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APPROXIMATION SOLUTION FOR NONLINEAR SET-VALUED MIXED RANDOM VARIATIONAL INCLUSIONS INVOLVING RANDOM NONLINEAR (A_ω, η_ω)-MONOTONE MAPPINGS

Hong Gang Li[1] **and Xian Bing Pan**[2]

[1]Institute of Applied Mathematics Chongqing University of Posts And TeleCommunications University, Chongqing 400065, China
e-mail: `lihg12@126.com`

[2]Yitong College of Chongqing University of Post And TeleCommunications, Chongqing 400065, China
e-mail: `panxb@cqupt.edu.cn`

Abstract. In this paper, we introduce and study a new class of nonlinear set-valued mixed random variational inclusions involving random nonlinear (A_ω, η_ω)-monotone Mappings in Hilbert spaces. Based on the generalized random resolvent operator associated with random nonlinear (A_ω, η_ω)-monotone mappings, an existence theorem of solutions for this kind of random nonlinear set-valued mixed variational inclusions is established and a new algorithm of approximation solution is suggested and discussed. The results presented in this paper generalize, improve, and unify some recent results in this field.

1. Introduction

Variational inclusions are an important and generalization of classical variational inequalities which have wide applications to many fields including, for example, mechanics, physics, optimization, control, and engineering sciences and in face, the problems for random variational inclusions(inequalities) are

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just so. Motivated and inspired by the recent research works in these fascinating areas, the random variational inclusion (inequalities, equalities, quasi-variational inclusions, quasi-complementarity) problems have been introduced and studied by Ahmad and Bazán [1], Chang [3], Chang and Huang [5], Ganguly and Wadhwa [9], Huang [10], Huang et al. [11], Khan et al. [14], Lan [15], Noor and Elsanosi [17]. Very recently, the problems of random fuzzy generalized variational inclusions involving random nonlinear mappings have been studied by Li [16], and Zhang and Bi [25] in Hilbert spaces.

On the other hand, monotonicity techniques were extended and applied in recent years because of their importance in the theory of variational inequalities, complementarity problems, and variational inclusions.

In 2003, Huang and Fang [12] introduced a class of generalized monotone mappings, maximal η-monotone mappings, and defined an associated resolvent operator. Using resolvent operator methods, which is a very important method to find solutions of variational inequality and variational inclusion problems, they developed some iterative algorithms to approximate the solution of a class of general variational inclusions involving maximal η-monotone operators. Huang and Fang's method extended the resolvent operator method associated with an η-subdifferential operator due to Ding and Luo [6].

Recently, Fang and Huang [7], Fang-Huang-Thompon [8], Verma [23] introduced H-monotone operator, (H, η)-monotone operator, and (A, η)-monotone operator, which are generalization of the classical monotone operator. They defined associated resolvent operator, established the Lipschitz continuity of the resolvent operator, studied some classes of variational inclusions in Hilbert spaces using those resolvent operators and constructed some algorithms for approximating solutions of those variational inclusions.

The main purpose of this paper is to introduce and study a new class of random nonlinear set-valued mixed variational inclusions involving random nonlinear (A_ω, η_ω)-monotone mappings in Hilbert spaces. Based on the generalized random resolvent operator associated with random nonlinear (A_ω, η_ω)-monotone mappings, an existence theorem of solutions for this kind of random nonlinear set-valued mixed variational inclusions is established and a new algorithm of approximation solution is suggested and discussed.

1.1. **Set-valued random mapping.**

Throughout this paper, we suppose that $(\Omega, \Re, \mu)$ is a complete σ-finite measure space and X is a separable real Hilbert space endowed with a norm $\|\cdot\|$ and an inner product $\langle \cdot, \cdot \rangle$. We denote by $\Im(X)$ the class of Borel σ-fields in X. Let 2^X and $CB(X)$ denote the family of all the nonempty subsets of X and the family of all the nonempty bounded closed sets of X, respectively. Let us recall the following definitions and some auxiliary results.

Definition 1.1. *A mapping $x : \Omega \to X$ is said to be measurable if, for any $B \in \Im(X)$, $\{\omega \in \Omega : x(\omega) \in B\} \in \Re$.*

Definition 1.2. *A mapping $f : \Omega \times X \to X$ is called a random mapping if, for any $x \in X, f(\omega, x) = y(\omega)$ is measurable. A random mapping f is said to be continuous (resp., linear, bounded) if for any $\omega \in \Omega$, the mapping $f(\omega, \cdot) : X \to X$ is continuous (resp., linear, bounded).*

Similarly, we can define a random mapping $h : \Omega \times X \times X \to X$. We shall write $x = x(\omega)$, $y = y(\omega)$, $f_\omega = f(\omega, x(\omega))$ and $h_\omega(x, y) = h(\omega, x(\omega), y(\omega))$ for all $\omega \in \Omega$ and $x(\omega), y(\omega) \in X$.

It is well-known that a measurable mapping is necessarily a random mapping.

