIONAL ANALYSIS USING ARTIFICIAL NEURAL NETWORKS

Naseer Ul Haq[1], Shakil Ahmad[2], Eiman[3], Sadique Ahmad[4], Sajid Shah[5] and Mohammed A. Elaffendi[6]

[1]Department of Mathematics, University of Malakand, Chakdara Dir (Lower), Khyber Pakhtunkhawa, Pakistan
e-mail: naseerulhaqpk100@gmail.com

[2]Department of Mathematics, University of Malakand, Chakdara Dir (Lower), Khyber Pakhtunkhawa, Pakistan
e-mail: ahmadshkil099@gmail.com

[3]Department of Mathematics, University of Malakand, Chakdara Dir (Lower), Khyber Pakhtunkhawa, Pakistan
e-mail: ehuzaifa@gmail.com

[4]EIAS Data Science and BlockChain Laboratory, College of Computer and Information Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia
e-mail: saahmad@psu.edu.sa

[5]EIAS Data Science and BlockChain Laboratory, College of Computer and Information Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia
e-mail: Sshah@psu.edu.sa

[6]EIAS Data Science and BlockChain Laboratory, College of Computer and Information Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia
e-mail: Affendi@psu.edu.sa

Abstract. A COVID-19 mathematical model with fractional order derivative is investigated. The model is solved using the Fractional Differential Transform Method (FDTM) to obtain a convergent semi-analytical solution. Also, the considered mathematical model is then studied by a sophisticated artificial neural network (ANN) architecture. This multi-layered network, trained via the Levenberg-Marquardt algorithm is used to perform robust data-driven validation across three strategic operational cases. Some graphical illustrations are given.

[0]Received September 8, 2025. Revised October 20, 2025. Accepted October 21, 2025.
[0]2020 Mathematics Subject Classification: 26A33, 03C65, 68T0, 92B20.
[0]Keywords: Mathematical model, neural network, approximate solution, FDTM, Levenberg-Marquardt algorithm.
[0]Corresponding author: Naseer Ul Haq(naseerulhaqpk100@gmail.com).

1. Introduction

Mathematical models are powerful tools to investigate various real world problems in terms of equations. This area has received significant interest over the last few decades. Particularly, the area devoted to mathematical biology has attracted considerable attention from researchers, and Various infectious diseases have been investigated using these models.

In the last few years, infectious disease caused by a coronavirus has infected a major portion of the human population worldwide. This disease cause respiratory illness . It was first reported in Wuhan, a city in China, in December 2019 [12], before spreading to almost every country, Coronaviruses are often found in cats, camels and bats [14]. These viruses can transfer between different animals species, and during this transfer, they may change. Eventually, the virus can come into contact with the human population. In the case of SARS-CoV-19, the first infected individuals are believed to have encountered the virus at a food market that sold meat, fish, and live animals [14]. The COVID-19 virus, SARS-CoV-2, enters the human body through the mouth, eyes or nose. It enters the human body either directly from the airborne droplets or indirectly by touching a contaminated surface and then one,s face. When an infected person talks, sneezes, cough, or breathes nearby, respiratory droplets carrying the virus discharged into the air. If these droplets are inhaled, another person can become infected [27]. COVID-19 symptoms can take several days to appear, although a person can be contagious during that incubation period. individuals are generally no longer contagious after 10 days of symptoms. To avoid spreading the virus, maintain a distance of at least 6 feet from other when possible, Wearing a mask, [27].

It is better to stay at home and isolated if someone has the symptoms of COVID-19. Peoples who live in or have recently visited any area where COVID-19 is widespread are at the greatest risk of infection. The incubation period, the time between getting an infection and showing symptoms, ranges from two to 14 days [16, 20]. Some people with COVID-19 experience severe symptoms, while others show only mild signs. Some infected people show no signs or symptoms. Common signs include tiredness, fever or chills, shortness of breath, cough, muscle aches, loss of taste or smell, headaches, sore throat, congestion or runny nose, nausea or vomiting and diarrhea. Researchers and scientists from all around the world were drawn to this pandemic due to its intensity. Many governments prohibited international travel, and shut down schools, shopping malls, and businesses [17]. The COVID-19 pandemic also causes serious economic damage worldwide. A large number of doctors and researchers participated the anti-pandemic fight and conducted research in their fields of expertise. They investigate COVID-19 from different perspectives, such as infectious diseases, microbiology, public environ mental occupational health, virology, political economics and sociology, etc. Recently different perspectives of COVID-19 have been studied by researchers, we refer to [15, 21, 22].

Different Mathematical modeling is used to investigate the transmission of infectious diseases such as COVID-19 [26]. Fractional calculus has garnered significant attention in recent decades. The field of fractional order differential equations (FODEs) in particular, has gained prominence due to its numerous applications in solving real world problems. Consequently, over the last few decades, numerous researchers have studied FODEs to obtain various types of solutions, including analytical, approximate, and iterative solutions (for more information, see [2, 4]). To adress these equations equations from various perspectives, other theoretical frameworks have also been developed.

