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APPROXIMATELY QUADRATIC MAPPINGS IN NON-ARCHIMEDEAN FUZZY NORMED SPACES

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Abstract. In this paper, we present the stability results and alternative stability results concerning the quadratic functional equation in non-Archimedean fuzzy normed spaces. As corollaries, we obtain the stability results of the quadratic functional equation in non-Archimedean normed spaces.

1. Introduction

A classical problem which was raised by Ulam [33] in the theory of functional equations is the following: "When is true that a function, which approximately satisfies a functional equation must be close to an exact solution of the equation?" If the problem accepts a unique solution, we say the equation is stable. In 1941, Hyers [14] considered the case of approximate additive mappings satisfying the Cauchy difference controlled by a positive constant in Banach spaces. Bourgin [5] and Aoki [1] treated this problem for additive mappings controlled by unbounded functions. In [30], Rassias provided a generalization of Hyers' theorem for linear mappings which allows the Cauchy difference to be unbounded. Găvruta [10] then generalized these theorems

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for additive mappings controlled by the unbounded Cauchy difference with regular conditions. Subsequently, the stability problem of various functional equations has been studied by a number of authors [12, 20, 21, 23]. Taking into consideration a lot of influence of Ulam and Hyers, the stability of functional equation is called by Hyers–Ulam stability. Hyers–Ulam stability of the quadratic functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \tag{1.1}$$

was first proved by Skof for mapping $f : E_1 \to E_2$, where E_1 is a normed space and E_2 is a Banach space [32]. In the papers [7, 8], Czerwik proved the Hyers–Ulam stability of the quadratic functional equation (1.1).

In particular, Kannappan [16] introduced the following functional equation

$$f(x+y)+f(y+z)+f(z+x)=f(x+y+z)+f(x)+f(y)+f(z) \tag{1.2}$$

and proved that a function on a real vector space is a solution of (1.2) if and only if there exist a symmetric biadditive function B and an additive function A such that $f(x)=B(x,x)+A(x)$.

In [2], the authors proved the generalized Hyers–Ulam stability of the functional equation

$$f(x-y)+f(y-z)+f(z-x)+f(x+y+z)=3[f(x)+f(y)+f(z)],$$

which is equivalent to the quadratic equation (1.1).

Recently, Kim and Shin [22] proved the general solution of the following quadratic functional equation

$$\begin{aligned} &f(x+ny)+f(y+nz)+f(z+nx) \\ &= nf(x+y+z)+(n^2-n+1)[f(x)+f(y)+f(z)] \end{aligned} \tag{1.3}$$

and investigated the Hyers–Ulam stability of the equation (1.3) in some spaces, where n $(\neq 0, \pm 1)$ is an integer.

In 1897, Hensel [13] discovered the p-adic numbers as a number theoretical analogues of power seires in complex analysis. The important examples of non-Archimedean spaces are p-adic numbers which do not satisfy the Archimedean property. During the last three decades the theory of non-Archimedean spaces has gained the interest of physicists for their research, in particular the problems that emerge in quantum physics, p-adic strings and superstrings [19].

Katsaras [17] introduced the concept of a fuzzy norm on a linear space in 1984, in the same year Wu and Fang [35] introduced a notion of fuzzy normed space to give a generalization of the Kolmogoroff normalized theorem for fuzzy topological linear spaces. In 1992, Felbin [9] introduced an alternative definition of a fuzzy norm on a linear space with an associated metric of Kaleva and Seikkala type [15]. Xiao and Zhu [34] found the lonear topological structures of fuzzy normed spaces. In 1994, Cheng and Mordeson introduced a definition of a fuzzy norm on a linear space in such a way that the corresponding induced fuzzy metric is of Kramosil and Michalek type [28]. In 2003, Bag and Samanta [3] modified the definition of Cheng and Mordeson [6] by removing a regular condition. Recently many various results have been investigated in this topic (see [24, 25, 26, 27] and references therein).