Definition 1.3. *A set-valued mapping $Q : \Omega \to 2^X$ is said to be measurable if, for any $B \in \Im(X)$, $Q^{-1}(B) = \{\omega \in \Omega : Q(\omega) \bigcap B \neq \emptyset\} \in \Re$.*

Definition 1.4. *A mapping $u : \Omega \to X$ is called a measurable selection of a set-valued measurable mapping $Q : \Omega \to 2^X$ if, for any $\omega \in \Omega$, $u(\omega)$ is measurable and $u(\omega) \in Q(\omega)$.*

Definition 1.5. *A set-valued mapping $Q : \Omega \times X \to 2^X$ is called a set-valued random mapping if, for any $x \in X$, $Q(\cdot, x)$ is measurable(denoted by Q_ω, or Q).*

1.2. **Random resolvent operator of (A_ω, η_ω)-monotone mapping.**

Definition 1.6. *The random mapping $\eta_\omega : \Omega \times X \times X \to X$ is said to be τ_ω-Lipschitz continuous if there exists a real-valued random variable $\tau_\omega > 0$ such that*

$$\|\eta_\omega(x(\omega), y(\omega))\| \leq \tau_\omega \|x(\omega) - y(\omega)\|,$$

for all $x(\omega), y(\omega) \in X$ and for all $\omega \in \Omega$.

Definition 1.7. *Let X be a separable real Hilbert Space, $\eta_\omega : \Omega \times X \times X \to X$ and $A_\omega, H_\omega : \Omega \times X$ be random single-valued mappings. Then a multi-valued random mapping $M_\omega : \Omega \times 2^X$ is said to be:*

(i) *$\hat{H}$-continuous if, for any $\omega \in \Omega, M_\omega(\cdot)$ is continuous in $\hat{H}(\cdot, \cdot)$ that is, there exists a real-valued random variable $\alpha_\omega > 0$ such that*

$$\hat{H}(M_\omega(x_1(\omega)), M_\omega(x_2(\omega))) \leq \alpha_\omega \|x_1(\omega) - x_2(\omega)\|,$$

$\forall x_1(\omega), x_2(\omega) \in X, \omega \in \Omega$ where $\hat{H}(\cdot, \cdot)$ is the Hausdorff metric on $CB(X)$ defined as follows: for any $A, B \in CB(X)$,

$$\hat{H}(A, B) = \max\{\sup_{x \in A} \inf_{y \in B} d(x, y), \sup_{y \in B} \inf_{x \in A} d(x, y)\};$$

(ii) *monotone if, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), x_1(\omega) - x_2(\omega)\rangle \geq 0,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*;*

(iii) η_ω*-monotone if, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), \eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq 0,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*;*

(iv) *strictly* η_ω*-monotone if, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), \eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq 0,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*, and the equality holds if and only if* $x_1(\omega) = x_2(\omega)$ *for all* $\omega \in \Omega$*;*

(v) $r_\omega-$*strongly* η_ω*-monotone if there exists a real-valued random variable* $r_\omega > 0$ *such that*

$$\langle u_1(\omega) - u_2(\omega), \eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq r_\omega\|x_1(\omega) - x_2(\omega)\|^2,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*;*

(vi) η_ω*-firmly nonexpansive if*

$$\|u_1(\omega) - u_2(\omega)\|^2 \leq \langle u_1(\omega) - u_2(\omega), \eta_\omega(x_1(\omega), x_2(\omega))\rangle,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*;*

(vii) (m_ω, η_ω)*-relaxed monotone if there exists a real-valued random variable* $m_\omega > 0$ *such that, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), \eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq -m_\omega\|x_1(\omega) - x_2(\omega)\|^2,$$

for all $x_1(\omega), x_2(\omega) \in X, u_1(\omega) \in M_\omega(x_1(\omega)), u_2(\omega) \in M_\omega(x_2(\omega))$*;*

(viii) *maximal monotone if, for any* $\omega \in \Omega$*, and any random variable* $\rho_\omega > 0$*, the* M_ω *is monotone and*

$$(I + \rho_\omega M_\omega)(X) = X,$$

where I *denotes the identity mapping on* X*;*

(ix) *maximal* η_ω*-monotone if, for any* $\omega \in \Omega$*, and any random variable* $\rho_\omega > 0$*, the* M_ω *is* η_ω*-monotone and*

$$(I + \rho_\omega M_\omega)(X) = X,$$

where I *denotes the identity mapping on* X*;*

(x) A_ω*-monotone if, for any* $\omega \in \Omega$ *and any random variable* $\rho_\omega > 0$*, the* M_ω *is* m_ω*-relaxed monotone and*

$$(A_\omega + \rho_\omega M_\omega)(X) = X;$$

(xi) H_ω*-monotone if, for any* $\omega \in \Omega$ *and any random variable* $\rho_\omega > 0$*, the* M_ω *is monotone and*

$$(H_\omega + \rho_\omega M_\omega)(X) = X;$$

(xii) (H_ω, η_ω)*-monotone if, for any* $\omega \in \Omega$ *and any random variable* $\rho_\omega > 0$*, the* M_ω *is* η_ω*-monotone and*

$$(H_\omega(\cdot) + \rho_\omega M_\omega(\cdot))(X) = X;$$

(xiii) (A_ω, η_ω)*-monotone if, for any* $\omega \in \Omega$ *and any random variable* $\rho_\omega > 0$*, the* M_ω *is* (m_ω, η_ω)*-relaxed monotone and*

$$(A_\omega + \rho_\omega M_\omega)(X) = X.$$

Definition 1.8. *Let* X *be a separable real Hilbert Space,* $A_\omega : \Omega \times X \to X$ *and* $F_\omega : \Omega \times X \times X \to X$ *be single-valued random mappings, and* $P_\omega : \Omega \times X \to 2^X$ *be a multi-valued random mapping.*