For analytical solutions researchers have explored various tools to find exact solutions. However it is very difficult to compute an exact or analytical solution for every FODE. Therefore, researchers have introduced various numerical or semi-analytical methods to solve such problems. Also, the integral transforms have been used to find approximate or exact solutions to FODEs. From 2000 to 2010, perturbation approaches were predominantly employed to solve FODEs (we refer to [28]). Decomposition techniques, which were primarily used to solve classical differential equations, have been expanded to handle FODEs in various ways. In order to address numerous issues in the aforementioned field, some authors have also used homotopy analysis transform approaches. Various engineering difficulties, such as circuit problems, have been solved using the DTM (details can be viewed in [28]). This method is mostly used to solve integral equations, partial differential equations, and classical equations. Additionally, several issues in the above field have been resolved using the aforementioned transform ([19]). Inspired by the aforementioned study, we applied the FDTM to the COVID-19 mathematical model for approximate solution.

The speedy evolution of Artificial Intelligence (AI) technologies has provided the healthcare industry with additional options, especially in the fields of early disease detection and treatment [24]. Recently, artificial neural network and deep neural networks (DNNs) analysis has attracted attention very well [25]. Using compartmental models for investigation of infectious disease we face the challenge of parameters estimation. Also, the artificial intelligence based models fail to describe the biological pattern of the dynamics. Recently, researchers have considered neural networks based on multi layers together with input output layers to overcome some limitation. For the COVID-19, a deep learning neural network was used to study the concerned epidemic

[18]. Recently, researchers are increasing use the said tools in investigations of various problems, we refer to [5, 23]. The deep learning neural network consists on hidden multiple layers together with input output layers. Analyzing and halting the spread of the COVID-19 pandemic also involved the use of artificial intelligence (AI), particularly deep neural network (DNN) models. A structure diagram of such artificial neural network is given in figure 1.

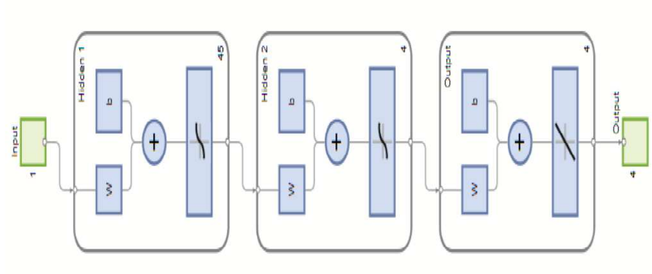


FIGURE 1. Diagram of artificial deep neural network.

In this work, we extend the COVID-19 model studied in [10] under fractional order derivative in the Caputo sense. For computational purposes, we use the fractional order differential transform technique. Keeping in mind the importance, in the suggested epidemic neural network model approach, the fractional differential transform method is used to solve the fractional differential equations and provide the equation values at a specific time, while the neural network is created to express the unknown parameters in the compartmental model. We use the Levenberg-Marquardt algorithm to investigate three different cases for our model.

2. Preliminaries

Here, we define the standard Caputo derivative as follows:

Definition 2.1. ([2]) For the function $g \in C[0, T]$ and $\delta > 0$, we define the Caputo derivative as

$$^{C}D^{\alpha}g(t) = D^{(n-\delta)}\frac{d^n}{dt^n}g(t) = \frac{1}{\Gamma(n-\delta)}\int_0^t (t-\tau)^{n-\delta-1}g^{(n)}(\tau)d\tau,$$

where $n = [\delta + 1],\ n - 1 < \delta \leq n$. If $0 < \delta < 1$, then we have

$$^{C}D^{\alpha}g(t) = D^{(1-\delta)}\frac{d}{dt}g(t) = \frac{1}{\Gamma(1-\delta)}\int_0^t (t-\tau)^{-\delta}g'(\tau)d\tau.$$

The FDTM is semi-numerical and analytical approach. It is considered as a traditional DTM reform.

Definition 2.2. ([13]) Let $g(t)$ be a $p - times$ differentiable function with respect to time t. Then the differential transform of the p^{th} derivative of $g(t)$ in one variable is a follows:

$$G(p) = \frac{1}{p!}\left(\frac{d^p g(t)}{dt^p}\right)_{t=t_0}, \tag{2.1}$$

where $g(t)$ is the original function and $G(p)$ is the transformed function.

Definition 2.3. ([13]) The inverse differential transform of $G(p)$ is defined by

$$g(t) = \sum_{p=0}^{\infty} G(p)(t - t_0)^p. \tag{2.2}$$

Clubbing (2.1) and (2.2) together, we get

$$g(t) = \sum_{p=0}^{\infty} \frac{(t - t_0)^p}{p!}\left(\frac{d^p g(t)}{dt^p}\right)_{t=t0}. \tag{2.3}$$

From (2.3), using Taylor series expansion we can deduce DTM, now (2.2) is written as

$$g(t) = \sum_{p=0}^{N} G(p)(t - t_0)^p,$$

where N is decided by the convergence of natural frequency.

Definition 2.4. ([13]) Differential transform for fractional operator is given as: if $y(x) =^C D^\alpha g(x)$, then

$$G(p) = \frac{\Gamma(\alpha + 1 + \frac{p}{q})}{\Gamma(\alpha + \frac{p}{q})} G(p + \alpha q),$$

where q is the order of fraction and $G(p)$ is the fractional differential transform of $g(x)$. Also

$$G(p) = \begin{cases} \frac{1}{\Gamma(1+\frac{p}{q})}\left[\frac{d^{\frac{p}{q}} g(x)}{dx^{\frac{p}{q}}}\right], \text{ for } p = 0, 1, 2, ..., (\alpha q - 1), \ \frac{p}{q} \in N, \\ 0, \text{ for } \frac{p}{q} \text{ does not belong to } N. \end{cases}$$

