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STABILITY OF CAUCHY ADDITIVE FUNCTIONAL EQUATION IN A LATTICE RANDOM NORMED SPACE

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Abstract. This study investigates the stability of the Cauchy additive functional equation

$$\jmath\left(\frac{\mathbb{M}-\eth}{n}+\aleph\right)+\jmath\left(\frac{\eth-\aleph}{n}+\mathbb{M}\right)+\jmath\left(\frac{\aleph-\mathbb{M}}{n}+\eth\right)=\jmath\left(\mathbb{M}+\eth+\aleph\right)$$

in lattice random normed spaces (LRN-S). Using lattice structure and random norm techniques, it is shown that any approximate solution under random perturbations is close to a true linear function. The results extend the classical Hyers-Ulam stability concept to uncertain and lattice-structured environments.

1. Introduction

Functional equations are used in various parts of mathematics, including: probability theory, measurement theory, geometry, statistics, mechanics, etc. Therefore, it is necessary to study them and pay attention to studying their properties. One of these properties is stability.

The origins of the functional equation stability can be traced to a problem posed by Ulam [10] in 1940. After that, studies on this topic, were conducted by many mathematicians for different functional equations in many spaces, including: the normed space, Banach space, Banach algebra, and other generalized spaces. Usually using two methods: the fixed point and the direct

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classical method. In most cases, they reached interesting and motivating results.

I have chosen some references through which you can view more details about the topic [1, 2, 3, 4, 6, 7, 8, 9]. In our paper, We employ both the direct approach and the fixed point technique to establish the generalized Hyers-Ulam stability of a CAFE:

$$\jmath\left(\frac{\mathbb{M}-\eth}{n}+\aleph\right)+\jmath\left(\frac{\eth-\aleph}{n}+\mathbb{M}\right)+\jmath\left(\frac{\aleph-\mathbb{M}}{n}+\eth\right)=\jmath\left(\mathbb{M}+\eth+\aleph\right) \quad (1.1)$$

for all $\mathbb{M}, y \in X$ in LRN-space.

2. Preliminaries

Definition 2.1. ([5]) An order set $l = (\varrho, \geq)$ is referred to as a complete lattice, if each nonempty subset A of ϱ admits the existence of both its least upper bound and greatest lower bound. Also, $\inf \varrho = 0_l$, $\sup \varrho = 1_l$. Δ_ϱ^+ is defined by

$\Delta_\varrho^+ = \{\iota | \iota : \mathbb{F} \cup \{-\infty, \infty\} \to \varrho, \iota(0) = 0_l, \iota(+\infty) = 1_l, \iota$ is anon-decreasing, left continuous on $\mathbb{F}\}$.

$\left(\Delta_\varrho^+, \geq_\varrho\right)$ is a partially order set ($\geq_\varrho$ is usual ordered).

The subspace D_ϱ^+ of Δ_ϱ^+ is defined by

$$D_\varrho^+ = \left\{P \in \Delta_\varrho^+ : \lim_{x\to+\infty} P(x) = 1_l\right\}.$$

Also, the function

$$\mathfrak{C}_\circ(t) = \begin{cases} 0_l, & if\ t \leq 0 \\ 1_l, & if\ t > 0 \end{cases}$$

is regarded as the greatest element in D_ϱ^+.

Definition 2.2. ([5]) Let ϱ be a complete lattice. A mapping $\top : \varrho^2 \to \varrho$ is termed a t-norm if it meets the following axiomatic conditions:

(1) $\top(\mathbb{M}, 1_l) = \mathbb{M}$ for all $\mathbb{M} \in \varrho$,
(2) $\top(\mathbb{M}, \eth) = \top(\eth, \mathbb{M})$ for all $(\mathbb{M}, \eth) \in \varrho^2$,
(3) $\top(\mathbb{M}, \top(\eth, v)) = \top(\top(\mathbb{M}, \eth), v)$ for all $(\mathbb{M}, \eth, v) \in \varrho^3$,
(4) if $\eth \leq \eth'$ then $\top(\mathbb{M}, \eth) \leq \top(\mathbb{M}, \eth')$ for all $(\mathbb{M}, \eth, \eth') \in \varrho^3$.

