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NEW APPLICATIONS OF GENERALIZED HYPERGEOMETRIC FUNCTIONS AND LINEAR OPERATOR WITH HURWITZ-LERCH-ZETA FUNCTION

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Abstract. The Hadamard product is employed in this study to describe a new class of analytic functions that include linear operator. Some of the properties related to this new class are also investigated. In the second result of this paper, applications on hypergeometric functions that employ the popular hypergeometric functions with the operator are included as well as achievement of some new results.

1. Introduction

Let Σ signify analytic meromorphic functions' class $f(z)$ which is normalized by

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \tag{1.1}$$

in the punctured disk $U^* = \{z : 0 < |z| < 1\}$. Also, $S^*(\beta)$ and $k(\beta)$ are employed, the subclasses of Σ that contain all meromorphic functions, respectively, include starlike of order β and convex of order β in U^*, $0 \leq \beta$ (see the

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recent works [13] and [18]).
For functions $f_m(z)(m = 1; 2)$ given below

$$f_m(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,m} z^n, \tag{1.2}$$

the Hadamard product (or convolution) for $f_1(z)$ and $f_2(z)$ can be expressed as follows:

$$(f_1 * f_2) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^n. \tag{1.3}$$

Here, we the general Hurwitz-Lerch-Zeta function, is defined by the series (see [[11], [12]]):

$$\Phi(z, k, a) = \frac{1}{a^k} + \sum_{n=1}^{\infty} \frac{z^n}{(n+a)^k} \tag{1.4}$$

$\left(a \in \mathbb{C}\backslash\mathbb{Z}_0^-, \ \mathbb{Z}_0^- = \{0, -1, -2, ...\}; \ k \in \mathbb{C} \text{ when } \mathrm{z} \in \mathrm{U} = \mathrm{U}^* \cup \{0\}; \ \Re(k) > 1 \text{ when } z \in \partial U\right).$

Important special cases of the function $\Phi(z, k, a)$ include, for example, the Reimann zeta function $\zeta(k) = \Phi(1, k, 1)$, the Hurwitz zeta function $\zeta(k, a) = \Phi(1, k, a)$, the Lerch zeta function $l_k(\zeta) = \Phi\left(\exp^{2\pi i\xi}, k, 1\right), (\xi \in \mathbb{R}, \Re(k) > 1)$, the polylogarithm $L_k^i(z) = z\Phi(z, k, a)$ and so on. The expositions [[16], [17]] provide recent results on $\Phi(z, k, a)$. By employing the following normalized function, Ghanim ([4], and also [5] and [6]) has introduced the function $G_{k,a}$, defined by:

$$\begin{aligned} G_{k,a}(z) &= (1+a)^k \left[\Phi(z, k, a) - a^k + \frac{1}{z(1+a)^k}\right] \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^k z^n, \ z \in U^*. \end{aligned} \tag{1.5}$$

In agreement with the functions $G_{k,a}(z)$ and utilising the Hadamard product for $f(z) \in \Sigma$, a new linear operator $L_{k,a}f(z)$: $\Sigma \to \Sigma$ is defined through the following series:

$$L_{k,a}f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^k |a_n| z^n, \ z \in U^*. \tag{1.6}$$

From (1.6), it follows that

$$z\left(L_a^k f(z)\right)' = aL_a^k f(z) - (a+1) \ L_a^k f(z). \tag{1.7}$$

The Hadamard product or convolution of the functions f given by (1.6) with the function $L_{k,a}g$ and $L_{k,a}h$ given, respectively, by

$$L_{k,a}g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^{k} |b_n| z^n,\ z \in U^*,\ g(z) \in \Sigma \tag{1.8}$$

and

$$L_{k,a}h(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^{k} |c_n| z^n,\ z \in U^*,\ h(z) \in \Sigma \tag{1.9}$$

can be presented as follows:

$$L_{k,a}\left(f * g\right)(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^{k} |a_n b_n| z^n,\ \ z \in U^* \tag{1.10}$$

and

$$L_{k,a}\left(f * h\right)(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^{k} |a_n c_n| z^n,\ \ z \in U^*. \tag{1.11}$$

The new class $\Sigma_k^a(\lambda, A, B)$ of meromorphic functions is introduced by applying the subordination definition, which can be given as:

