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THE COMPLEX EE TECHNIQUE FOR ERROR FUNCTION AND THEIR APPLICATIONS

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Abstract. In this paper, integral transforms of the error function were used to evaluate improper integrals. We will use the complex Emad-Elaf (complex EE transform) and demonstrate its ability to evaluate improper integrals containing error functions. The results obtained are much simpler than those obtained using previous transforms and techniques, as they are illustrated via multiple examples within the work. Some properties of the complex EE transform that were needed in our work and were not previously proven for the complex EE transform have been demonstrated and proven within the body of the work.

1. Introduction

The importance of diffusion processes and probability theory, as well as their relationship to the error function errf(x) and the complementary error function errfc(x) [19, 20], makes these functions immensely significant in various scientific fields, including physics [8], thermodynamics, and materials science. The error function is outstanding in finding the exact analytical solution of partial differential equations, particularly in the heat conduction (or diffusion) equation that involves temperature distribution, heat flow, or mass transfer

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within semi-infinite mediums over time [1]. For those reasons, error and complementary error functions have become such a rich field for mathematicians to develop methods and techniques, including novel integral transforms [2, 7, 9, 16, 18], to simplify the process of solving the complex differential and integral equations [12, 13, 14, 15, 17] that govern these applied science models.

Based on the above, many researchers have proposed integral transforms that result from modifications of Laplace and Fourier transforms to enhance real-world applications with more precise results and easier, faster methods than previous ones. Examples of these transforms include SEE, Complex SEE, Rangaig, AL-Tememe, Emad-Falih, and many others [1, 2, 5–18].

The proposition of the complex EE transform in solving the error function is considered a better solving technique than other most common integral transforms, such as the Laplace transform [3], due to its generalization that came from its kernel, which represents a more general than Laplace, where the Laplace kernel is a special case of the complex EE transform; if and s is a negative real parameter then complex EE transform became the Laplace transform. However, the Fourier transform is considered more general than the complex EE transform from the perspective of the period of the transform, but in relation to the kernel, the complex EE transform is more general than the Fourier transform [4].

2. Some useful properties of the complex EE transform technique

The complex EE technique is defined as:

$$E_\alpha^c\{f(t)\} = F^c(is^\alpha) = \int_0^\infty f(t)e^{-is^\alpha t}dt,$$

where s is a complex parameter, $\mathrm{Im}(s^\alpha) < 0$, $\alpha \in \mathbb{Z}$ and $i \in \mathbb{C}$ $(i^2 = -1)$.

2.1 Linearity Property [5]. The complex EE technique of two functions $f_1(t)$ and $f_2(t)$ is:

$$E_\alpha^c\{\beta f_1(t) \pm \gamma f_2(t)\} = \beta E_\alpha^c\{f_1(t)\} \pm \gamma E_\alpha^c\{f_2(t)\},$$

where β and γ are any constants.

2.2 Change of Scale Property [5]. If the complex EE transform of function $f(t)$ is:

$$E_\alpha^c\{f(t)\} = F^c(is^\alpha) = \int_0^\infty f(t).e^{-is^\alpha t}dt.$$

So, the complex EE technique of function $f(\lambda t)$ is given by

$$E_\alpha^c\{f(\lambda t)\} = \int_0^\infty f(\lambda t)e^{-is^\alpha t}dt.$$

Now, put $N = \lambda t$, then $t = \frac{N}{\lambda}$ and $\lambda dt = dN$ and $dt = \frac{dN}{\lambda}$,

$$E_\alpha^c\{f(\lambda t)\} = \frac{1}{\lambda}\int_0^\infty f(N)e^{-i\frac{s^\alpha N}{\lambda}}dN$$

and

$$E_\alpha^c\{f(\lambda t)\} = \frac{1}{\lambda}F^c\left(\frac{is^\alpha}{\lambda}\right).$$

Thus, if $E_\alpha^c\{f(t)\} = F^c(is^\alpha)$, then

$$E_\alpha^c\{f(\lambda t)\} = \frac{1}{\lambda}F^c\left(\frac{is^\alpha}{\lambda}\right).$$