Definition 2.5. ([13]) The inverse DT of $G(p)$ is given by

$$G(p) = \sum_{p=0}^{\infty} \frac{\Gamma(\alpha + 1 + \frac{p}{q})}{\Gamma(\alpha + \frac{p}{q})} G(p + \alpha q)(t)^{\frac{p}{\alpha}}.$$

3. Formulation of proposed model

The entire population is further divided into classes using the following model. Those with a high risk of contracting an infection are placed in the susceptible class S, those who have close contact with the COVID-19 environment are placed in the exposed class E, those who exhibit COVID-19 symptoms are placed in the infected class I, and those in the R recovered class are recovered individuals. We extend the model studied by Din,et.al [10] under fractional order derivative in the Caputo sense by using differential transformation method. The model can be described as follows:

$$\begin{cases} D^{\delta}S(t) = \gamma - KS(t)I(t)(1+\alpha I(t)) - \xi S(t), \\ D^{\delta}E(t) = KS(t)I(t)(1+\alpha I(t)) - (\xi+\zeta)E(t), \\ D^{\delta}I(t) = \eta + \zeta E(t) - (\mu+\xi+\beta)I(t), \\ D^{\delta}R(t) = \beta I(t) - \xi R(t), \end{cases} \tag{3.1}$$

where $S(0) = S_0$, $E(0) = E_0$, $I(t) = T_0$, $R(0) = R_0$. Here, the symbol γ represents the constant of recruitment. The letter $\delta \in (0,1]$ stands for fractional order. The remaining parameters are described in Table 1.

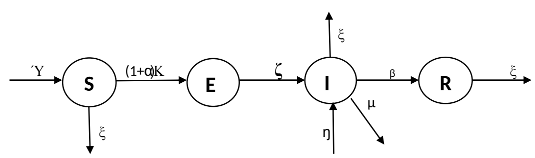


Figure 2. A Flow Chart of the Proposed Model (3.1).

4. Existence and positivity of solution

In this section, we establish some existence results. Let $0 \leq \mathfrak{t} \leq T < \infty$ and defined

$$X = C[0,T]$$

be the Banach space with norm

$$\|W\| = \sup_{\mathfrak{t}\in[0,T]} \Big|W(t)\Big|. \tag{4.1}$$

Then we write system as

$${}^{C}D^{\alpha}W(t) = F(t, W(t)),\ \mathfrak{t} \in [0,T],\ W(0) = W_0. \tag{4.2}$$

Theorem 4.1. *The solution of* (4.2) *is equivalent to the following integral equation*

$$W(t) = W_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}F(\tau, W(\tau))d\tau. \tag{4.3}$$

Proof. By following [22], we can prove that (4.2) is equivalent to (4.3). □

(H1) For constant $\mathbf{L}_F > 0$ and $W, \bar{W} \in X$, we have

$$|F(t, W) - F(t, \bar{W})| \leq \mathbf{L}_F[|W - \bar{W}|.$$

Theorem 4.2. *The solution of* (4.2) *is unique under the* (H1) *and the conditions*

$$\frac{\mathbf{L}_F}{\Gamma(\alpha+1)} \leq 1$$

holds.

Proof. Let $A : X \to X$ be defined by

$$AW = W_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} F(\tau, W(\tau))d\tau.$$

Then for $W, \bar{W} \in X$, we have

$$\begin{aligned} \|A(W) - A(\bar{W})\| &= \sup_{t\in\mathcal{T}} \left| AW(t) - A\bar{W}(t) \right| \\ &\leq \sup_{t\in\mathcal{T}} \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} |A(t, W(t)) - A(\tau, \bar{W}(\tau))| d\tau \\ &\leq \frac{\mathbf{L}_F T^\alpha}{\Gamma(\alpha+1)(\theta)} \|W - \bar{W}\|. \end{aligned} \tag{4.4}$$

Hence A is contraction operator and therefore in view of Banach the concerned problem (4.3) has unique solution and so the considered model (5.1) has a unique solution. □

Theorem 4.3. *The solution* (S, E, I, R) *of model* (3.1) *for positive initial conditions with* $t > 0$ *is positive for all* $t > 0$.

Proof. From model (3.1), we see

$$\begin{cases} D^\delta S(t)\Big|_{S=0} = \gamma > 0, \\ D^\delta E(t)\Big|_{E=0} = KS(t)I(t)(1 + \alpha I(t)) > 0, \\ D^\delta I(t)\Big|_{I=0} = \eta + \zeta E(t) > 0, \\ D^\delta R(t)\Big|_{R=0} = \beta I(t) > 0. \end{cases} \tag{4.5}$$

From first equation of (4.5), we have on using Laplace transform that

$$S(t) > S(0)E_\delta(-\xi t^\delta) > 0, \ \forall t > 0.$$

Hence $S(t) > 0$, for all $t > 0$. In the same way we can prove that $E(t) > 0, I(t) > 0, R(t) > 0$ for all $t > 0$. □

5. Construction of algorithm

To compute the algorithm for required approximate solution of proposed model, using Differential transform on both sides of system (3.1), we have

$$\begin{cases} D^\delta S(t) = \gamma - KS(t)I(t)(1+\alpha I(t)) - \xi S(t), \\ D^\delta E(t) = KS(t)I(t)(1+\alpha I(t)) - (\xi+\zeta)E(t), \\ D^\delta I(t) = \eta + \zeta E(t) - (\mu+\xi+\beta)I(t), \\ D^\delta R(t) = \beta I(t) - \xi R(t) \\ \delta \in (0,1]. \end{cases} \tag{5.1}$$