Definition 1.1. A function $f \in \Sigma$ with the form (1.1) is considered to be in the class $\Sigma_k^a(\lambda, A, B)$ if it can satisfy the following subordination property:

$$\lambda \frac{L_{k,a}\left(f * g\right)(z)}{L_{k,a}\left(f * h\right)(z)} \prec \lambda - \frac{(A-B)\,z}{1+Bz},\ \ z \in U^*, \tag{1.12}$$

where $-1 \leq B < A \leq 1$, $\lambda > 0$, with conditions $|b_n| \geq |c_n| \geq 0$ and $L_{k,a}\left(f * h\right)(z) \neq 0$.

As for the second result of this paper on applications involving hypergeometric functions, we need to utilize the well known Gaussian hypergeometric function needs to be used. We represent $\tilde{\phi}(\alpha, \beta; z)$ as the function's class, given by

$$\tilde{\phi}\left(\alpha, \beta; z\right) = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{(\alpha)_{n+1}}{(\beta)_{n+1}} z^n, \tag{1.13}$$

where $\beta \neq 0, -1, -2, ...,$ and $\alpha \in \mathbb{C} \setminus \{0\}$ and $(\lambda)n = \lambda(\lambda+1)_{n+1}$ signifies the Pochhammer symbol. It is clear that

$$\tilde{\phi}\left(\alpha, \beta; z\right) = \frac{1}{z} {}_2F_1\left(1, \alpha, \beta; z\right),$$

where

$$ {}_2F_1\left(b,\alpha,\beta;z\right)=\sum_{n=0}^{\infty}\frac{(b)_n\,(\alpha)_n}{(\beta)_n}\frac{z^n}{n!}. $$

The meromorphic functions with the generalized hypergeometric functions have been considered recently by several authors; see, for example [1], [2], [3], [7], [8], [9], [10], [14] and [15].

A new linear operator $L_{k,a}(\alpha,\beta)$ on Σ is defined, which corresponds to the functions $G_{k,a}(z)$ given in (1.5) and employing the Hadamard product for $f(z)\in\Sigma$, through the follow series:

$$ \begin{aligned} L_a^k\left(\alpha,\beta\right)f(z) &= \phi\left(\alpha,\beta;z\right)*G_{k,a}\left(z\right)\\ &= \frac{1}{z}+\sum_{n=1}^{\infty}\frac{(\alpha)_{n+1}}{(\beta)_{n+1}}\left(\frac{1+a}{n+a}\right)^k\left|a_n\right|z^n,\ z\in U^*. \end{aligned} \tag{1.14} $$

Recently, many others considered the generalized hypergeometric and meromorphic functions along with the above operator. It follows from (1.14) that

$$ z\left(L_a^k\left(\alpha,\beta\right)f(z)\right)'=\alpha\left(L_a^k\left(\alpha+1,\beta\right)f(z)\right)-\left(\alpha+1\right)L_a^k\left(\alpha,\beta\right)f(z). \tag{1.15} $$

The following form is taken by the subordination relation (1.12) in conjunction with (1.14):

$$ \lambda\frac{L_a^k\left(\alpha+1,\beta\right)f(z)}{L_a^k\left(\alpha,\beta\right)f(z)}\prec\lambda-\frac{\left(A-B\right)z}{1+Bz},\ 0\le B<A\le 1,\ \lambda>0. \tag{1.16} $$

Definition 1.2. A function $f\in\Sigma$ in the form (1.1) is considered to be in the class $\Sigma_k^a\left(\lambda,\alpha,\beta,A,B\right)$ if it can satisfy the above subordination relation (1.16).

2. Characterization and other related properties

Here, we will prove the characterization property that generates an essential and the sufficient condition so that a function $f\in\Sigma$ in the form (1.1) can fit to the class $\Sigma_k^a\left(\lambda,A,B\right)$ of meromorphically analytic functions.