2.3 Shifting Property [5].

$$\begin{aligned} E_\alpha^c\{e^{\beta t}f(t)\} &= \int_0^\infty e^{\beta t}f(t)e^{-is^\alpha t}dt \\ &= \int_0^\infty f(t)e^{-i(i\beta+s^\alpha)t}dt \\ &= F^c(i\beta + s^\alpha). \end{aligned}$$

Theorem 2.1. ([5]) **The Complex EE Technique of The Derivatives of The Function of** $f(t)$**:** *The Complex EE Technique of The Derivatives of The Function of $f(t)$ are the followings:*

(i) $E_\alpha^c\{f'(t)\} = -f(0) + is^\alpha F^c(is^\alpha)$.
(ii) $E_\alpha^c\{f''(t)\} = -f'(0) - is^\alpha f(0) - s^{2\alpha}F^c(is^\alpha)$.
(iii) $E_\alpha^c\{f'''(t)\} = -f''(0) - is^\alpha f'(0) - (is^\alpha)^2 f(0) + (is^\alpha)^3F^c(is^\alpha)$.

In general

$$\begin{aligned} E_\alpha^c\{f^{(m)}(t)\} = (is^\alpha)^m F^c(is^\alpha) - f^{(m-1)}(0) - is^\alpha f^{(m-2)}(0) \\ - (is^\alpha)^2 f^{(m-3)}(0) - \cdots - (is^\alpha)^{(m-2)}f'(0) \\ - (is^\alpha)^{(m-1)}f(0), \end{aligned}$$

where m is a positive integer number.

Theorem 2.2. The Complex EE Technique of Integral of a function $f(t)$*: If the complex EE transform technique of function $f(t)$ is $E_\alpha^c\{f(t)\} = F^c(s^\alpha)$, then*

$$E_\alpha^c\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{is^\alpha}F^c(is^\alpha) = \frac{-i}{s^\alpha}F^c(is^\alpha).$$

Proof. Assume that $G(t) = \int_0^t f(\tau)d\tau$ so $G'(t) = f(t)$ and $G(0) = 0$. From the applying the complex EE technique property of the derivative of a function, we obtain

$$\begin{aligned} E_\alpha^c\{G'(t)\} &= \int_0^\infty G'(t)e^{-is^\alpha t}dt = is^\alpha F^c(is^\alpha) - G(0) \\ &= is^\alpha E_\alpha^c\{G(t)\} \end{aligned}$$

and

$$E_\alpha^c\{G(t)\} = \frac{1}{is^\alpha}E_\alpha^c\{G'(t)\} = \frac{1}{is^\alpha}E_\alpha^c\{f(t)\} = \frac{1}{is^\alpha}F^c(is^\alpha) = -\frac{i}{s^\alpha}F^c(is^\alpha).$$

Then,

$$E_\alpha^c\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{-i}{s^\alpha}F^c(is^\alpha).$$

□

Theorem 2.3. The Complex EE **Technique** $(tf(t))$: *If the complex EE technique of function* $f(t)$ *is*

$$E_\alpha^c\{f(t)\} = F^c(is^\alpha) = \int_0^\infty f(t)e^{-is^\alpha t}dt\,.$$

Then,

$$E_\alpha^c\{tf(t)\} = -\left[\frac{1}{i\alpha s^{\alpha-1}}\frac{d}{ds}F(is^\alpha)\right] = \frac{i}{\alpha s^{\alpha-1}}\frac{d}{ds}F(is^\alpha)\,.$$

Proof. By applying the EE technique, we obtain

$$E_\alpha^c\{f(t)\} = \int_0^\infty f(t)e^{-is^\alpha t}dt = F^c(is^\alpha)\,,$$

implies that

$$\begin{aligned} \frac{d}{ds}F^c(is^\alpha) &= \frac{d}{ds}\left[\int_0^\infty f(t)e^{-is^\alpha t}dt\right] \\ &= \int_0^\infty f(t)\frac{d}{ds}e^{-is^\alpha t}dt \\ &= \int_0^\infty f(t)e^{-is^\alpha t}(-it\cdot\alpha s^{\alpha-1})dt \\ &= -i\alpha s^{\alpha-1}\int_0^\infty tf(t)e^{-is^\alpha t}dt \\ &= -i\alpha s^{\alpha-1}E_\alpha^c\{tf(t)\}. \end{aligned}$$