After applying FDTM, we get

$$\begin{cases} \dfrac{\Gamma(\alpha+1+\frac{k}{\beta})}{\Gamma(\alpha+\frac{k}{\beta})} S(k+\alpha\beta) = \gamma\delta(K) - K\sum_{l}^{k} I(l)S(k-l) \\ -\alpha K \sum_{k_3}^{k_4}\sum_{k_2}^{k_3}\sum_{k_1}^{k_2} S(k_1)I(k_2-k_1)I(k_3-k_2) - \xi S(K), \\ \\ \dfrac{\Gamma(\alpha+1+\frac{k}{\beta})}{\Gamma(\alpha+\frac{k}{\beta})} E(k+\alpha\beta) = K\sum_{l}^{k} I(l)S(k-l) \\ -\alpha K \sum_{k_3}^{k_4}\sum_{k_2}^{k_3}\sum_{k_1}^{k_2} S(k_1)I(k_2-k_1)I(k_3-k_2) - (\zeta+\xi)E(K), \\ \\ \dfrac{\Gamma(\alpha+1+\frac{k}{\beta})}{\Gamma(\alpha+\frac{k}{\beta})} I(k+\alpha\beta) = \eta\delta(K) + \zeta E(K) - (\mu+\xi+\beta)I(K), \\ \\ \dfrac{\Gamma(\alpha+1+\frac{k}{\beta})}{\Gamma(\alpha+\frac{k}{\beta})} R(k+\alpha\beta) = \beta I(K) - \xi R(K). \end{cases} \tag{5.2}$$

On further simplification, we have

$$\begin{cases} S(k+\alpha\beta) = \dfrac{\Gamma(\alpha+\frac{k}{\beta})}{\Gamma(\alpha+1+\frac{k}{\beta})}[\gamma\delta(K) - K\sum_{l}^{k} I(l)S(k-l) \\ -\alpha K \sum_{k_3}^{k_4}\sum_{k_2}^{k_3}\sum_{k_1}^{k_2} S(k_1)I(k_2-k_1)I(k_3-k_2) - \xi S(K)], \\ E(k+\alpha\beta) = \dfrac{\Gamma(\alpha+\frac{k}{\beta})}{\Gamma(\alpha+1+\frac{k}{\beta})}[K\sum_{l}^{k} I(l)S(k-l) \\ -\alpha K \sum_{k_3}^{k_4}\sum_{k_2}^{k_3}\sum_{k_1}^{k_2} S(k_1)I(k_2-k_1)I(k_3-k_2) - (\zeta+\xi)E(K)], \\ I(k+\alpha\beta) = \dfrac{\Gamma(\alpha+\frac{k}{\beta})}{\Gamma(\alpha+1+\frac{k}{\beta})}[\eta\delta(K) + \zeta E(K) - (\mu+\xi+\beta)I(K)], \\ R(k+\alpha\beta) = \dfrac{\Gamma(\alpha+\frac{k}{\beta})}{\Gamma(\alpha+1+\frac{k}{\beta})}[\beta I(K) - \xi R(K)] \end{cases} \tag{5.3}$$

and so on. Evaluating the series (5.3), we get solution as

$$\begin{cases} S(t) = \sum_{p=0}^{\infty} S(p)(t)^{\frac{p}{\alpha}}. \\ E(t) = \sum_{p=0}^{\infty} E(p)(t)^{\frac{p}{\alpha}}. \\ I(t) = \sum_{p=0}^{\infty} I(p)(t)^{\frac{p}{\alpha}}. \\ R(t) = \sum_{p=0}^{\infty} R(p)(t)^{\frac{p}{\alpha}}. \end{cases} \tag{5.4}$$

6. Numerical interpretation

This part is related to demonstrate graphically the solutions of different compartments using various fractional orders values. We take the initial data [10] described by $\Big(S(0), E(0), I(0), R(0)\Big) = (32.37, 12, 0.001523, 0.005025)$. The parameters values are given in table 1

Symbol	Description	Real value
γ	birth rate	0.250281
η	rate of positive infection	0.006656
K	infection rate	0.000024
α	immunity lose rate	0.01182
ξ	natural death rate	0.0000004×10^6
ζ	exposed rate	0.016
μ	infection death rate	0.025
β	recovery rate	0.75

TABLE 1. Description of the parameters [10] of the Model (5.1).

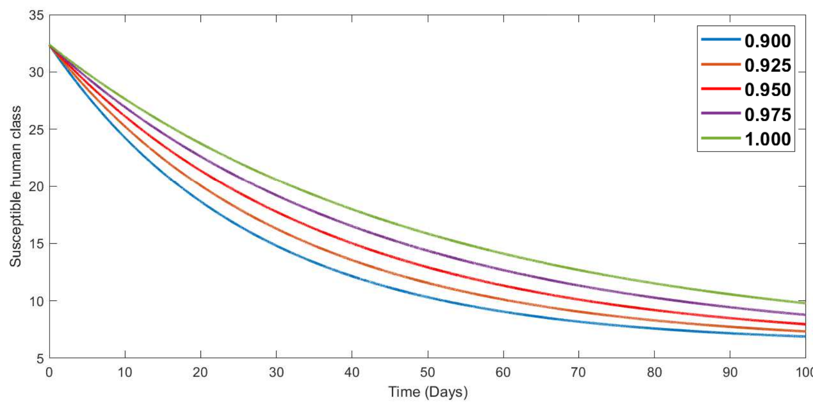


FIGURE 3. Dynamical behavior of susceptible class.