Definition 2.3. ([5]) A LRN-space is triple $(X, P, \top)$, where X is a vector space, while $\top$ represents a t-norm on ϱ, and $P : X \times \mathbb{F} \to D_\varrho^+$ is a mapping that achieves:

(1) $P(\mathbb{M}, t) = \mathfrak{C}_\circ(t)$ for all $t > 0$ if and only if $\mathbb{M} = 0$,

(2) $P(\alpha\mathbb{M}, t) = P\left(\mathbb{M}, \frac{t}{|\alpha|}\right)$ for all $0 \neq \mathbb{M} \in X, t \geq 0$,

(3) $P(\mathbb{M} + \eth, t + s) \geq_{\varrho} \mathsf{T}(P(\mathbb{M}, t), P(\eth, s))$ for all $\mathbb{M}, \eth \in X$ and $t, s \geq 0$.

Definition 2.4. ([5]) Suppose ϱ is a complete lattice. The function $N : \varrho \to \varrho$ is referred to as a negation if N is decreasing and $N(0_l) = 1_l$, and $N(1_l) = 0_l$. This function is called involutive, if and only if $N(N(\alpha)) = \alpha$ for all $\alpha \in \varrho$.

Definition 2.5. ([5]) Let (X, P, T) be an LRN-space, consider $\{\mathbb{M}_n\}$ to be an arbitrary sequence in X.

(1) $\{\mathbb{M}_n\}$ is converges to $\mathbb{M} \in X$, if for all $\varepsilon > 0$, $r \in \varrho/\{0_l, 1_l\}$, there exist $N \in \mathbb{Z}^+$ such that for all $n \geq N$,

$$P(\mathbb{M}_n - \mathbb{M}, \varepsilon) >_{\varrho} N(r).$$

(2) $\{\mathbb{M}_n\}$ is a Cauchy sequence in X, if for all $\varepsilon > 0$ and $r \in \varrho/\{0_l, 1_l\}$, there exist $N \in \mathbb{Z}^+$ such that

$$P(\mathbb{M}_n - \mathbb{M}_m, \varepsilon) >_{\varrho} N(r), \text{for all } n \geq m \geq N.$$

(3) X is regarded as complete whenever each Cauchy sequence in X has a limit in X.

3. Stability of CAFE (1.1) via direct method

We will discuss the stability of CAFE in LRN space. In this paper, X is a real linear space, $(\mathfrak{Y}, P, T)$ denotes as a complete LRN space. We note that

$$1_l = \sup \varrho \geq_{\varrho} N(k) \quad \text{for all } k \in \varrho/\{0_l, 1_l\}.$$

Theorem 3.1. *Let $l = (\varrho, \geq)$ be a complete lattice and $\jmath : X \to \mathfrak{Y}, \jmath(tx)$ be continuous in $t \in \mathbb{F}$ for each fixed $x \in X$. With $\jmath(0) = 0$ and $\mathfrak{C} : X^3 \to D_{\varrho}^{+}$ such that*

$$P\left(\jmath\left(\frac{\mathbb{M} - \eth}{n} + \aleph\right) + \jmath\left(\frac{\eth - \aleph}{n} + \mathbb{M}\right) + \jmath\left(\frac{\aleph - \mathbb{M}}{n} + \eth\right) - \jmath(\mathbb{M} + \eth + \aleph), t\right) \geq_{\varrho} \mathfrak{C}(\mathbb{M}, \eth, \aleph, t). \tag{3.1}$$

For all $\mathbb{M}, \eth, \aleph \in X, t > 0$. If

$$\lim_{n\to\infty} T_{i=1}^{\infty}\left(\mathfrak{C}\left(3^{n+i-1}\mathbb{M}, 3^{n+i-1}\mathbb{M}, 3^{n+i-1}\mathbb{M}, 3^{n}t\right)\right) = 1_l \tag{3.2}$$

and

$$\lim_{n\to\infty} \mathfrak{C}(3^n\mathbb{M}, 3^n\eth, 3^n\aleph, 3^n t) = 1_l \tag{3.3}$$

for all $\mathbb{M}, \gamma, v \in X, t > 0$, *then a unique linear mapping* $J : X \to \mathfrak{Y}$, *exists such that*

$$P(\jmath(\mathbb{M}) - J(\mathbb{M}), t) \geq_\varrho T_{\mathrm{i}=1}^{\infty}\left(\mathfrak{C}\left(3^{\mathrm{i}-1}\mathbb{M}, 3^{\mathrm{i}-1}\mathbb{M}, 3^{\mathrm{i}-1}\mathbb{M}, t\right)\right) \mathbb{M} \in X, t > 0. \tag{3.4}$$