(i) *A single-valued random mapping* F_ω *is said to be* (μ_ω, ν_ω)*-Lipschitz continuous, if there exist two random variables* $\mu_\omega, \nu_\omega : \Omega \to (0, +\infty)$ *such that*

$$\begin{aligned}\|F_\omega(x_1(\omega), y_1(\omega)) - F_\omega(x_2(\omega), y_2(\omega))\| \le & \mu_\omega \|x_1(\omega) - x_2(\omega)\| \\ & + \nu_\omega \|y_1(\omega) - y_2(\omega)\|,\end{aligned}$$

$\forall x_i(\omega), y_i(\omega) \in X, i = 1, 2;$

(ii) *A single-valued random mapping* F_ω *is said to be* ψ_ω*-*P_ω*-strongly monotone with respect to* A_ω *in the first argument of* $F_\omega(\cdot, \cdot)$*, if there exist a random variables* $\psi_\omega : \Omega \to (0, +\infty)$ *such that*

$$\langle F_\omega(x_1, \cdot) - F_\omega(x_2, \cdot), A_\omega(y_1) - A_\omega(y_2)\rangle \ge \psi_\omega \|x_1 - x_2\|^2,$$

$\forall x_i(\omega) \in X, y_i \in P_\omega (i = 1, 2), \omega \in \Omega;$

Definition 1.9. *Let* X *be a separable real Hilbert Space,* $\eta_\omega : \Omega \times X \times X \to X$ *be a random single-valued mapping,* $A_\omega : \Omega \times X \to X$ *be a strictly* η_ω*-monotone single-valued mapping and* $M_\omega : \Omega \times X \to 2^X$ *be a* (A_ω, η_ω)*-monotone mapping. The random resolvent operator* $R^{A_\omega,\eta_\omega}_{\rho_\omega,M_\omega} : \Omega \times X \to X$ *is defined by*

$$R^{A_\omega,\eta_\omega}_{\rho_\omega,M_\omega}(y) = (A_\omega + \rho_\omega M_\omega)^{-1}(y),$$

for all $\omega \in \Omega$*,* $y = y(\omega) \in X$*, and* $\{\omega \in \Omega : 0 < \rho_\omega \in B\} \in \Re$*.*

Lemma 1.10. *([4]) Let* X *be a separable real Hilbert Space,* $G_\omega : \Omega \times X \to CB(X)$ *be a* $\hat{H}$*-continuous random set-valued mapping. then for any measurable mapping* $x : \Omega \to X$*, the set-valued mapping* $G_\omega(x(\omega)) : \Omega \times X \to CB(X)$ *is measurable.*

Lemma 1.11. *([4]) Let $G_\omega, P_\omega : \Omega \times X \to CB(X)$ be two measurable set-valued mappings, $\varepsilon > 0$ be a constant and $x : \Omega \to X$ be a measurable selection of P_ω. Then there exists a measurable selection $y : \Omega \to X$ of G_ω such that for any $\omega \in \Omega$,*

$$\|x(\omega) - y(\omega)\| \leq (1+\varepsilon)\hat{H}(G_\omega(\cdot), P_\omega(\cdot)).$$

Lemma 1.12. *([20]) Let X be a separable real Hilbert Space, $\eta_\omega : \Omega\times X\times X \to X$ be τ_ω-Lipschtiz continuous random mapping, $A_\omega : \Omega \times X \to X$ be an r_ω-strongly η_ω-monotone random mapping, and $M_\omega(y) : \Omega \times X \to 2^X (\forall y \in X)$ be an (A_ω, η_ω)-monotone random mapping. Then the generalized random resolvent operator $R^{A_\omega,\eta_\omega}_{\rho_\omega,M_\omega} : X \to X$ is $\tau_\omega/(r_\omega - m_\omega\rho_\omega)$-Lipschitz continuous, that is,*

$$\|R^{A_\omega,\eta_\omega}_{\rho_\omega,M_\omega}(x) - R^{A_\omega,\eta_\omega}_{\rho_\omega,M_\omega}(y)\| \leq \frac{\tau_\omega}{r_\omega - m_\omega\rho_\omega}\|x-y\|,$$

for all $x, y \in X, \omega \in \Omega$, where $\rho_\omega, r_\omega, m_\omega : \Omega \to (0, +\infty)$ are real-valued measurable, and $0 < \rho_\omega < \frac{r_\omega}{m_\omega}$.

2. Random variational inclusion problem and Algorithm for random the approximation solution of the problem

2.1. **Random variational inclusion problem.**

Let $A_\omega, f_\omega : \Omega\times X \to X$, $\eta_\omega : \Omega\times X\times X \to X$ and $F_\omega : \Omega\times X\times X \to X$ be single-valued random mappings, and G_ω, P_ω be a multi-valued random mappings. Let $M_\omega : \Omega\times X \to X2^X$ is a set-valued random mapping such that for each $\omega \in \Omega$, and $x \in X$ $M_\omega(x, \cdot) : X \times X \to 2^X$ is (A_ω, η_ω)-monotone random mapping and $range(P_\omega) \bigcap dom(M_\omega(x, \cdot)) \neq \emptyset$. We introduce and study the following problem for a new class of nonlinear set-valued mixed random variational inclusions Involving Random (A_ω, η_ω)-monotone mappings.