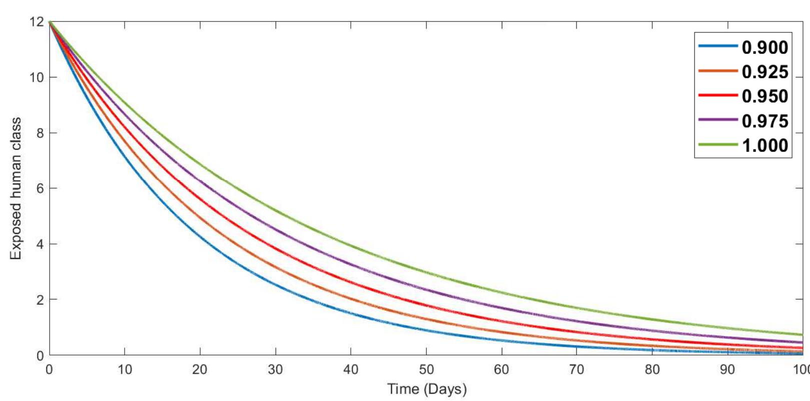


FIGURE 4. Dynamical behavior of exposed class.

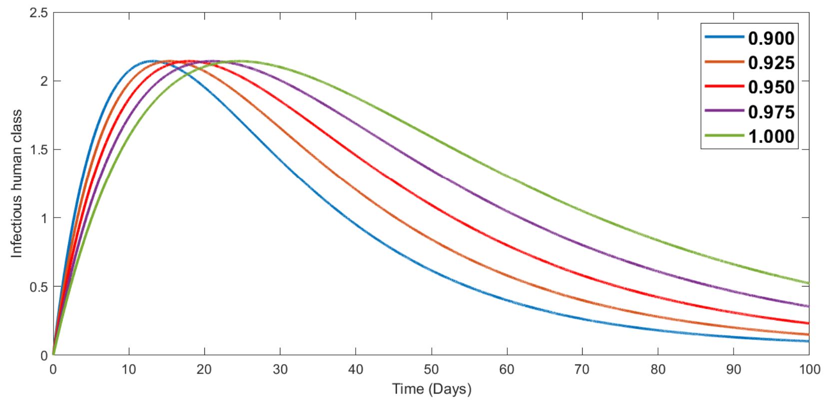


FIGURE 5. Dynamical behavior of infected class.

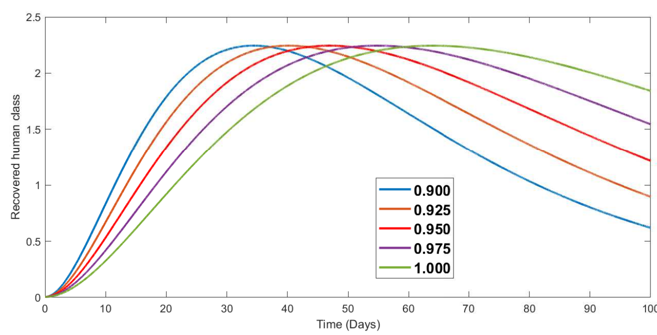


FIGURE 6. Dynamical behavior of recovered class.

We have presented approximate solutions of various compartment of the proposed model in Figures 3-6 by taking different fractional order. We see that the decline in susceptible class as well as in exposed is occurring with different rate. At smaller fractional order the decay process is faster as compared to greater order. In same line growth in infection is faster at greater fractional order. The same behavior can be observed for recovered class also. We see that as $\delta \to 1$, the curves tending to the integer order curve in Figures 3-6.

7. Neural network analysis

In this section, we extend our analysis based on artificial neural network using Levenberg-Marquardt algorithm [18]. We have discussed three cases in this regards to investigate the proposed analysis. The corresponding structure of the used neural network is given in figure 7.

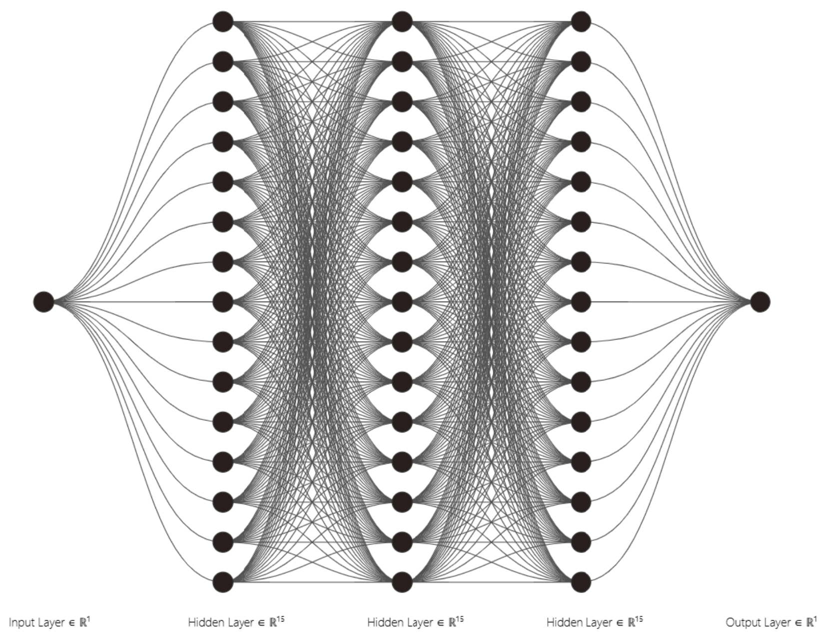


Figure 7. Structure of used artificial neural network with multi layers.