Proof. Putting $\mathbb{M} = \eth = \aleph$ in (3.1),

$$P\left(3f(\mathbb{M}) - 3\frac{\jmath(3\mathbb{M})}{3}, t\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, t),$$

$$P\left(\left(\jmath(\mathbb{M}) - \frac{\jmath(3\mathbb{M})}{3}\right), \frac{t}{|3|}\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, t),$$

$$P\left(\jmath(\mathbb{M}) - \frac{\jmath(3\mathbb{M})}{3}, t\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t), \tag{3.5}$$

$$P\left(\frac{\jmath(3^n\mathbb{M})}{3^n} - \frac{\jmath\left(3^{n+1}\mathbb{M}\right)}{3^{n+1}}, t\right) \geq_\varrho \mathfrak{C}\left(3^n\mathbb{M}, 3^n\mathbb{M}, 3^n\mathbb{M}, 3^{n+1}t\right), \tag{3.6}$$

$$P\left(\frac{\jmath(3^n\mathbb{M})}{3^n} - \frac{\jmath\left(3^{n+1}\mathbb{M}\right)}{3^{n+1}}, \frac{t}{3^{n+1}}\right) \geq_\varrho \mathfrak{C}(3^n\mathbb{M}, 3^n\mathbb{M}, 3^n\mathbb{M}, t) \tag{3.7}$$

for all $\mathbb{M} \in X, t > 0$. Since $\sum_{k=1}^{n} \frac{1}{3^k} < 1$, by the triangle inequality, we have

$$\begin{aligned}
P\left(\frac{\jmath(3^n\mathbb{M})}{3^n} - \jmath(\mathbb{M}), t\right) &\geq_\varrho P\left(\frac{\jmath(3^n\mathbb{M})}{3^n} - \jmath(\mathbb{M}), \sum_{k=1}^{n}\frac{t}{3^k}\right) \\
&\geq_\varrho T_{k=0}^{n-1}\left(P\left(\frac{\jmath\left(3^{k+1}\mathbb{M}\right)}{3^{(k+1)}} - \frac{\jmath\left(3^k\mathbb{M}\right)}{3^k}, \frac{t}{3^{k+1}}\right)\right) \\
&\geq_\varrho T_{k=0}^{n-1}\left(\mathfrak{C}\left(3^k\mathbb{M}, 3^k\mathbb{M}, 3^k\mathbb{M}, t\right)\right) \\
&= T_{\mathrm{i}=1}^{n}\left(\mathfrak{C}\left(3^{\mathrm{i}-1}\mathbb{M}, 3^{\mathrm{i}-1}\mathbb{M}, 3^{\mathrm{i}-1}\mathbb{M}, t\right)\right).
\end{aligned} \tag{3.8}$$

By changing the value of $\mathbb{M}$ with $3^m\mathbb{M}$ in (3.8), we can show that $\left\{\frac{\jmath(3^n\mathbb{M})}{3^n}\right\}$ is convergent in $\mathfrak{Y}$,

$$\begin{aligned}
&P\left(\frac{\jmath(3^{n+m}\mathbb{M})}{3^{(n+m)}} - \frac{\jmath(3^m\mathbb{M})}{3^m}, t\right) \\
&\geq_\varrho T_{\mathrm{i}=1}^{n}\left(\mathfrak{C}\left(3^{\mathrm{i}+m-1}\mathbb{M}, 3^{\mathrm{i}+m-1}\mathbb{M}, 3^{\mathrm{i}+m-1}\mathbb{M}, 3^m t\right)\right).
\end{aligned} \tag{3.9}$$

When n, m tend to ∞, and using the condition (3.2), then it is clear that the sequence $\left\{\frac{\jmath(3^n\mathbb{M})}{3^n}\right\}$ is convergent sequence. Consider $J(\mathbb{M}) = \lim_{n\to\infty} \frac{\jmath(3^n\mathbb{M})}{3^n}$ for all $\mathbb{M} \in X$.