In this paper, we study the Hyers–Ulam stability and alternative the Hyers–Ulam stability for the functional equation (1.3) in the setting of non-Archimedean fuzzy normed spaces. As corollaries, we obtain the stability results and alternative stability results of the quadratic functional equation (1.3) in non-Archimedean normed spaces.

2. Preliminaries

In this section we recall some notations and definitions of a non-Archimedean fuzzy normed spaces

Definition 2.1. Let $\mathbb{K}$ be a field. A non-Archimedean absolute value on $\mathbb{K}$ is a functional $|\cdot| : \mathbb{K} \to [0, \infty)$ such that for any $r, s \in \mathbb{K}$ we have

(1) $|r| = 0$ if and only if $r = 0$;
(2) $|rs| = |r||s|$;
(3) $|r + s| \leq \max\{|r|, |s|\}$.

The condition (3) is called the strong triangle inequality. Clearly, $|1| = |-1| = 1$ and $n \leq 1$ for all $n \in \mathbb{N}$. We always assume in addition that $|\cdot|$ is non trivial, *i.e.*, that there exists an $r_0 \in \mathbb{K}$ such that $|r_0| \neq 0, 1$.

Definition 2.2. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$;
(ii) $\|rx\| = |r|\|x\|$;
(iii) the strong triangle inequality (ultrametric); namely,
$\|x + y\| \leq \max\{\|x\|, \|y\|\} \quad (x, y \in X)$.

Then $(X, \|\cdot\|)$ is called a non-Archimedean space.

Example 2.3. Let p be a prime number. For any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = \frac{a}{b}p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x, y) = |x - y|_p$ is denoted by $\mathbb{Q}_p$ which is called the p-adic number field. In fact, $\mathbb{Q}_p$ is the set of all formal series $x = \sum_{k \geq n_x}^{\infty} a_k p^k|_p = p^{-n_x}$, where $|a_k| \leq p - 1$ are integers. The addition and multiplication between any two elements of $\mathbb{Q}_p$ are defined naturally. The norm $|\sum_{k\geq n_x}^{infty} a_k p^k|_p| = p^{-n_x}$ is a non-Archimedean norm on $\mathbb{Q}_p$ and it makes $\mathbb{Q}_p$ a locally compact field (see [11, 29]).

Definition 2.4. Let X be a linear space over a non-Archimedean field $\mathbb{K}$. A function $N : X \times \mathbb{R} \to [0, 1]$ is said to be a non-Archimedean funzzy norm on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$:

(N1) $N(x, c) = 0$ for all $c \leq 0$;
(N2) $x = 0$ if and only if $N(x, c) = 1$ for all $c > 0$;
(N3) $N(cx, t) = N(x, \frac{t}{|c|})$;
(N4) $N(x + y, \max\{s, t\}) \geq \min\{N(x, s), N(y, t)\}$;
(N5) $\lim_{t\to\infty} N(x, t) = 1$.

A non-Archimedean fuzzy normed space is a pair (X, N), where X is a linear space and N is a non-Archimedean fuzzy norm on X. If (N4) holds then so is

$$N(x + y, t) \geq \min\{N(x, t), N(y, t)\}$$

for all $x, y \in X, t \in \mathbb{R}$.

Example 2.5. Let $(X, \|\cdot\|)$ be a non-Archimedean normed space. For all $x \in X$, consider

$$N(x, t) = \begin{cases} \dfrac{t}{t + \|x\|}, & \text{if} \quad t > 0, \\ 0, & \text{if} \quad t \leq 0. \end{cases}$$

Then (X, N) is a non-Archimedean fuzzy normed space.

Example 2.6. Let $(X, \|\cdot\|)$ be a non-Archimedean normed space. For all $x \in X$, consider

$$N(x, t) = \begin{cases} 0, & \text{if} \quad t \leq \|x\|, \\ 1, & \text{if} \quad t > \|x\|. \end{cases}$$

Then (X, N) is a non-Archimedean fuzzy normed space.

Definition 2.7. Let (X, N) be a non-Archimedean fuzzy normed space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x \in X$, such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In that case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $\lim_{n\to\infty} x_n = x$.