Then,

$$E_\alpha^c\{tf(t)\} = \frac{\frac{-d}{ds}F^c(is^\alpha)}{i\alpha s^{\alpha-1}} = -\frac{1}{i\alpha s^{\alpha-1}}\frac{d}{ds}F^c(is^\alpha)$$
$$= \frac{i}{\alpha s^{\alpha-1}}\frac{d}{ds}F^c(is^\alpha).$$

□

Theorem 2.4. ([5]) **Convolution Property for Complex** EE **Transform**: *If* $E_\alpha^c\{f_1(t)\} = F_1^c(is^\alpha)$ *and* $E_\alpha^c\{f_2(t)\} = F_2^c(is^\alpha)$*, then*

$$E_\alpha^c\{f_1(t) * f_2(t)\} = E_\alpha^c\{f_1(t)\} * E_\alpha^c\{f_2(t)\}$$
$$= F_1^c(is^\alpha)\cdot F_2^c(is^\alpha).$$

3. The complex EE integral technique for elementary functions

We can see the followings [5].

(1) $E_\alpha^c\{k\} = \frac{-ki}{s^\alpha}$, where k is any constant.

(2) $E_\alpha^c\{t^n\} = \frac{n!}{(is)^{(n+1)\alpha}}$, n is a +ive integer.

$$E_\alpha^c\{t^n\} = \frac{\Gamma(n+1)}{(is)^{(n+1)\alpha}} \text{ , } n > -1, \text{ wher } \Gamma(\cdot) \text{ is a gamma function.}$$

(3) $E_\alpha^c\{e^{\beta t}\} = -\left[\frac{\beta}{\beta^2+s^{2\alpha}} + i\frac{s^\alpha}{\beta^2+s^{2\alpha}}\right], \beta \in \mathbb{R}$.

(4) $E_\alpha^c\{\sin(\beta t)\} = \frac{-\beta}{s^{2\alpha}-\beta^2}, \beta \in \mathbb{R}$.

(5) $E_\alpha^c\{\cos(\beta t)\} = \frac{-is^\alpha}{s^{2\alpha}-\beta^2}, \beta \in \mathbb{R}$.

(6) $E_\alpha^c\{\sinh(\beta t)\} = \frac{-\beta}{s^{2\alpha}+\beta^2}, \beta \in \mathbb{R}$.

(7) $E_\alpha^c\{\cosh(\beta t)\} = \frac{-is^\alpha}{s^{2\alpha}+\beta^2}, \beta \in \mathbb{R}$.

4. Several essential particulars of error and complementary error functions

We define error and Complementary error functions as follows ([19],[20]).

$$\mathrm{err}f(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}dx\,. \tag{4.1}$$

$$\mathrm{err}f_c(t) = \frac{2}{\sqrt{\pi}}\int_t^\infty e^{-x^2}dx\,. \tag{4.2}$$

The following properties are presented to several essential particulars of both the error and complementary error functions ([19],[20]).

(1) The Sum of Error and Complementary Error Function is Unity

$$\mathrm{err}f(t) + \mathrm{err}f_c(t) = 1\,.$$

(2) Error function is an odd function

$$\mathrm{err}f(-t) = -\mathrm{err}f(t)\,.$$

(3) The value of the error function at $t = 0$ is equal to 0, that is $\mathrm{err}f(0) = 0$.
(4) The value of the complementary error function at $t = 0$ is equal to 1, that is $\mathrm{err}f_c(0) = 1$.
(5) The domain of error function and complementary function is the open interval $(-\infty, \infty)$.
(6) $\mathrm{err}f(t) \to 1$ when $t \to \infty$.
(7) $\mathrm{err}f_c(t) \to 0$ when $t \to \infty$.