Now, put $3^m\mathbb{M}$, $3^m\eth, 3^m\aleph$ instead of $\mathbb{M},\eth,\aleph$ in (3.1), we derive

$$P\left(D_l(\mathbb{M},\eth,\ \aleph),t\right)\geq_\varrho \mathfrak{C}\left(3^m\mathbb{M},3^m\eth,3^m\aleph,3^m t\right)\mathbb{M},\eth,\aleph\in X,m\in N,t>0. \tag{3.10}$$

As m tends to ∞, $D_l\left(\mathbb{M},\eth,\aleph\right)=0$, hence $J\left(\mathbb{M}\right)$ satisfies (3.1) for all $\mathbb{M}\in X$, and it is linear. If n tends to ∞ in (3.8), we get (3.4). Let J' be an another mapping which meets the conditions (3.1) and (3.4), and put $\acute{t}=3^n t$. Then

$$\begin{aligned}
P\left(J\left(\mathbb{M}\right)-J'\left(\mathbb{M}\right)\acute{t}\right) &= P(\frac{1}{3^n}(J\left(3^n\mathbb{M}\right)-J'\left(3^n\mathbb{M}\right)),3^n t)\\
&= \ P\left(J\left(3^n\mathbb{M}\right)-J'\left(3^n\mathbb{M}\right)),3^{2n}t\right)\\
&\geq_\varrho T(P\left(J\left(3^n\mathbb{M}\right)-\jmath\left(3^n\mathbb{M}\right),3^n t\right),\\
&\qquad P\left(\jmath\left(3^n\mathbb{M}\right)-J'\left(3^n\mathbb{M}\right),3^n t\right))\\
&\geq_\varrho T\Bigg(T_{\mathrm{i}=1}^{\infty}\left(\mathfrak{C}\left(3^{\mathrm{i}+n-1}\mathbb{M},3^{\mathrm{i}+n-1}\mathbb{M},3^{\mathrm{i}+n-1}\mathbb{M},3^n t\right)\right),\\
&\qquad T_{\mathrm{i}=1}^{\infty}\left(\mathfrak{C}\left(3^{\mathrm{i}+n-1}\mathbb{M},3^{\mathrm{i}+n-1}\mathbb{M},3^{\mathrm{i}+n-1}\mathbb{M},3^n t\right)\right)\Bigg)
\end{aligned}$$

for any given $x\in X,n\in\mathbb{N},t>0$, by taking n tends to ∞ in (3.4), we have $J'=J$. □

Corollary 3.2. *Let* $(\varrho,\geq)$ *be a complete lattice,* $\jmath:X\to\mathfrak{Y},\jmath\left(tx\right)$ *continuous in* t *for each fixed* $x\in X,\jmath\left(0\right)=0$*, if* $T=T_m,T_p$ *or* T_D *and*

$$\left(\jmath\left(\frac{\mathbb{M}-\eth}{n}+\aleph\right)+\jmath\left(\frac{\eth-\aleph}{n}+\mathbb{M}\right)+\jmath\left(\frac{\aleph-\mathbb{M}}{n}+\eth\right)-\jmath\left(\mathbb{M}+\eth+\aleph\right),t\right)$$
$$\geq_\varrho \frac{t}{t+\delta\|x_\circ\|},\quad x,\eth,v\in X,t>0.$$

Proof. In Theorem 3.1, take $\mathfrak{C}\left(\mathbb{M},y,v,t\right)=\frac{t}{t+\delta\|x_\circ\|},t>0$. The result appears. □