The neural network is a function which takes the values as input and processes them through hidden lyes to output which we can describe as follows:

$$\mathbb{F} : R^m \to R^n$$

by

$$\mathbb{F}\mathbf{u} = \mathbf{v}, \ \mathbf{u} \in R^m, \ \mathbf{v} \in R^n,$$

where m and n are the dimensions of input and output respectively. The notations $\mathbf{u}$ and $\mathbf{v}$ denote the input and output respectively. We use 45 hidden layers together with input output layers to investigate the neural network analysis of our results for three different cases. We have recorded the mean squared error (MSE) by training, validation, testing and also recorded numbers of best epoch with their maximum values by using 47 neurons. The corresponding results are given in table 2. We use $MSE(Training) = E_1, MSE(Validation) = E_2, MSE(Testing) = E_3$, Best Epoch=B, Final Epoch=F.

Case	E_1	E_2	E_3	B	F
Case 1	8.077677e-04	3.994641e-03	6.060872e-03	14	44
Case 2	3.641964e-03	1.608150e-03	2.650752e-03	15	45
Case 3	1.078978e-03	4.203209e-03	5.729591e-03	11	41

Table 2. MSE Performance Comparison for ANN Training Cases

In figures 8, 9, and 10, we have presented the graphical illustrations of all three cases to analyze the validation, best performance and best function out fit as well as the error histograms for all three cases. The corresponding numbers of epoches, and their maximum values and MSE analysis for validation, training and testing have also recorded in table 2. The maximum numbers of neurons we have used is 47 with three multi hidden layers together with input output layers.

8. Some explanation and concluding remarks

This research has successfully developed and analyzed a nonlinear fractional-order SEIR model to capture the dynamics of COVID-19 transmission. The model incorporates a saturated incidence rate to reflect more realistic disease spread patterns. The Caputo fractional derivative was employed to better represent the memory effects and hereditary properties inherent in epidemiological processes.

The Fractional Differential Transform Method proved effective in obtaining semi-analytical solutions for the proposed model. Numerical simulations demonstrated the significant influence of the fractional order parameter on system dynamics, particularly showing how variations in this parameter affect the temporal evolution of susceptible, exposed, infected, and recovered populations. The solutions exhibited good convergence properties with relatively few series terms.

A key innovation of this work lies in the successful integration of mathematical modeling with deep learning methodologies. The implementation of an artificial neural network trained with the Levenberg-Marquardt algorithm yielded excellent agreement with the analytical solutions, achieving low mean squared error across all validation cases. This hybrid approach provides a

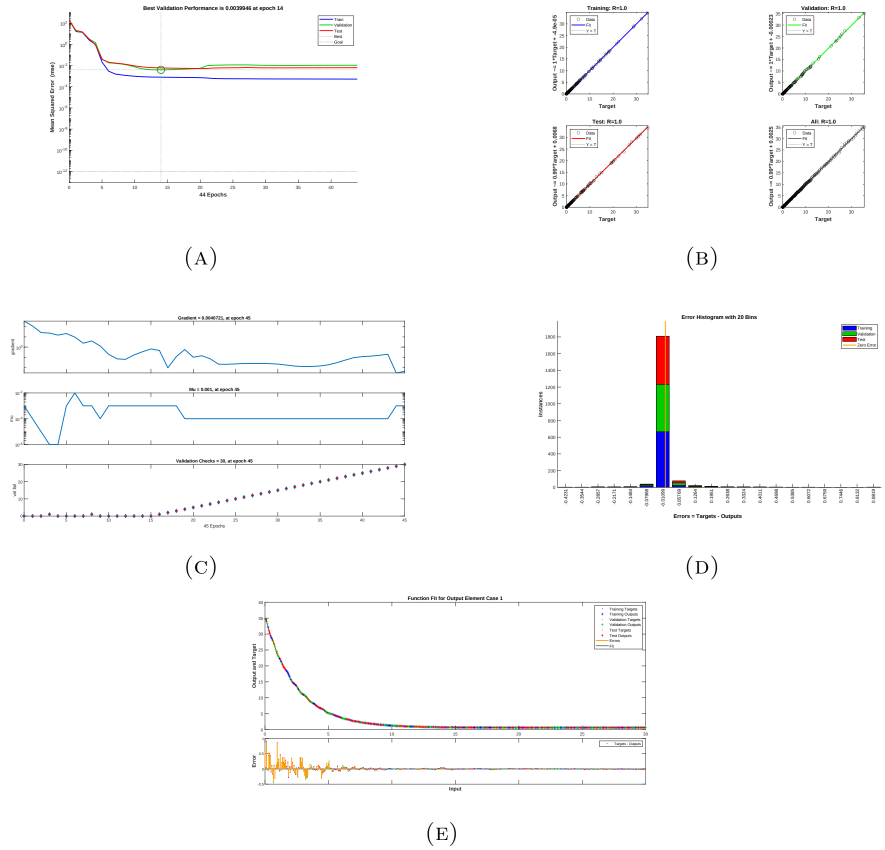
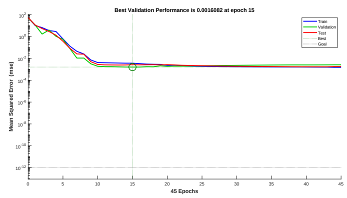


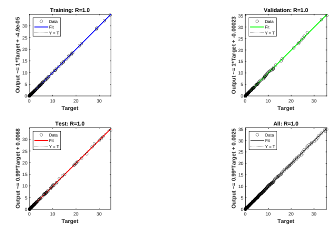
Figure 8. Classification of data analysis for proposed model using artificial neural network Case-1 (a) data validation , (b) regression coefficient, (c) performance(d) error histogram(e) best function fit for case 1.

powerful framework for both simulation and parameter estimation in complex epidemiological systems.