5. The complex EE technique of error function

By equation (4.1), and $x \geq 0$, we have

$$\begin{aligned}\mathrm{err}f\left(x^{\frac{1}{2}}\right) = \mathrm{err}f(\sqrt{x}) &= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{x}} e^{-t^2} dt \\ &= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{x}} \left[1 - \frac{t^2}{1!} + \frac{t^4}{2!} - \frac{t^6}{3!} + \frac{t^8}{4!} - \cdots\right] dt.\end{aligned}$$

After integrating each term of the series e^{-x^2}, we get

$$\frac{2}{\sqrt{\pi}} \left[t - \frac{t^3}{3 \cdot 1!} + \frac{t^5}{5 \cdot 2!} - \frac{t^7}{7 \cdot 3!} + \frac{t^9}{9 \cdot 4!} - \cdots\right]_0^{\sqrt{x}}. \tag{5.1}$$

By employing the complex EE technique on both sides of above equation, we get

$$E_\alpha^c\{\mathrm{err}f(\sqrt{x})\} = \frac{2}{\sqrt{\pi}} E_\alpha^c \left\{x^{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{3 \cdot 1!} + \frac{x^{\frac{5}{2}}}{5 \cdot 2!} - \frac{x^{\frac{7}{2}}}{7 \cdot 3!} + \frac{x^{\frac{9}{2}}}{9 \cdot 4!} - \cdots\right\}, \tag{5.2}$$

where $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

After substituting the functions with equivalents for this transform of the above equation and performing simple algebraic calculations and the use of expansion formulas, we obtain

$$E_\alpha^c\left\{\mathrm{err}f\left(x^{\frac{1}{2}}\right)\right\} = E_\alpha^c\{\mathrm{err}f(\sqrt{x})\} = \frac{-1}{s^\alpha\sqrt{s^\alpha + 1}}\,. \tag{5.3}$$

6. The complex EE technique for complementary error function

We have,

$$\mathrm{err}f(\sqrt{x}) + \mathrm{err}f_c(\sqrt{x}) = 1\,,$$

that is,

$$\mathrm{err}f_c(\sqrt{x}) = 1 - \mathrm{erf}(\sqrt{x})\,. \tag{6.1}$$

Using EE technique on both sides of above equation, we get:

$$E^c_\alpha\{\mathrm{err}f_c(\sqrt{x})\} = E^c_\alpha\{1\} - E^c_\alpha\{\mathrm{err}f(\sqrt{x})\}\,, \tag{6.2}$$

it implies that

$$\begin{aligned} E^c_\alpha\{\mathrm{err}f_c(\sqrt{x})\} &= -\frac{i}{s^\alpha} - \frac{-1}{s^\alpha\sqrt{s^\alpha+1}} = \frac{-i}{s^\alpha} + \frac{1}{s^\alpha\sqrt{s^\alpha+1}} \\ &= \frac{-i\sqrt{s^\alpha+1}+1}{s^\alpha\sqrt{s^\alpha+1}} = \frac{1-i\sqrt{s^\alpha+1}}{s^\alpha(\sqrt{s^\alpha+1})}. \end{aligned}$$

Then,

$$E^\alpha_c\{\mathrm{err}f_c(\sqrt{x})\} = \frac{1-i\sqrt{s^\alpha+1}}{s^\alpha(\sqrt{s^\alpha+1})}\,. \tag{6.3}$$

7. Applications

In this section of paper, we introduce some important applications to explain the effect of the complex EE transform technique on error func-tion to calculate the improper integral, which contain error function.

Application 7.1. Calculate the improper integral for

$$I = \int_{t=0}^{\infty} e^{-t}\mathrm{err}f(\sqrt{t})dt\,.$$

We have

$$E^c_\alpha\{\mathrm{err}f(\sqrt{t})\} = \int_0^\infty \mathrm{err}f(\sqrt{t})e^{-is^\alpha t}dt$$

with

$$E^c_\alpha\{\mathrm{err}f(\sqrt{t})\} = \frac{-1}{s^\alpha(\sqrt{s^\alpha+1})}. \tag{7.1}$$