Corollary 3.3. *Let* $\left(\varrho=[0,1],\geq\right)$ *be a complete lattice,* $\jmath:X\to\mathfrak{Y},\jmath\left(tx\right)$ *be continuous in* t *for each fixed* $\mathbb{M}\in X$*,* $\jmath\left(0\right)=0$*. If* $T=T_m$ *or* $T=T_p$ *and*

$$P\left(\jmath\left(\frac{\mathbb{M}-\eth}{n}+\aleph\right)+\jmath\left(\frac{\eth-\aleph}{n}+\mathbb{M}\right)+\jmath\left(\frac{\aleph-\mathbb{M}}{n}+\eth\right)-\jmath\left(\mathbb{M}+\eth+\aleph\right),t\right)$$
$$\geq_\varrho \frac{t}{t+\alpha\left(\|\mathbb{M}\|^r+\|\eth\|^r+\|\aleph\|^r\right)},\quad x,\eth,v\in X,t>0,$$

then there exists a unique linear mapping $J:X\to\mathfrak{Y}$ *such that*

$$P\left(\jmath\left(\mathbb{M}\right)-J\left(\mathbb{M}\right),t\right)>_\varrho T_{i=1}^{\infty}\left(\frac{t}{t+3\alpha\left(\|3^{i-1}\mathbb{M}\|^r\right)}\right)\quad \text{for all } \mathbb{M}\in X,t>0.$$

Proof. In Theorem 3.1, if we take

$$\mathfrak{C}(\mathbb{M}, y, t) = \frac{t}{t + \alpha\left(\|\mathbb{M}\|^r + \|\eth\|^r + \|\aleph\|^r\right)}$$

for all $\mathbb{M}, \eth, \aleph \in X, t > 0$. $T = T_m$ or T_p and $r < 0$. This completes the proof for all $\alpha > 0$. □

4. Stability of CAFE (1.1) via fixed point technique

Theorem 4.1. *Let $l = (\varrho, \geq_\varrho)$ be a complete lattice and $\jmath : X \to \mathfrak{Y}, \jmath(tx)$ be continuous in t, for each fixed $x \in X$ with $\jmath(0) = 0$ and $\mathfrak{C} : X^3 \to D_\varrho^+$ mapping with the property*

$$\mathfrak{C}(3\mathbb{M}, 3\eth, 3v, \alpha t) \geq_\varrho \mathfrak{C}(\mathbb{M}, \eth, v, t) \tag{4.1}$$

for all $x, \eth, v \in X, t > 0,\ 1 \leq \alpha < 3$. If

$$P\left(\jmath\left(\frac{\mathbb{M} - \eth}{n} + \aleph\right) + \jmath\left(\frac{\eth - \aleph}{n} + \mathbb{M}\right) + \jmath\left(\frac{\aleph - \mathbb{M}}{n} + \eth\right) - \jmath(\mathbb{M} + \eth + \aleph), t\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \eth, \aleph, t) \tag{4.2}$$

and

$$\lim_{n\to\infty} \mathfrak{C}(3^n\mathbb{M}, 3^n\eth, 3^n\aleph, 3^n t) = 1_l, \tag{4.3}$$

then there exists a unique linear mapping $C : X \to \mathfrak{Y}$ such that

$$C(x) = \lim_{n\to\infty} \frac{\jmath(3^n\mathbb{M})}{3^n} \text{ for all } x \in X \tag{4.4}$$

and

$$P(\jmath(\mathbb{M}) - C(\mathbb{M}), t) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, (3 - \alpha)t). \tag{4.5}$$

Proof. Putting $\mathbb{M} = \eth = \aleph$ in Theorem 3.1, we getting

$$P\left(3\left(\jmath(\mathbb{M}) - \frac{\jmath(3\mathbb{M})}{3}\right), t\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, t),$$

$$P\left(\jmath(\mathbb{M}) - \frac{\jmath(3\mathbb{M})}{3}, t\right) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t). \tag{4.6}$$

Let $E = \{\iota : X \to \mathfrak{Y}, \iota(0) = 0\}$,

$$d_G(\iota, h) = \inf\left\{u \in \mathbb{F}^+ : P(\iota(\mathbb{M}) - h(\mathbb{M}), ut) \geq_\varrho \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t)\right\} \tag{4.7}$$

for all $x \in X, t > 0$. Then (E, d_G) is a complete metric space [6].

Let $J : E \to E$ such that

$$J(\iota(\mathbb{M})) = \frac{\iota(3\mathbb{M})}{3} \text{ for all } \mathbb{M} \in X.$$

Now, we will prove that J is strictly contractive with Lipschitz constant $k = \frac{\alpha}{3}$, for all $1 \le \alpha < 3$.