For a given element $g_\omega : \Omega \to X$ and any real-valued random variable $k_\omega > 0$, finding measurable mappings $x = x(\omega), z = z(\omega), y = y(\omega) : \Omega \to X, z \in G_\omega(x)$, and $y \in P_\omega(x)$ such that

$$g_\omega \in F_\omega(x, f_\omega(z)) + k_\omega M_\omega(y, z), \tag{2.1}$$

which is a called nonlinear set-valued mixed random variational inclusions involving random nonlinear (A_ω, η_ω)-monotone mappings(short down: nonlinear random SVMVI.) in Hilbert spaces. And a solution of the problem (2.1) is called a random solution.

For a suitable choice of $A_\omega, \eta_\omega, F_\omega, f_\omega, M_\omega, P_\omega$ and the space X, a number of known classes of variational inclusions and variational inequalities can be obtained as special cases of the general set-valued mixed quasi-variational inclusions (2.1).

Special cases:

(i) If $k_\omega = 1, G_\omega(x) \equiv \{x\}$, and $M_\omega(\cdot,\cdot) = \partial\phi(\cdot)$ is the subdifferential of a lower semi-continuous and η-subdifferentiable function $\phi : X \to R\bigcup\{+\infty\}$. Let for any $v \in X$, $\eta_\omega(v, g_\omega) = v - g_\omega(u, x)$, $F_\omega(v, f_\omega(z)) = f_\omega(x) - p_\omega(v)$, and taking $g_\omega = v \in X(\forall\omega \in \Omega)$, then problem (3.1) becomes the following problem: Find measurable mappings $u, x, y : \Omega \to X$, such that for each $\omega \in \Omega, v \in X$, hold

$$x(\omega) \in G_\omega(u), \quad y(\omega) \in P_\omega(u),$$

and

$$\langle f_\omega(x) - p_\omega(y), v -_\omega (u, x)\rangle \geq \phi(g_\omega(u, x)) - \phi(v), \tag{2.2}$$

for all $\omega \in \Omega$, and each measurable mappings $u(\omega), v(\omega) \in X$, which is random generalized nonlinear variational inclusions for random mappings in Hilbert space. The form of the problem (2.2) was studied by Zhang and Bi [25] when all the fuzzy mappings are taken as general determine mappings in the problem.

(ii) If in the problem (2.2), $P_\omega = p_\omega(x), f_\omega(z) \equiv z$, $M_\omega(\cdot,\cdot) \equiv \partial\phi(\omega,\cdot) : \Omega \times X \to X$ is subdifferentiable, and $\phi(\omega,\cdot)$ is the indicator function of a nonempty closed convex set K in K defined in the form:

$$\phi(y) = \begin{cases} 0 & if\ y \in K, \\ \infty & otherwise, \end{cases}$$

then the problem (2.3) becomes the problem of finding measurable mappings $x, u : \Omega \to X$ such that $u \in T_\omega(x)$ and

$$\langle f_\omega(x) + u(\omega) - g_\omega, y - p_\omega(x)\rangle \geq 0, \tag{2.3}$$

for all $\omega \in \Omega, y \in K$.

(iii) If $g_\omega = 0, k_\omega = 1, f_\omega(z) \equiv z, P_\omega(x) = I, M_\omega(\cdot,\cdot) = M_\omega(\cdot)$, which is identity mapping in the X, then the problem (2.1) is described as: determine an element $x \in X, z \in Q_\omega$ such that

$$0 \in F_\omega(x, z_\omega(x)) + M_\omega(x) \tag{2.4}$$

which the form of the problem (2.4) have been studied by Verma[23] when the random variables (functions, mappings) all are taken as general variables (functions, mappings) in the problem.

(iv) If $g(\omega) = 0, k_\omega = 1, f_\omega(z) \equiv z, A_\omega = H_\omega, P_\omega(x) = I, M_\omega(\cdot,\cdot) = M_\omega(\cdot)$, which is identity mapping in the X, then the problem (2.1) is described as: determine an element $x \in X, z \in Q_\omega$ such that

$$0 \in F_\omega(x, z_\omega(x)) + M_\omega(x). \tag{2.5}$$

(v) If $M_\omega(\cdot,\cdot) = M_\omega(\cdot), f_\omega(z) \equiv z$, then the problem (2.1) becomes the problem (2.1) in Li[16] which the form of the problem (2.1) have been studied by Fang Huang and Thompson [8] when the random variables (functions, mappings) all are taken as general variables (functions, mappings) in the problem.

Furthermore, a number of known classes of variational inclusions and variational inequalities in Chang [3], Huang [10], Noor and Elsanosi [17] have been studied as special cases of the problem (2.1) when the random variables (functions, mappings) all are taken as general variables (functions, mappings) in the problem. These types of variational inclusions can enable us to study many important nonlinear problems arising in mechanics, physics, optimization and control, nonlinear programming, economics, finance, regional, structural, transportation, elasticity, and applied sciences in a general and unified framework.