Theorem 2.1. *The function $f\in\Sigma$ is considered to be a member of the class $\Sigma_k^a\left(\lambda,A,B\right)$ if and only if it satisfy*

$$ \sum_{n=1}^{\infty}\left(\frac{1+a}{n+a}\right)^k\left(\lambda\left|b_n\right|\left(1+B\right)-\left|c_n\right|\left(\lambda\left(1+B\right)+A-B\right)\right)\left|a_n\right|\le\left(A-B\right). \tag{2.1} $$

For the function $f_n(z)$, the equality is attained, given by

$$f_n(z) = \frac{1}{z} + \frac{(A-B)(n+a)^k}{(1+a)^k(\lambda|b_n|(1+B) - |c_n|(\lambda(1+B)+A-B))} z^n. \quad (2.2)$$

Proof. Let f be in the form (1.1) belong to the class $\Sigma_k^a(\lambda, A, B)$. Then, with regards to (1.10) to (1.11), we find that

$$\left| \frac{\lambda \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^k a_n (|b_n| - |c_n|) z^{n+1}}{(A-B) - \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^k a_n (\lambda B|b_n| + \{(A-B) - \lambda B\}|c_n|) z^{n+1}} \right| \quad (2.3)$$

$$\leq \frac{\lambda \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^t |a_n| (|b_n| - |c_n|) |z^{n+1}|}{(A-B) - \sum_{n=1}^{\infty} \left(\frac{1+a}{n+a}\right)^t |a_n| (\lambda B|b_n| + \{(A-B) - \lambda B\}|c_n|) |z^{n+1}|} \leq 1.$$

Putting $|z| = r(0 < r < 1)$, and considering the fact that the denominator remains positive in the above inequality due to constraints stated in (1.12) for all $r \in (0,1)$, by letting $z \to 1$, we easily arrived at the desired inequality (2.1).

Conversely, assuming that in the simplified form (2.3), the inequality (2.1) holds true, we can easily show that

$$\left| \frac{\lambda\{((f*g)(z)) - ((f*h)(z))\}}{\lambda B((f*g)(z)) + \{\lambda(A-B) - \lambda B\}((f*h)(z))} \right| < 1, \ z \in U^*,$$

which corresponds equally to our theorem condition, so that $f \in \Sigma_k^a(\lambda, A, B)$. This completes the proof. □

From Theorem 2.1 we get the following result:

Corollary 2.2. *If the function $f \in \Sigma$ belongs to the class $\Sigma_k^a(\lambda, A, B)$, then*

$$|a_n| \leq \frac{(A-B)(n+a)^k}{(1+a)^k(\lambda|b_n|(1+B) - |c_n|(\lambda(1+B)+A-B))}, \quad (2.4)$$

$n \geq 1$, where, for the functions $f_n(z)$, the equality holds true, given by (2.2).

The following growth and distortion properties can be stated for the class $\Sigma_t^a(\lambda, A, B)$.

Theorem 2.3. *If the function f defined by* (1.1) *is in the class $\Sigma_k^a(\lambda, A, B)$, then for $0 < |z| = r < 1$, we get*

$$\begin{aligned}
&\frac{1}{r}-\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}r\\
&\leq |f(z)| \\
&\leq \frac{1}{r}+\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}r
\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}
&\frac{1}{r^2}-\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}\\
&\leq \left|f'(z)\right| \\
&\leq \frac{1}{r^2}+\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}.
\end{aligned} \tag{2.6}$$

Proof. Since $f \in \Sigma_k^a(\lambda, A, B)$, Theorem 2.1 readily gives inequality

$$\sum_{n=1}^{\infty}\left|a_n\right| \leq \frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}. \tag{2.7}$$

Thus, for $0<|z|=r<1$ and employing (2.7), we have

$$\begin{aligned}
|f(z)| &= \frac{1}{|z|}+\sum_{n=1}^{m}\left|a_n\right||z|^n \\
&\leq \frac{1}{r}+r\sum_{n=1}^{m}\left|a_n\right| \\
&\leq \frac{1}{r}+\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}r
\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}
|f(z)| &= \frac{1}{|z|}-\sum_{n=1}^{m}\left|a_n\right||z|^n \\
&\geq \frac{1}{r}-r\sum_{n=1}^{m}\left|a_n\right| \\
&\geq \frac{1}{r}-\frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}r.
\end{aligned} \tag{2.9}$$

Also from Theorem 2.1, we get

$$\sum_{n=1}^{\infty}n\left|a_n\right| \leq \frac{(A-B)}{(\lambda\left|b_1\right|(1+B)-\left|c_1\right|(\lambda(1+B)+A-B))}. \tag{2.10}$$