Assume that $\iota, h \in E$ such that $d_G(\iota, h) = \delta$.

$$P(\iota(\mathbb{M}) - h(\mathbb{M}), \delta t) \ge_{\varrho} \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t),$$

$$\begin{aligned} P\left(J(\iota(\mathbb{M})) - J(h(\mathbb{M})), \frac{\alpha}{3}\delta t\right) &= P\left(\frac{\iota(3\mathbb{M})}{3} - \frac{h(3\mathbb{M})}{3}, \frac{\alpha}{3}\delta t\right) \\ &= P(\iota(3\mathbb{M}) - h(3\mathbb{M}), \alpha\delta t) \\ &\ge_{\varrho} \mathfrak{C}(3\mathbb{M}, 3\mathbb{M}, 3\mathbb{M}, 3\alpha t) \\ &\ge_{\varrho} \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t). \end{aligned} \tag{4.8}$$

Then

$$d_G(J(\iota(\mathbb{M})), J(h(\mathbb{M}))) \le \frac{\alpha}{3}\delta,$$

thus

$$d_G(J(\iota(\mathbb{M})), J(h(\mathbb{M}))) \le \frac{\alpha}{3} d_G(\iota(\mathbb{M}), h(\mathbb{M}))$$

and J is a strict contraction. From (4.6) and (4.7),

$$P(J(\jmath(\mathbb{M})) - \jmath(\mathbb{M}), t) \ge_{\varrho} \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t) \text{ and } d_G(Jf(\mathbb{M}), \jmath(\mathbb{M})) \le 1. \tag{4.9}$$

Then, there exists $C : X \to \mathfrak{Y}$ satisfying:

(1) The point C that is fixed under J, that is, $J(C(\mathbb{M})) = C(\mathbb{M})$, for all $\mathbb{M} \in X$.

$$\frac{C(3\mathbb{M})}{3} = C(\mathbb{M}),$$

thus

$$C(3\mathbb{M}) = 3C(\mathbb{M}) \quad \text{for all } \mathbb{M} \in X. \tag{4.10}$$

C is the unique fixed point of J within the set

$$M = \{\iota \in E : d_G(\iota, \jmath) < \infty\}.$$

Hence, C represents the unique mapping satisfying (4.10) with some $u \in (0, \infty)$ meeting

$$P(\jmath(\mathbb{M}) - C(\mathbb{M}), ut) \ge_{\varrho} \mathfrak{C}(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t). \tag{4.11}$$

(2) $d_G(J^n(\jmath(\mathbb{M})), C(\mathbb{M})) \to 0$, n tends to $\infty, \mathbb{M} \in X$. This implies the equality,

$$\lim_{n\to\infty} \frac{\jmath(3^n\mathbb{M})}{3^n} = C(\mathbb{M}) \quad \text{for all } \mathbb{M} \in X. \tag{4.12}$$

Since $\jmath$ additive and continuous then C is an linear mapping.

(3)

$$d_G\left(\jmath\left(\mathbb{M}\right), C\left(\mathbb{M}\right)\right) \leq \frac{1}{1-\frac{\alpha}{3}} d_G\left(\jmath\left(\mathbb{M}\right), J\left(\jmath\left(\mathbb{M}\right)\right)\right)$$
$$\leq \frac{3}{3-\alpha}.$$

Thus

$$P\left(\jmath\left(\mathbb{M}\right) - C\left(\mathbb{M}\right), \frac{3}{3-\alpha}t\right) \geq_{\varrho} \mathfrak{C}\left(\mathbb{M}, \mathbb{M}, \mathbb{M}, 3t\right),$$
$$P\left(\jmath\left(\mathbb{M}\right) - C\left(\mathbb{M}\right), t\right) \geq_{\varrho} \mathfrak{C}\left(x, x, x, \left(3-\alpha\right)t\right).$$

Now, in (4.2) substitute $3^n x$ for x and $3^n y$ for y. We have,

$$P\left(D_C(\mathbb{M}, \eth,\ \aleph), t\right) \geq_{\varrho} \mathfrak{C}\left(3^m\mathbb{M}, 3^m\eth, 3^m\aleph, 3^m t\right) \mathbb{M}, y, v \in X, m \in N, t > 0. \tag{4.13}$$