2.2. **Algorithm for random the approximation solution of the Random variational inclusion problem.**

New, we transfer the problem (2.1) into a fixed point problem.

Lemma 2.1. *Measurable $(x, z, y) : \Omega \to X$ is random solution of nonlinear set-valued mixed random variational inclusions the problem (2.1) if and only if for each $\omega \in \Omega$, holds the following relation*

$$y = R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega}[A_\omega(y) + \rho_\omega g_\omega - \rho_\omega F_\omega(x, f_\omega(z))], \tag{2.6}$$

where $z \in G_\omega, y \in P_\omega$, $\rho, k : \Omega \to (0, +\infty)$ are two real-valued random variables, and $R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega} = (A_\omega + \rho_\omega k_\omega M_\omega)^{-1}$ is a random resolvent operator in a Hilbert Space X.

Proof. The proof directly follows from the definition of $R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega}$ and so it is omitted. □

Based on Lemma 2.1 and Nadler [18], we can develop a new algorithm of approximation solution for solving the nonlinear set-valued mixed random variational inclusions (2.1) with random nonlinear (A_ω, η_ω)-monotone mappings as follows:

Algorithm 2.2. *Let $G_\omega, P_\omega : \Omega \times X \to CB(X)$ be set-valued random mappings, $A_\omega, f_\omega : \Omega \times X \to X$, $\eta_\omega, F_\omega : \Omega \times X \times X \to X$ single-valued random mappings and $M_\omega : \Omega \times X \times X \to 2^X$ a set-valued random mapping such that, for each fixed $\omega \in \Omega$ and any measurable mapping $x : \Omega \times X \to X$, $M_\omega(x, \cdot) : \Omega \times X \to 2^X$ is a (A_ω, η_ω)-monotone mapping and $range(P_\omega) \bigcap dom M_\omega(\cdot, z) \neq \emptyset (\forall z \in G_\omega)$.*

Step 1. Initialize*:*

For any given $x_0 : \Omega \to X$*, the multi-mappings* $G_\omega(x_0(\cdot)), P_\omega(x_0(\cdot)) : \Omega \times X \to CB(X)$ *are both measurable by Lemma 1.10, and so there exist measurable selections* $z_0 \in G_\omega(x_0(\cdot))$ *and* $y_0 \in P(\cdot, x_0(\cdot))$ *(*[13]*). Set*

$$x_1(\omega) = (1-\varpi)x_0+\varpi[x_0-y_0+R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega,M_\omega}(A_\omega(y_0)+\rho_\omega g_\omega-\rho_\omega F_\omega(x_0, f_\omega(z_0)))]+e_0,$$

where $k_\omega, \rho_\omega, A_\omega, M_\omega, F_\omega$ *are the same as in Lemma 4.1,* $1 > \varpi > 0$ *is a constant, and* $e_0 = e_0(\omega) : \Omega \to X$ *is a measurable function which takes into account a possible inexact computation of the proximal point. Then, it is easy to know that* $x_1 : \Omega \to X$ *is a measurable mapping. Since* $z_0 \in G_\omega(x_0(\cdot))$ *and* $y_0 \in P_\omega(x_0(\cdot))$*, by Lemma 1.11, there exist measurable selections* $z_1 \in G_\omega(x_1(\cdot))$ *and* $y_1 \in P_\omega(x_1(\cdot))$ *such that, for all* $\omega \in \Omega$*,*

$$\|z_0 - z_1\| \leq (1 + \frac{1}{1})\hat{H}(G_\omega(x_0(\cdot)), G_\omega(x_1(\cdot))),$$
$$\|y_0 - y_1\| \leq (1 + \frac{1}{1})\hat{H}(P_\omega(x_0(\cdot)), P_\omega(x_1(\cdot))).$$

Step 2. Iterative*:*

By induction, we can get three measurable sequences $\{x_n\}, \{z_n\}$*, and* $\{y_n\}$ *from* Ω *to* x *inductively satisfying*

$$\begin{cases} x_{n+1} = (1-\varpi)x_n + \varpi[x_n - y_n \\ +R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega,M_\omega}(A_\omega(y_n) + \rho_\omega g_\omega - \rho_\omega F_\omega(x_n, f_\omega(z_n)))] + e_n, \\ \|z_n - z_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(G_\omega(x_n), G_\omega(x_{n+1})), \\ \|y_n - y_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(P_\omega(x_n), P_\omega(x_{n+1})), \end{cases} \tag{2.7}$$

for $n = 0, 1, 2, \cdots$*, where* $e_n = e_n(\omega) \in Xn \geq 0)$ *is a random error to take into account a possible inexact computation of the proximal point.*

Step 3. Condition for stopping algorithm*:*

If x_{n+1}, z_{n+1}*, and* y_{n+1} *satisfy (2.7) sufficiently accurate, stop; otherwise, set* $n := n + 1$ *and return to Step 2.*

Remark 2.3. *If we choose suitable eta,* A, F, G, P, f *and* M*, then the Algorithm 2.2 reduce to a number of known algorithms for solving some classes of variational inequalities and variational inclusions (see, for example,* [5], [9]*–*[11]*,* [16]*,* [25]*).*

Now, if we prove the existence of solutions of problem (2.1) and the convergence of iterative sequences $\{x_n\}, \{y_n\}, \{z_n\}$ generated by the Algorithm

2.2, then the (x_n, z_n, y_n) will be a random approximation solution of problem (2.1).