A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if $\lim_{n\to\infty} N(x_{n+p} - x_n, t) = 1$ for all $t > 0$ and $p = 1, 2, 3, \cdots$. Due to the fact

$$N(x_{n+p} - x_n, t) \geq \min\{N(x_{n+p} - x_{n+p-1}, t), \cdots, N(x_{n+1} - x_n, t)\}$$

the sequence $\{x_n\}$ is Cauchy if for each $\varepsilon > 0$ and each $t > 0$ there exists n_0 such that for all $n \geq n_0$ we have $N(x_{n+1} - x_n, t) > 1 - \varepsilon$. We will frequently use this criterion in this paper. It is easy to show that every convergent sequence in a (non-Archimedean) fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the (non-Archimedean) fuzzy normed space is caled a (non-Archimedean) fuzzy Banach space.

3. Hyers-Ulam stability of (1.3) in non-Archimedean fuzzy normed spaces

In this section, we investigate the Hyers–Ulam stability for functional equation (1.3) in non-Archimedean fuzzy normed spaces. Throughout this paper, we will assume that $\mathbb{K}$ is a non-Archimedean field, X is a vector space over $\mathbb{K}$, (Y, N) is a non-Archimedean fuzzy Banach space over $\mathbb{K}$ and (Z, N') is a fuzzy normed space.

For the sake of convenience, given mapping $f : X \to Y$, we introduce a difference operator Df as follows :

$$\begin{aligned} Df(x,y,z) &= f(x+ny) + f(y+nz) + f(z+nx) \\ &\quad -nf(x+y+z) - (n^2-n+1)[f(x)+f(y)+f(z)] \end{aligned}$$

for all $x, y, z \in X$, where $n(\neq 0, \pm 1)$ is a fixed integer.

We introduce the following lemma which was proved in [22].

Lemma 3.1. *Let V and W be real vector spaces. If a mapping $f : V \to W$ satisfies the functional equation* (1.3)*, then f is a quadratic functional equation.*

We present a main theorem, which concerns the Hyers-Ulam stability of a quadratic functional equation in non-Archimedean fuzzy normed spaces.

Theorem 3.2. *Let $\alpha > |n|^2$ be fixed real number and $\phi : X^3 \to Z$ be a mapping*

$$N'(\phi(n^{-1}x, n^{-1}y, n^{-1}z), t) \geq N'(\phi(x, y, z), \alpha t) \tag{3.1}$$

for all $x, y, z \in X$ and all $t > 0$. If $f : X \to Y$ with $f(0) = 0$ is a mapping satisfying

$$N(Df(x, y, z), t) \geq N'(\phi(x, y, z), t) \tag{3.2}$$

for all $x, y, z \in X$ and $t > 0$, then there exists a unique quadratic mapping $Q : X \to Y$, such that

$$N(f(x) - Q(x), t) \geq N'(\phi(x, 0, 0), \alpha t) \tag{3.3}$$

for all $x \in X$ *and* $t > 0$.

Proof. Letting y, z by 0 in (3.2), respectively, we get

$$N(f(nx) - n^2 f(x), t) \geq N'(\phi(x, 0, 0), t) \tag{3.4}$$

for all $x \in X$ and $t > 0$. Replacing x by $n^{-(m+1)}x$ in (3.4) and using inequality (3.1), we obtain

$$\begin{aligned} N(f(n^{-m}x) - n^2 f(n^{-m-1}x), t) &\geq N'(\phi(n^{-m-1}x, 0, 0), t) \\ &\geq N'(\phi(x, 0, 0), \alpha^{m+1}t) \end{aligned}$$

for all $x \in X$ and $t > 0$, $m \in \mathbb{N}$. Thus it follows that

$$\begin{aligned} &N(n^{2m} f(n^{-m}x) - n^{2m+2} f(n^{-m-1}x), t) \\ &= N\Big(f(n^{-m}x) - n^2 f(n^{-m-1}x), \frac{1}{|n|^{2m}} t\Big) \\ &\geq N'\Big(\phi(x, 0, 0), \frac{\alpha^{m+1} t}{|n|^{2m}}\Big). \end{aligned}$$