Taking $s^\alpha = 1$ in the equation (7.1), using the complex EE technique, we get

$$\int_0^\infty \mathrm{err}f(\sqrt{t})e^{-it}dt = \frac{-1}{\sqrt{2}}\,.$$

Then,

$$\int_0^\infty \mathrm{err}f(\sqrt{t})e^{-it}dt = -\frac{1}{\sqrt{2}}\,.$$

Application 7.2. Calculate the improper integral for

$$I = \int_{t=0}^{\infty} te^{-2t}\mathrm{err}f(\sqrt{t})dt\,.$$

We have

$$E^c_\alpha\{t\ \mathrm{err}f(\sqrt{t})\} = \int_0^{\infty} t\ \mathrm{err}f(\sqrt{t})e^{-is^\alpha t}dt\,, \tag{7.2}$$

it implies that

$$E^c_\alpha\{t\ \mathrm{err}f(\sqrt{t})\} = -\left[\frac{1}{i\alpha s^{\alpha-1}}\frac{dF^c(is^\alpha)}{ds}\right] = \frac{i}{\alpha s^{\alpha-1}}\frac{dF^c(is^\alpha)}{ds}$$

and

$$E^c_\alpha\{\mathrm{err}f(\sqrt{t})\} = \frac{-1}{s^\alpha\sqrt{s^\alpha+1}}\,.$$

Hence we have

$$\begin{aligned} E^c_\alpha\{t\ \mathrm{err}f(\sqrt{t})\} &= \left[\frac{i}{\alpha s^{\alpha-1}}\frac{d}{ds}\right]\cdot\frac{-1}{s^\alpha\sqrt{s^\alpha+1}} \\ &= \frac{-i}{\alpha s^{\alpha-1}}\cdot\frac{1}{s^\alpha\sqrt{s^\alpha+1}}\frac{d}{ds} \\ &= \frac{-i}{\alpha}\left[\frac{d}{ds}\frac{1}{s^{2\alpha-1}\sqrt{s^\alpha+1}}\right]\,. \end{aligned}$$

After some simple calculations, we get

$$E^c_\alpha\{t\ \mathrm{err}f(\sqrt{t})\} = \frac{-1}{i\alpha}\left[\frac{\alpha s^\alpha}{2(s^{2\alpha})(s^\alpha+1)^{\frac{3}{2}}} + \frac{(2\alpha-1)}{s^{2\alpha}\sqrt{s^\alpha+1}}\right]\,. \tag{7.3}$$

From Eq. (7.2) and Eq. (7.3), we obtain

$$\begin{aligned} \int_{t=0}^{\infty} t\ \mathrm{err}f(\sqrt{t})e^{-is^\alpha t}dt &= \frac{-1}{i\alpha}\left[\frac{\alpha s^\alpha}{2(s^{2\alpha})(s^\alpha+1)^{\frac{3}{2}}} + \frac{(2\alpha-1)}{s^{2\alpha}\sqrt{s^\alpha+1}}\right] \\ &= \left[\frac{-s^\alpha}{2i(s^{2\alpha})(s^\alpha+1)^{\frac{3}{2}}} - \frac{(2\alpha-1)}{i\alpha s^{2\alpha}\sqrt{s^\alpha+1}}\right]\,. \end{aligned}$$

Now, if $s^\alpha = 2$ is taking in the above equation, we get

$$\int_{t=0}^{\infty} t\ \mathrm{err}f(\sqrt{t})e^{-is^{\alpha}t}dt = \left[\frac{-2}{2i(4)(2+1)^{\frac{3}{2}}} - \frac{(2\alpha-1)}{4i\alpha\sqrt{3}}\right]$$
$$= \left[\frac{-1}{4i(3)^{\frac{3}{2}}} - \frac{(2\alpha-1)}{4i\alpha\sqrt{3}}\right]$$
$$= \left[\frac{i}{12\sqrt{3}} + i\frac{(2\alpha-1)}{4\sqrt{3}\alpha}\right].$$

Application 7.3. Find the improper integral for

$$I = \int_0^{\infty} e^{-t}\left[\int_0^t \mathrm{err}f(\sqrt{x})dx\right]dt\,.$$

We have,

$$E_{\alpha}^{c}\left\{\int_0^t \mathrm{err}f(\sqrt{x})dx\right\} = \int_0^{\infty} e^{-is^{\alpha}t}\left[\int_0^t \mathrm{err}f(\sqrt{x})dx\right]dt \tag{7.4}$$

and

$$E_{\alpha}^{c}\{\mathrm{err}f(\sqrt{t})\} = \frac{-1}{s^{\alpha}(\sqrt{s^{\alpha}+1})}\,.$$