3. Existence and convergence theorem of solution of the Random variational inclusion problem

In this section, we shall prove the existence of solution for problem (2.1) and the convergence of the iterative sequences generated by the Algorithm 2.2.

3.1. **Existence and convergence theorem.**

Theorem 3.1. *Let X be a real separable Hilbert space, $\eta_\omega : \Omega \times X \times X \to X$ be a τ_ω-Lipschtiz continuous mapping, $A_\omega : \Omega \times X \to X$ be a r_ω-strongly η_ω-monotone mapping and α_ω-Lipschitz continuous. Let G_ω, $P_\omega : \Omega \times X \to CB(X)$ be $\hat{H}$-Lipschitz continuous set-valued random mappings with random variables $\zeta_\omega, \chi_\omega$, respectively, and $f_\omega : \Omega \times X \to X$ be ξ_ω-Lipschitz continuous. Let P_ω be a β_ω-strongly monotone random mapping, $F_\omega : \Omega \times X \times X \to X$ be Lipschitz continuous with random variables (μ_ω, ν_ω), and F_ω be ψ_ω-P_ω-strongly monotone with respect to A_ω in the first argument of $F_\omega(\cdot, \cdot)$, and let $g_\omega : \Omega \to X$ be a real random variable. Let $M_\omega : \Omega \times X \times X \to 2^X$ be a set-valued random mapping such that for each measurable $y \in X$, $M_\omega(y, \cdot) : X \to 2^X$ is a (A_ω, η_ω)-monotone mapping, and $range(P_\omega) \bigcap domM_\omega(y) \neq \emptyset$. If for any $x, y, z \in X$,*

$$\|R^{A_\omega,\eta_\omega}_{k_\omega\rho_\omega,M_\omega(\cdot,x)}(z) - R^{A_\omega,\eta_\omega}_{k_\omega\rho_\omega,M_\omega(\cdot,y)}(z)\| \leq \delta_\omega \|x - y\|, \tag{3.1}$$

and

$$\begin{cases} |\rho - \frac{\psi_\omega - l_\omega r_\omega(k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)}{\tau_\omega^2\mu^2 - (k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)}| \\ < \frac{\{[\psi_\omega - l_\omega r_\omega(k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)]^2 - [\tau_\omega^2\mu^2 - (k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)^2][\tau_\omega^2\alpha_\omega^2\chi_\omega^2 - l_\omega^2 r_\omega^2]\}^{\frac{1}{2}}}{\tau_\omega^2\mu^2 - (k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)^2}, \\ \tau_\omega^2\mu^2 > (k_\omega m_\omega l_\omega + \tau_\omega\xi_\omega\nu_\omega\zeta_\omega)^2, \\ l_\omega = 1 - \sqrt{1 - 2\beta_\omega + \chi_\omega^2} - \delta_\omega\zeta_\omega < 1, \end{cases} \tag{3.2}$$

and

$$\lim_{n\to\infty} \|e_n(\omega)\| = 0, \quad \sum_{n=1}^{\infty} \|e_n(\omega) - e_{n-1}(\omega)\| < \infty, \quad \forall \omega \in \Omega, \tag{3.3}$$

then the random iterative sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\} : \Omega \to X$ generated by Algorithm 4.2 converge strongly to random variables x^*, y^*, and $z^* : \Omega \to X$, respectively, (x^*, y^*, z^*) is a random solution of the problem (3.1), and (x_n, y_n, z_n) is a random approximation solution of the problem (3.1).

Proof. From Algorithm 2.2, Lemma 2.1 and (2.8)–(2.10), for any $\omega \in \Omega$, and $0 < \varpi < 1$, we have

$$\begin{aligned}
&\|x_{n+1} - x_n\| \\
&\le (1-\varpi)\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\| + \varpi\|x_n - x_{n-1} - (y_n - y_{n-1})\| \\
&\quad +\varpi\|R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega(y_n,z_n)}(A_\omega(y_n) + \rho_\omega g_\omega - \rho_\omega F_\omega(x_n, f_\omega(z_n))) \\
&\quad -R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega(y_{n-1},z_{n-1})}(A_\omega(y_{n-1}) + \rho_\omega g_\omega - \rho_\omega F_\omega(x_{n-1}, f_\omega(z_{n-1})))\| \\
&\le (1-\varpi)\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\| \\
&\quad +\varpi\{\|x_n - x_{n-1} - (y_n - y_{n-1})\| + \delta_\omega\|z_n - z_{n-1}\| \\
&\quad +\frac{\tau_\omega}{r_\omega - k_\omega\rho_\omega m_\omega}[\|A_\omega(y_n) - A_\omega(y_{n-1}) \\
&\quad -\rho_\omega(F_\omega(x_n, f_\omega(z_n)) - F_\omega(x_{n-1}, f_\omega(z_n)))\| \\
&\quad +\rho(\omega)\|F_\omega(x_{n-1}, f_\omega(z_n)) - F_\omega(x_{n-1}, f_\omega(z_{n-1}))\|]\}. \qquad (3.4)
\end{aligned}$$