According to the fact $\lim_{m\to\infty} N'(\phi(x, 0, 0), \frac{\alpha^{m+1}}{|n|^{2m}}) = 1$, above inequality shows that $\{n^{2m} f(n^{-m}x)\}$ is a Cauchy sequence in the non-Archimedean fuzzy Banach space (Y, N). Thus, we may define a mapping $Q : X \to Y$ as

$$Q(x) := \lim_{m\to\infty} n^{2m} f(n^{-m}x),$$

that is, $\lim_{m\to\infty} N(n^{2m} f(n^{-m}x) - Q(x), t) = 1 \quad (x \in X)$. For each $m \geq 1$, $x \in X$ and $t > 0$,

$$\begin{aligned} N(f(x) - n^{2m} f(n^{-m}x), t) &= N\Big(\sum_{k=0}^{m-1} m^{2k} f(m^{-k}x) - m^{2k+2} f(m^{-k-1}x), t\Big) \\ &\geq \min \bigcup_{k=0}^{m-1} \{N(n^{2k} f(n^{-k}x) - n^{2k+2} f(n^{-k-1}x), t)\} \\ &= N'(\phi(x, 0, 0), \alpha t). \end{aligned}$$

We conclude the estimation (3.3) of f by Q holds for all $x \in X$ and $t > 0$.

Now we claim that the mapping Q is quadratic mapping. Setting (x, y, z) $:= (n^{-m}x, n^{-m}y, n^{-m}z)$ in (3.1), we see that

$$\begin{aligned} N(n^{2m} Df(n^{-m}x, n^{-m}y, n^{-m}z), t) &= N\left(Df(n^{-m}x, n^{-m}y, n^{-m}z), \frac{1}{|n|^{2m}} t\right) \\ &\geq N'\left(\phi(n^{-m}x, n^{-m}y, n^{-m}z), \frac{1}{|n|^{2m}} t\right) \\ &\geq N'\left(\phi(x, y, z), \frac{\alpha^{m+1}}{|n|^{2m}} t\right) \end{aligned}$$

for all $x, y, z \in X$ and $t > 0, m \in \mathbb{N}$. Thus it follows that

$$\begin{aligned} N(DQ(x,y,z),t) \geq\ & \min\{N(Q(x+ny) - n^{2m}f(n^{-m}(x+ny)),t), \\ & N(Q(y+nz) - n^{2m}f(n^{-m}(y+nz)),t), \\ & N(Q(z+nx) - n^{2m}f(n^{-m}(z+nx)),t), \\ & N(nQ(x+y+z) - n^{2m+1}f(n^{m}(x+y+z)),t), \\ & N((n^2-n+1)Q(x) - (n^2-n+1)n^{2m}f(n^{-m}x),t), \\ & N((n^2-n+1)Q(y) - (n^2-n+1)n^{2m}f(n^{-m}y),t), \\ & N((n^2-n+1)Q(z) - (n^2-n+1)n^{2m}f(n^{-m}z),t), \\ & N(n^{2m}Df(n^{-m}x, n^{-m}y, n^{-m}z),t)\} \\ \geq\ & \min\{N(Q(x+ny) - n^{2m}f(n^{-m}(x+ny)),t), \\ & N(Q(y+nz) - n^{2m}f(n^{-m}(y+nz)),t), \\ & N(Q(z+nx) - n^{2m}f(n^{-m}(z+nx)),t), \\ & N\left(Q(x+y+z) - n^{2m}f(n^{m}(x+y+z)), \frac{1}{n}t\right), \\ & N\left(Q(x) - n^{2m}f(n^{-m}x), \frac{1}{n^2-n+1}t\right), \\ & N\left(Q(y) - n^{2m}f(n^{-m}y), \frac{1}{n^2-n+1}t\right), \\ & N\left(Q(z) - n^{2m}f(n^{-m}z), \frac{1}{n^2-n+1}t\right), \\ & N'\left(\phi(x,y,z), \frac{\alpha^{m+1}}{|n|^{2m}}t\right)\right\} \end{aligned}$$

for all $x, y, z \in X$ and all positive integers m. Taking the limit as $m \to \infty$, one see that Q satisfies (1.3). By Lemma 3.1, Q is quadratic.