Taking m tends to ∞, we show that $C\left(x\right)$ satisfying (4.2) for all $x, y, v \in X$. □

Corollary 4.2. *Assume that* $l = \left(\varrho = [0,1], \geq_{\varrho}\right)$ *is a complete lattice,* $\jmath : X \to \mathfrak{Y}, \jmath(tx)$ *is continuous in* t*, for each fixed* $x \in X$*,* $\jmath(0) = 0$ *for all* $\mathbb{M}, \eth, v \in X, t > 0$ *and*

$$P\left(\jmath\left(\frac{\mathbb{M}-\eth}{n} + \aleph\right) + \jmath\left(\frac{\eth-\aleph}{n} + \mathbb{M}\right) + \jmath\left(\frac{\aleph-\mathbb{M}}{n} + \eth\right) - \jmath\left(\mathbb{M}+\eth+\aleph\right), t\right) \geq_{\varrho} \frac{t}{t+\delta\|x_{\circ}\|}.$$

Then there exists a unique linear mapping $C : X \to \mathfrak{Y}$ *such that*

$$C\left(\mathbb{M}\right) = \lim_{n\to\infty} \frac{\jmath\left(3^n\mathbb{M}\right)}{3^n} \mathbb{M} \in X$$

and

$$P\left(\jmath\left(\mathbb{M}\right) - C\left(\mathbb{M}\right), t\right) \geq_{\varrho} \frac{\left(3-\alpha\right)t}{\left(3-\alpha\right)t + \delta\|x_{\circ}\|},\quad 1 \leq \alpha < 3.$$

Proof. In Theorem 4.1, if we choose $\mathfrak{C}\left(\mathbb{M}, \eth, v, t\right) = \frac{t}{t+\delta\|x_{\circ}\|}, \delta > 0$, then we have the desired result. □

Corollary 4.3. *Assume that* $l = \left(\varrho = [0,1], \geq_{\varrho}\right)$ *is a complete lattice,* $\jmath : X \to \mathfrak{Y}\jmath(tx)$ *is continuous in* t*, for each fixed* $x \in X, \jmath\left(0\right) = 0$ *and* $\mathbb{M}, \eth, v \in X, t > 0$

$$P\left(\jmath\left(\frac{\mathbb{M}-\eth}{n} + \aleph\right) + \jmath\left(\frac{\eth-\aleph}{n} + \mathbb{M}\right) + \jmath\left(\frac{\aleph-\mathbb{M}}{n} + \eth\right) - \jmath\left(\mathbb{M}+\eth+\aleph\right), t\right) \geq_{\varrho} \frac{t}{t+\gamma\left(\|\mathbb{M}\|^r + \|\eth\|^r + \|v\|^r\right)}.$$

Then there exists a unique linear mapping $C : X \to \mathfrak{Y}$ *such that*

$$C\left(\mathbb{M}\right) = \lim_{n\to\infty} \frac{\jmath\left(3^n \mathbb{M}\right)}{3^n}$$

and for $3^r \leq \alpha < 3,\ r < 1$,

$$P\left(\jmath\left(\mathbb{M}\right) - C\left(\mathbb{M}\right), t\right) \geq_{\varrho} \frac{\left(3-\alpha\right)t}{\left(3-\alpha\right)t + 3\gamma\|x\|^r}.$$

Proof. In Theorem 4.1, let

$$\mathfrak{C}\left(x, \eth, v, t\right) = \frac{t}{t + \gamma\left(\|\mathbb{M}\|^r + \|\eth\|^r + \|v\|^r\right)} \text{ for all } \gamma \geq 0$$

such that $r < 1$. Thus, the conditions of theorem will be fulfilled and the result will be achieved. □

5. Conclusion

This study demonstrates that any approximate solution of the CA equation in LRN-spaces under random perturbations is close to a true linear function. By employing both the direct method and the fixed-point method, the Hyers-Ulam stability concept is effectively extended to lattice-structured random environments. These results provide a solid foundation for further research on more complex functional equations and other types of random perturbations in LRN-space.

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