Since P_ω is $\hat{H}$-Lipschitz continuous with χ_ω and β_ω-strongly monotone in the second argument of $P_\omega(\cdot)$, by Algorithm 2.1, we obtain

$$\begin{aligned}
&\|x_n - x_{n-1} - y_n - y_{n-1}\|^2 \\
&= \|x_n - x_{n-1}\|^2 - 2\langle y_n - y_{n-1}, x_n - x_{n-1}\rangle + \|y_n - y_{n-1}\|^2 \\
&\le \|x_n - x_{n-1}\|^2 + (1+n^{-1})^2\hat{H}^2(P_\omega(x_n), P_\omega(x_{n-1})) - 2\beta_\omega\|x_n - x_{n-1}\|^2 \\
&\le [1 - 2\beta_\omega + (1+n^{-1})^2\chi_\omega^2]\|x_n - x_{n-1}\|^2. \qquad (3.5)
\end{aligned}$$

Since F_ω is $\hat{H}$-Lipschitz continuous with (μ_ω, ν_ω), and ψ_ω-P_ω-strongly monotone with respect to A_ω in the first argument of $F_\omega(\cdot,\cdot)$, we have

$$\begin{aligned}
&\|A_\omega(y_n) - A_\omega(y_{n-1}) - \rho_\omega(F_\omega(x_n, f_\omega(z_n)) - F_\omega(x_{n-1}, f_\omega(z_n)))\|^2 \\
&\le \|A_\omega(y_n) - A_\omega(y_{n-1})\|^2 + \rho_\omega^2\|F_\omega(x_n, f_\omega(z_n)) - F_\omega(x_{n-1}, f_\omega(z_n))\|^2 \\
&\quad -2\rho_\omega\langle(F_\omega(f_\omega(x_n), z_n) - F_\omega(f_\omega(x_{n-1}), z_n)), A_\omega(y_n) - A_\omega(y_{n-1})\rangle \\
&\le \alpha_\omega^2\|y_n - y_{n-1}\|^2 + \rho_\omega^2\mu_\omega^2\|x_n - x_{n-1}\|^2 - 2\rho_\omega\psi_\omega\|x_n - x_{n+1}\|^2 \\
&\le [\alpha_\omega^2(1+n^{-1})^2\chi_\omega^2 + \rho_\omega^2\mu_\omega^2 - 2\rho_\omega\psi_\omega]\|x_n - x_{n-1}\|^2. \qquad (3.6)
\end{aligned}$$

Further, by assumptions, we have

$$\begin{aligned}
\|F_\omega(x_{n-1}, f_\omega(z_n)) \;-\; & F_\omega(x_{n-1}, f_\omega(z_{n-1}))\| \\
& \le \nu_\omega\xi_\omega\zeta_\omega(1+n^{-1})\|x_n - x_{n-1}\| \qquad (3.7)
\end{aligned}$$

and

$$\|z_n - z_{n-1}\| \le \zeta_\omega(1+n^{-1})\|x_n - x_{n-1}\|. \qquad (3.8)$$

From (3.6)–(3.8), it follows that

$$\begin{aligned}\|x_{n+1}-x_n\| &\le (1-\varpi+\varpi h_n)\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\| \\ &= \theta_n\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\|, \end{aligned} \tag{3.9}$$

where $\theta_n = 1-\varpi+\varpi h_n$ and

$$\begin{aligned} h_n &= [1-2\beta_\omega+(1+n^{-1})^2\chi_\omega^2]^{\frac{1}{2}}+\delta_\omega\zeta_\omega(1+n^{-1}) \\ &\quad +\frac{\tau_\omega}{r_\omega-k_\omega\rho_\omega m_\omega}[(\alpha_\omega^2(1+n^{-1})^2\chi_\omega^2+\rho_\omega^2\mu_\omega^2-2\rho_\omega\psi_\omega)^{\frac{1}{2}} \\ &\quad +\rho_\omega\xi_\omega\nu_\omega\zeta_\omega(1+n^{-1})]. \end{aligned}$$

Let $\theta = 1-\varpi+\varpi h$ and

$$\begin{aligned} h &= [1-2\beta_\omega+\chi_\omega^2]^{\frac{1}{2}} \\ &\quad +\delta_\omega\zeta_\omega+\frac{\tau_\omega}{r_\omega-k_\omega\rho_\omega m_\omega}[(\alpha_\omega^2\chi_\omega^2+\rho_\omega^2\mu_\omega^2-2\rho_\omega\psi_\omega)^{\frac{1}{2}}+\rho_\omega\nu_\omega\xi_\omega\zeta_\omega]. \end{aligned}$$

We have that $h_n \to h$ and $\theta_n \to \theta$ as $n \to \infty$. It follows from condition (3.2) and $0<\varpi<1$ that $0<\theta<1$ and hence there exists $N_0>0$ and $\theta_* \in (\theta, 1)$ such that $\theta_n < \theta_*$ for all $n \ge N_0$. Therefore, by (3.9), we have

$$\|x_{n+1}-x_n\| \le \theta_*\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\|, \forall n \ge N_0.$$