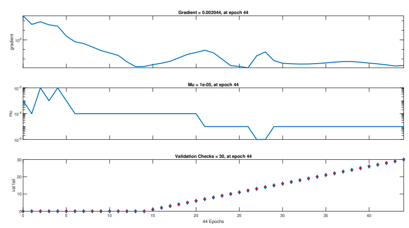
The main contributions of this research include the development of a new fractional-order epidemic model with nonlinear incidence rate, the creation of an efficient computational scheme combining FDTM with neural networks, and the demonstration of this framework's effectiveness through comprehensive numerical experiments. Future work will focus on extending the model to include additional compartments such as vaccinated and asymptomatic populations, incorporating spatial heterogeneity through partial differential equations, and adapting the methodology to other infectious diseases. Further investigation



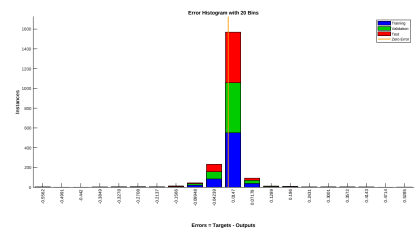
(A)



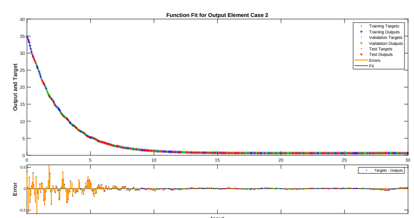
(B)



(C)



(D)

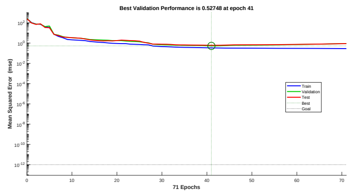


(E)

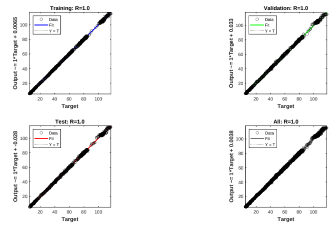
FIGURE 9. Classification of data analysis for proposed model using artificial neural network Case-2 (a) data validation , (b) regression coefficient, (c) performance(d) error histogram(e) best function fit for case 2.

into real-time parameter estimation using adaptive neural networks and the application of this framework to optimal control problems in public health policy represents promising research directions.

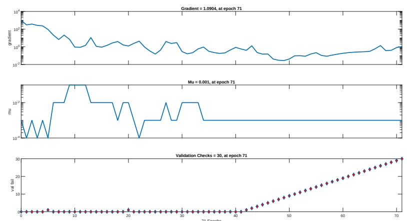
This research establishes a solid foundation for combining advanced mathematical modeling with machine learning techniques in epidemiology, providing valuable tools for understanding disease dynamics and informing public health decisions.



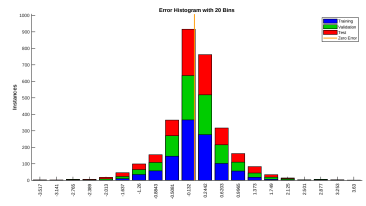
(A)



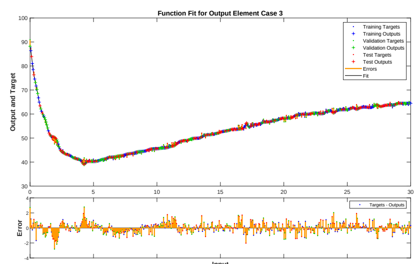
(B)



(C)



(D)



(E)

FIGURE 10. Classification of data analysis for proposed model using artificial neural network Case-3 (a) data validation , (b) regression coefficient, (c) performance(d) error histogram(e) best function fit for case 3.

Acknowledgments: The authors would like to thank Prince Sultan University for their valuable support and paying the Article Processing Charges of this publication.

REFERENCES

[1] R. Abazari and A. Borhanifar, *Numerical study of the solution of the Burgers and coupled Burgers equations by a differential transformation method,* Compu. Math. Appl., **59**(8) (2010), 2711–2722.

[2] T. Abdeljawad, N. Khan, B. Abdalla, A. Al-Jaser, M. Alqudah and K. Shah, *A mathematical analysis of human papilloma virus (HPV) disease with new perspectives of fractional calculus,* Alexandria Eng. J., **125** (2025), 575–599.

[3] F.S. Akinboro, S. Alao and F.O. Akinpelu, *Numerical solution of SIR model using differential transformation method and variational iteration method,* General Math. Notes, **22**(2) (2014), 82–92.

[4] M.A. Alqudah, K. Shah, T. Abdeljawad and F. Mofarreh, *Study on nonlinear fractional order conformable model of infectious disease using updated analytical techniques,* Fractals, **33**(04) (2025), 2540080.