To show the uniqueness of Q, we assume that there exists a quadratic mapping $Q' : X \to Y$ which satisfies the inequality

$$N(f(x) - Q'(x), t) \geq N'(\phi(x,0,0), \alpha t)$$

for all $x \in X$ and $t > 0$. Then, since Q and Q' are quadratic mappings, we see from the equality $Q(n^{-m}x) = n^{-2m}Q(x)$ and $Q'(n^{-m}x) = n^{-2m}Q'(x)$ that

$$\begin{aligned} N(Q(x) - Q'(x), t) \geq\ & \min\{N(Q(x) - n^{2m}f(n^{-m}x), t), \\ & N(n^{2m}f(n^{-m}x) - Q'(x), t)\} \end{aligned}$$

$$\begin{aligned} &\geq \min\Big(N\Big(Q(n^{-m}x) - f(n^{-2m}x), \frac{t}{|n|^{2m}}\Big), \\ &\qquad N\Big(f(n^{-m}x) - Q'(n^{-m}x), \frac{t}{|n|^{m}}\Big)\Big) \\ &\geq N'\Big(\phi(x,0,0), \frac{\alpha^{m+1}}{|n|^{2m}}t\Big) \end{aligned}$$

for all $x \in X, t > 0, m \in \mathbb{N}$. By taking $m \to \infty$, we complete the proof. □

Corollary 3.3. *Let X be a linear space and $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose $f : X \to Y$ with $f(0) = 0$ satisfies the condition*

$$\|Df(x,y,z)\| \leq \phi(x,y,z) \quad (x,y,z \in X),$$

and $\phi : X^3 \to [0,\infty)$ is a mapping such that

$$\phi(n^{-1}x, n^{-1}y, n^{-1}z) \leq \alpha^{-1}\phi(x,y,z) \quad (x,y \in X),$$

where α is a positive real number with $\alpha > |n|^2$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$\|f(x) - Q(x)\| \leq \frac{1}{\alpha}\phi(x,0,0)$$

for all $x \in X$.

Proof. Let $Z = \mathbb{R}$ with the following fuzzy norm

$$N'(z,t) = \begin{cases} \dfrac{t}{t + \|z\|}, & \text{if} \quad t > 0, z \in Z, \\ 0, & \text{if} \quad t \leq 0, z \in Z, \end{cases}$$

and also define the following fuzzy norm

$$N(y,t) = \begin{cases} \dfrac{t}{t + \|y\|}, & \text{if} \quad t > 0, y \in Y, \\ 0, & \text{if} \quad t \leq 0, y \in Y. \end{cases}$$

By the Example 2.5, N' is a fuzzy norm of $\mathbb{R}$ and N is a non-Archimedean fuzzy norm on Y. We can easily check that all conditions of Theorem 3.2 are equipped. Using Theorem 3.2, we arrive at the desired conclusion. □

Corollary 3.4. *Let X be a linear space and $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose $f : X \to Y$ with $f(0) = 0$ satisfies the condition*

$$\|Df(x,y,z)\| \leq \theta(\|x\|^p + \|y\|^p + \|z\|^p) \quad (x,y,z \in X),$$

where $p \in (0,2)$ and $\theta > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$\|f(x) - Q(x)\| \leq \frac{\theta}{|n|^p}\|x\|^p$$

for all $x \in X$.

Proof. The proof follows immediately by taking $\phi : X^3 \to [0, \infty)$ is defined by

$$\phi(x, y, z) := \theta(\|x\|^p + \|y\|^p + \|z\|^p)$$

for all $x, y, z \in X$ and choosing $\alpha = |n|^{2p}$ in Corollary 3.3. □

Next, we are going to prove an alternative stability theorem of the functional equation (1.3) in non-Archimedean fuzzy normed spaces.