By applying the property of E technique of integral of a function, we obtain

$$E_{\alpha}^{c}\left\{\int_0^t \mathrm{err}f(\sqrt{x})dx\right\} = \frac{-i}{s^{\alpha}}F^{c}(is^{\alpha}) = \frac{-i}{s^{\alpha}}\cdot\frac{-1}{s^{\alpha}(\sqrt{s^{\alpha}+1})}$$
$$= \frac{i}{s^{2\alpha}(\sqrt{s^{\alpha}+1})}. \tag{7.5}$$

Now, Via equations (7.4) and (7.5), we have

$$\int_{t=0}^{\infty} e^{-is^{\alpha}t}\left[\int_0^t \mathrm{err}f(\sqrt{x})dx\right]dt = \frac{i}{s^{2\alpha}\sqrt{s^{\alpha}+1}}\,.$$

Taking $s^{\alpha}=1$, in the above equation, we get:

$$I = \int_0^{\infty} e^{-t}\left[\int_0^t \mathrm{err}f(\sqrt{x})dx\right]dt = \frac{i}{(1)\sqrt{1+1}} = \frac{i}{\sqrt{2}}\,.$$

Application 7.4. Calculate the following improper integral:

$$I = \int_{t=0}^{\infty} e^{-2t}\left[\frac{d}{dt}\mathrm{errf}(2\sqrt{t})\right]dt.$$

We have

$$E_{\alpha}^{c}\{\mathrm{errf}(\sqrt{t})\} = \frac{-1}{s^{\alpha}\sqrt{s^{\alpha}}+1}.$$

By changing the property of EE technique of derivative, we get

$$E_\alpha^c\{\operatorname{errf}(2\sqrt{t})\} = \frac{-2}{s^\alpha\sqrt{s^\alpha}+4}.$$

By applying the property of derivative of EE technique, we get

$$E_\alpha^c\left\{\frac{d}{dt}\operatorname{errf}(2\sqrt{t})\right\} = \frac{1}{4}\cdot\frac{-2i}{s^\alpha\sqrt{s^\alpha}+4}.$$

From the definition of the complex EE technique, we get

$$\begin{aligned} E_\alpha^c\left\{\frac{d}{dt}\operatorname{errf}(2\sqrt{t})\right\} &= \int_0^\infty e^{-is^\alpha t}\left(\frac{d}{dt}\operatorname{errf}(2\sqrt{t})\right)dt \\ &= \frac{-i}{2s^\alpha\sqrt{s^\alpha}+4}. \end{aligned}$$

Now taking $s^\alpha = 2$ in the above equation, we obtain:

$$\int_0^\infty e^{-is^\alpha t}\left[\frac{d}{dt}\operatorname{errf}(2\sqrt{t})\right]dt = \frac{-i}{4\sqrt{6}}.$$

Application 7.5. Find the following improper integral

$$I = \int_{t=0}^\infty e^{-5t}[\operatorname{errf}(\sqrt{t}) * \operatorname{errf}(\sqrt{t})]dt.$$

Using the convolution property of the complex EE technique, we get

$$E_\alpha^c\{\operatorname{errf}(\sqrt{t}) * \operatorname{errf}(\sqrt{t})\} = F^c(is^\alpha)$$

and

$$F^c(is^\alpha) = \frac{-1}{s^\alpha\sqrt{s^\alpha+1}} * \frac{-1}{s^\alpha\sqrt{s^\alpha+1}} = \frac{1}{s^{2\alpha}(s^\alpha+1)}.$$

Now, from the definition of complex EE integral transforma, we get

$$F^c(is^\alpha) = \frac{1}{s^{2\alpha}(s^\alpha+1)}.$$

If $s^\alpha = 5$, then $I = \frac{1}{25(5+1)} = \frac{1}{150}$.

8. Conclusions

The importance of the error function errf(x) leads to the suggestion of many solutions. This work applied the complex EE integral transfor-mation as a valued tool in solving the error function included in the im-proper integrals in which their solution is quite complicated and prone to errors; however, the application of the complex EE transform, which is proven in many applications in this work on improper error integrals, proves its efficiency in finding the exact solution to these applications without the lengthy, full-scale calculations that can be associated with the more classical methods of solution.

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