[5] M. Alqudah, K. Shah, F. Mofarreh and K. Abdeljawad, *Mathematical modeling of psychological disease by using artificial intelligence tools,* Fractals, **2025** (2025), doi: https://doi.org/10.1142/S0218348X25501014.

[6] A. Arikoglu and I. Ozkol, *Solution of fractional differential equations by using differential transform method,* Chaos, Solitons Fractals, **34**(5) (2007), 1473–1481.

[7] F. Ayaz, *Solutions of the system of differential equations by differential transform method,* Appl. Math. Comput., **147**(2) (2004), 547–567.

[8] F. Ayaz, *Applications of differential transform method to differential algebraic equations,* Appl. Math. Comput., **152**(3) (2004), 649–657.

[9] N. Das, *Study of Properties of Differential Transform Method for Solving the Linear Differential Equation,* Asian J. Eng. Appl. Tech., **8**(2) (2019), 50–56.

[10] R. ud Din, A.R. Seadawy, K. Shah, A. Ullah and D. Baleanu, *Study of global dynamics of COVID-19 via a new mathematical model,* Results Phys., **19** (2020), 103468.

[11] D. Fisher and D. Heymann, *Q&A: The novel coronavirus outbreak causing COVID-19,* BMC medicine, **18** (2020), 1–3.

[12] J.L. Hurwitz, *B cells, viruses, and the SARS-CoV-2/COVID-19 pandemic of 2020,* Viral Immunol., **33**(4) (2020), 251–252.

[13] B. Ibis, M. Bayram and A.G. Agargun, *Applications of fractional differential transform method to fractional differential-algebraic equations,* European Journal of Pure and Applied Mathematics, **4**(2) (2011), 129–141.

[14] F.W.C. Jasper, K.H. Kok and Z. Zheng, *Genomic characterization of the 2019 novel human-pathogenic coronavirus isolated from patients with acute respiratory disease in Wuhan, Hubei,* China. Emerg. Microbes Infect, **9**(1) (2020), 221–236.

[15] X. Lin, Y. Matsunaka and H. Shiraishi, *Estimating the effective reproduction number of COVID-19 via the chain ladder method.* Jap. J. Stat. Data Sci., **7(2)** (2024), 861–893.

[16] C. McAloon, Á. Collins, K. Hunt, A. Barber, A.W. Byrne, F. Butler and S.J. More, *Incubation period of COVID-19: a rapid systematic review and meta-analysis of observational research,* BMJ open, **10**(8) (2020), e039652.

[17] S.S. Musa, W. Mkandawire, T. Inekwe, Y. Dong, A. Grozdani, H. Hong and A. Colubri, *The effect of Behavioral Factors and Intervention Strategies on Pathogen Transmission: Insights from a Two-Week Epidemic Game at Wenzhou-Kean University in China.* medRxiv, **2024**(12) (2024).

[18] X. Ning, L. Jia, Y. Wei, X.A. Li and F. Chen, *Epi-DNNs: Epidemiological priors informed deep neural networks for modeling COVID-19 dynamics,* Comput. Biology Medicine, **158** (2023), 106693.

[19] O.J. Peter and M.O. Ibrahim, *Application of differential transform method in solving a typhoid fever model,* Int. J. Math. Sci. Opti. Theory Appl., **2017** (2017), 250–260.

[20] J.A. Quesada, A. López-Pineda, V.F. Gil-Guillén, J.M. Arriero-Marín, F. Gutiérrez and C. Carratala-Munuera, *Incubation period of COVID-19: A systematic review and meta-analysis,* Revista Clínica Española (English Edition), **221**(2) (2021), 109–117.

[21] K. Shah, T. Abdeljawad, I. Mahariq and F. Jarad, *Qualitative analysis of a mathematical model in the time of COVID-19,* BioMed Research Int., **2020**(1) (2020), 5098598.

[22] K. Shah, M. Sher, H. Rabai'ah, A. Ahmadian, S. Salahshour and B.A. Pansera, *Analytical and qualitative investigation of COVID-19 mathematical model under fractional differential operator,* Math. Methods Appl. Sci., **46**(7) (2023), 8223–8242.

[23] K. Shah, T. Abdeljawad, B. Abdalla and Z. Ali, *Analyzing a coupled dynamical system of materials recycling in chemostat systems with artificial deep neural network,* Model. Earth Syst. Envir., **11**(5) (2025), 1–14.

[24] I. Shaheen, N. Javaid, Z. Ali, I. Ahmed, F.A Khan and D. Pamucar,*A trustworthy and patient privacy-conscious framework for early diabetes prediction using Deep Residual Networks and proximity-based data.* Biomedical Signal Processing Cont., **112**(2025), 108361.

[25] M. Sher, K. Shah, Z. Ali, T. Abdeljawad, and M. Alqudah, *Using deep neural network in computational analysis of coupled systems of fractional integro-differential equations.* J. Comput. Appl. Math., (2025) 116912.

[26] S. Ullah, M.S. Arif, R.U. Din and I. Ullah, *Stability and Numerical Analysis of SEIVR COVID-19 Fractional and Integral Order Model under Saturated Incidence Rate,* Int. J. Appl. Comput. Math., **11**(3) (2025): 105.

[27] World Health Organization, *Advice on the use of masks in the context of COVID-19:* interim guidance, 6 April 2020.

[28] J.K. Zhou, *Differential Transformation and Its Applications for Electrical Circuits,* Huazhong Univ. Press: Wuhan, China, 1986.