Hence, we have

$$\begin{aligned} \left|f'(z)\right| &= \frac{1}{|z|^2} + \sum_{n=1}^{m} n\,|a_n|\,|z|^{n-1} \\ &\leq \frac{1}{r^2} + \sum_{n=1}^{m} n\,|a_n| \\ &\leq \frac{1}{r^2} + \frac{(A-B)}{(\lambda\,|b_1|\,(1+B) - |c_1|\,(\lambda\,(1+B) + A - B))} \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} \left|f'(z)\right| &= \frac{1}{|z|^2} - \sum_{n=1}^{m} n\,|a_n|\,|z|^{n-1} \\ &\geq \frac{1}{r^2} - \sum_{n=1}^{m} n\,|a_n| \\ &\geq \frac{1}{r^2} - \frac{(A-B)}{(\lambda\,|b_1|\,(1+B) - |c_1|\,(\lambda\,(1+B) + A - B))}. \end{aligned} \tag{2.12}$$

This finishes the proof of Theorem 2.3. □

Next, the radii of meromorphically starlikeness and meromorphically convexity of the class $\Sigma_k^a(\lambda, A, B)$, is determined, given by Theorems 2.4 and 2.5 in the following:

Theorem 2.4. *If the function f as defined by* (1.1) *is within the class* $\Sigma_k^a(\lambda, A, B)$*. Then f is considered to be meromorphically starlike of order δ in the disk $|z| < r_1$, where*

$$r_1 = \inf_{n\geq 1} \left\{ \frac{(1-\delta)\,(\lambda\,|b_n|\,(1+B) - |c_n|\,(\lambda\,(1+B) + A - B))}{(n+2-\delta)\,(A-B)} \right\}^{\frac{1}{n+1}}. \tag{2.13}$$

For the function $f_n(z)$, the equality is attained given by (2.2)*.*

Proof. It is sufficient to prove that

$$\left| \frac{z\,(f(z))'}{f(z)} + 1 \right| \leq 1 - \delta. \tag{2.14}$$

For $|z| < r_1 = (0 < r_1 < 1)$, we have

$$\left|\frac{z\,(f(z))'}{f(z)} + 1\right| = \left|\frac{\sum_{n=1}^{\infty}(n+1)\left(\frac{1+a}{n+a}\right)^k a_n z^n}{\frac{1}{z} + \sum_{n=1}^{\infty}\left(\frac{1+a}{n+a}\right)^k a_n z^n}\right|$$
$$= \left|\frac{\sum_{n=1}^{\infty}(n+1)\left(\frac{1+a}{n+a}\right)^k a_n z^{n+1}}{1 + \sum_{n=1}^{\infty}\left(\frac{1+a}{n+a}\right)^k a_n z^{n+1}}\right| \tag{2.15}$$
$$\leq \frac{\sum_{n=1}^{\infty}(n+1)\left(\frac{1+a}{n+a}\right)^k |a_n|\,|z|^{n+1}}{1 - \sum_{n=1}^{\infty}\left(\frac{1+a}{n+a}\right)^k |a_n|\,|z|^{n+1}}.$$

Hence (2.15) holds true for

$$\sum_{n=1}^{\infty}(n+1)\left(\frac{1+a}{n+a}\right)^k |a_n|\,|z|^{n+1} \leq (1-\delta)\left(1 - \sum_{n=1}^{\infty}\left(\frac{1+a}{n+a}\right)^k |a_n|\,|z|^{n+1}\right)$$

or

$$\frac{\sum_{n=1}^{\infty}(n+2-\delta)\left(\frac{1+a}{n+a}\right)^k |a_n|\,|z|^{n+1}}{(1-\delta)} \leq 1. \tag{2.16}$$

With the help of (2.1) and (2.16), it would be true to state that for fixed n

$$\frac{(n+2-\delta)\left(\frac{1+a}{n+a}\right)^k |z|^{n+1}}{(1-\delta)} \leq \frac{\left(\frac{1+a}{n+a}\right)^k \left(\lambda\,|b_n|\,(1+B) - |c_n|\,(\lambda\,(1+B) + A - B)\right)}{(A-B)}, \quad (n \geq 1)\,. \tag{2.17}$$