Theorem 3.5. *Let $\alpha > |n|^2$ be fixed real number and $\phi : X \times X \to Z$ be a mapping with,*

$$N'(\phi(nx, ny, nz), t) \geq N'(\phi(x, y, z), \alpha^{-1}t) \tag{3.5}$$

for all $x, y, z \in X$ and all $t > 0$. If $f : X \to Y$ with $f(0) = 0$ is a mapping satisfying

$$N(Df(x, y, z), t) \geq N'(\phi(x, y, z), t) \tag{3.6}$$

for all $x, y, z \in X$ and $t > 0$, then there exists a unique quadratic mapping $Q : X \to Y$, such that

$$N(f(x) - Q(x), t) \geq N'(\phi(x, 0, 0), |n|^2 t) \tag{3.7}$$

for all $x \in X$ and $t > 0$.

Proof. As the similar pattern of the proof of Theorem 3.2, we obtain the following inequality

$$N\Big(\frac{1}{n^{2m+2}} f(n^{m+1}x) - \frac{1}{n^{2m}} f(n^m x), t\Big) \geq N'(\phi(x, 0, 0), \frac{|n|^{2m}}{\alpha^m} t)$$

for all $x \in X$ and $t > 0$. Since $\lim_{m\to\infty} N'(\phi(x, 0, 0), \frac{|n|^{2m}}{\alpha^m} t) = 1$, above inequality shows that $\{n^{-2m} f(n^m x)\}$ is a Cauchy sequence in a non-Archimedean fuzzy Banach space (Y, N). Therefore, we may define a mapping $Q : X \to Y$ as

$$Q(x) := \lim_{m\to\infty} n^{-2m} f(n^m x),$$

that is, $\lim_{m\to\infty} N(n^{-2m} f(n^m x) - Q(x), t) = 1$ for all $x \in X, t > 0$. For each $n \geq 1, x \in X$ and $t > 0$,

$$\begin{aligned} N(f(x) - n^{-2m} f(n^m x), t) &= N\Big(\sum_{i=0}^{m-1} n^{-2i} f(n^i x) - n^{-2i-2} f(n^{i+1}x), t\Big) \\ &\geq \min \bigcup_{i=0}^{m-1} \{N(n^{-2i} f(n^i x) - n^{-2i-2} f(n^{i+1}x), t)\} \\ &= N(\phi(x, 0, 0), |n|^2 t). \end{aligned}$$

It follows that

$$\begin{aligned} N(f(x)-T(x),t) &\geq \min\{N(f(x)-n^{-2m}f(n^m x),t), \\ &\qquad N(n^{-2m}f(n^m x)-Q(x),t)\} \\ &\geq N(\phi(x,0,0),|n|^2 t). \end{aligned}$$

Thus the estimation (3.7) of f by Q holds for all $x \in X$ and $t > 0$. The rest of the proof is similar to the that of Theorem 3.2. □

Corollary 3.6. *Let X be a linear space and $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose $f : X \to Y$ with $f(0) = 0$ satisfies the condition*

$$\|Df(x,y,z)\| \leq \phi(x,y,z) \quad (x,y,z \in X),$$

and $\phi : X^3 \to [0,\infty)$ is a mapping such that

$$\phi(nx,ny,nz) \leq \alpha\phi(x,y,z) \quad (x,y,z \in X),$$

where α is a positive real number with $\alpha > |n|^2$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$\|f(x)-Q(x)\| \leq \frac{1}{|n|^2}\phi(x,0,0)$$

for all $x \in X$.

Corollary 3.7. *Let X be a linear space and $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose $f : X \to Y$ with $f(0) = 0$ satisfies the condition*

$$\|Df(x,y,z)\| \leq \theta(\|x\|^p + \|y\|^p + \|z\|^p) \quad (x,y,z \in X),$$

where $p \in (2,\infty)$ and $\theta > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$\|f(x)-Q(x)\| \leq \frac{\theta}{|n|^2}\|x\|^p$$

for all $x \in X$.

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