Solving (2.17) for $|z|$, we get

$$|z| < \left\{\frac{(1-\delta)\left(\lambda\,|b_n|\,(1+B) - |c_n|\,(\lambda\,(1+B) + A - B)\right)}{(n+2-\delta)\,(A-B)}\right\}^{\frac{1}{n+1}}. \tag{2.18}$$

This completes the proof of Theorem 2.4. □

Theorem 2.5. *If the function f which is defined by (1.1) is in the class $\Sigma_t^a(\lambda, A, B)$, then f is considered to be meromorphically convex of order δ in the disk $|z| < r_2$, where*

$$r_2 = \inf_{n\geq 1}\left\{\frac{(1-\delta)(\lambda|b_n|(1+B) - |c_n|(\lambda(1+B)+A-B))}{n(n+2-\delta)(A-B)}\right\}^{\frac{1}{n+1}}. \quad (2.19)$$

For the function $f_n(z)$, the equality is attained given by (2.2).

Proof. By employing an identical technique used in the proof of Theorem 2.4, we can show that

$$\left|\frac{z(f(z))''}{(f(z))'} + 2\right| \leq 1-\delta. \quad (2.20)$$

For $|z| < r_2$ with the help of Theorem 2.1, Theorem 2.5 can be asserted. □

3. Applications involving generalized hypergeometric functions

Theorem 3.1. *The function $f \in \Sigma$ is considered as a member of the class $\Sigma_k^a(\lambda, \alpha, \beta, A, B)$ if and only if it can satisfy*

$$\sum_{n=1}^{\infty}(\lambda|b_n|(1+B) - |c_n|(\lambda(1+B)+A-B))\frac{(\alpha)_{n+1}}{(b)_{n+1}}\left(\frac{1+a}{n+a}\right)^k|a_n| \leq (A-B). \quad (3.1)$$

For the function $f_n(z)$ the equality is attained given by

$$f_n(z) = \frac{1}{z}+\sum_{n=1}^{\infty}\frac{(A-B)(n+a)^k(\beta)_{n+1}}{(\lambda|b_n|(1+B)-|c_n|(\lambda(1+B)+A-B))(1+a)^k(\alpha)_{n+1}}z^n, \quad (3.2)$$

for $n \geq 1$.

Proof. By employing the same technique as in the proof of Theorem 2.1 as well as Definition 1.2, Theorem 3.1 can be proved. □

(3.1) and (3.2) along with Definition 1.2, can be utilised to deduce the following consequences of Theorem 3.1.

Corollary 3.2. *If the function $f \in \Sigma$ belongs to the class $\Sigma_k^a(\lambda, \alpha, \beta, A, B)$, then*

$$|a_n| \leq \frac{(A-B)(n+a)^k(\beta)_{n+1}}{(\lambda|b_n|(1+B)-|c_n|(\lambda(1+B)+A-B))(1+a)^k(\alpha)_{n+1}} \quad (3.3)$$

for $n \geq 1$, where for the functions $f_n(z)$, the equality holds true given by (3.2).

Corollary 3.3. *If the function f as defined by* (1.1) *is within the class* $\Sigma_k^a(\lambda,\alpha,\beta,A,B)$, *then f is considered as meromorphically starlike of order δ in the disk $|z|<r_3$, where*

$$r_3=\inf_{n\geq 1}\left\{\frac{(1-\delta)\left(\lambda\left|b_n\right|(1+B)-\left|c_n\right|(\lambda(1+B)+A-B)\right)}{(n+2-\delta)(A-B)}\right\}^{\frac{1}{n+1}}. \tag{3.4}$$

For the function $f_n(z)$, the equality is attained given by (3.2).

Corollary 3.4. *If the function f as defined by* (1.1) *is within the class* $\Sigma_k^a(\lambda,\alpha,\beta,A,B)$, *then f is considered as meromorphically convex of order δ in the disk $|z|<r_4$, where*

$$r_4=\inf_{n\geq 1}\left\{\frac{(1-\delta)\left(\lambda\left|b_n\right|(1+B)-\left|c_n\right|(\lambda(1+B)+A-B)\right)}{n(n+2-\delta)(A-B)}\right\}^{\frac{1}{n+1}}. \tag{3.5}$$

For the function $f_n(z)$, the equality is attained given by (3.2).

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