Without loss of generality, we assume

$$\|x_{n+1}-x_n\| \le \theta_*\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\|, \forall n \ge 1,$$

Hence, for any $m>n>0$, we have

$$\|x_m-x_n)\| \le \sum_{i=n}^{m-1}\|x_{i+1}-x_i\| \le \sum_{i=n}^{m-1}\theta_*^i\|x_1-x_0\|+\sum_{i=n}^{m-1}\sum_{j=1}^{i}\theta_*^{i-j}\|e_j-e_{j-1}\|.$$

It follows from conditions (3.2)–(3.3) that $\|x_m-x_n\| \to 0$, as $n \to \infty$, and so $\{x_n\}$ is a Cauchy sequence in X. Let $x_n \to x^*$ as $n \to \infty$. By the Lipschitz continuity of $G_\omega(\cdot)$ and $P_\omega(\cdot)$, we obtain

$$\begin{aligned} \|z_{n+1}-z_n\| &\le (1+n^{-1})\hat{H}(G_\omega(x_{n+1}),G_\omega(x_n)) \le \zeta_\omega(1+n^{-1})\|x_{n+1}-x_n\|, \\ \|y_{n+1}-y_n\| &\le (1+n^{-1})\hat{H}(P_\omega(x_{n+1}),P_\omega(x_n)) \le \chi_\omega(1+n^{-1})\|x_{n+1}-x_n\|. \end{aligned}$$

It follows that $\{z_n\}$ and $\{y_n\}$ are both Cauchy sequences in X. We assume that $z_n \to z^*$ and $y_n \to y^*$, respectively. Noticing that $z_n \in G_\omega(x_n)$, we have

$$\begin{aligned} d(z^*,G_\omega(x^*)) &\le \|z^*-z_n\|+d(z_n,G_\omega(x^*)) \\ &\le \|z^*-z_n\|+\hat{H}(G_\omega(x_n),G_\omega(x^*)) \\ &\le \|z^*-z_n\|+\zeta_\omega\|x_n-x^*\| \to 0(n\to\infty). \end{aligned}$$

Hence $d(z^*,G_\omega(x^*))=0$ and so $z^* \in G_\omega(x^*)$. Similarly, we can prove that $y^* \in P_\omega(x^*))$.

By the Lipschitz continuity of $G_\omega(\cdot)$ and $P_\omega(\cdot)$, Lemma 2.1, the condition (4.1) and $\lim_{n\to\infty}\|e_n(\omega)\| = 0$, we have

$$\begin{aligned} x^*(\omega) &= (1-\varpi)x^*(\omega) + \varpi[x^*(\omega) - y^*(\omega) \\ &\quad + R^{A_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega}(A_\omega(y^*(\omega)) + \rho_\omega g_\omega - \rho_\omega F_\omega(x^*(\omega), f_\omega(z^*(\omega))))]. \end{aligned}$$

By Lemma 2.1, we know that (x^*, z^*, y^*) is a solution of problem (2.1). This completes the proof. □

From Theorem 3.1, we have the following theorem.

3.2. **Algorithm for random the approximation solution of the problem (2.5).**

From Algorithm 2.2, we can get algorithm for solving problems (2.5) as follows:

Algorithm 3.2. *Let $G_\omega, P_\omega : \Omega \times X \to CB(X)$ be the multi-valued random mappings , $H_\omega : \Omega \times X \to X$, $\eta_\omega, F_\omega : \Omega \times X \times X \to X$ and $f_\omega = I_\omega$ be single-valued random mappings, and let $M_\omega : \Omega \times X \to 2^X$ be a multi-valued random mapping such that for each fixed $\omega \in \Omega$, and for any a measurable mapping $z : \Omega \times X \to X$, $M_\omega(z) : \Omega \times X \to 2^X$ be an (H_ω, η_ω)-monotone mapping and $range(P_\omega) \bigcap domM_\omega(\cdot) \neq \emptyset$.*

Step 1. Initialize*:*

For any given $x_0 : \Omega \to X$, the multi-mappings $G_\omega(x_0(\cdot)), P_\omega(x_0(\cdot)) : \Omega \times X \to CB(X)$ all are measurable by Lemma (1.10). Set

$$x_1(\omega) = (1-\varpi)x_0 + \varpi[x_0 - y_0 + R^{H_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega}(H_\omega(y_0) + \rho_\omega g_\omega - \rho_\omega F_\omega(x_0, z_0))] + e_0,$$

where $k_\omega, \rho_\omega, A_\omega, M_\omega, F_\omega$ are the same as in Lemma 2.1, $1 > \varpi > 0$ is a constant, and $e_0 = e_0(\omega) : \Omega \to X$ is a measurable function which is an random error to take into account a possible inexact computation of the proximal point. Since $z_0 \in G_\omega(x_0(\cdot)), y_0 \in P_\omega(x_0(\cdot))$, by Lemma 1.11, there exist measurable selections $z_1 \in G_\omega(x_1(\cdot))$ and $y_1 \in P_\omega(x_1(\cdot))$ such that, for all $\omega \in \Omega$,

$$\|z_0 - z_1\| \le (1 + \frac{1}{1})\hat{H}(G_\omega(x_0(\cdot)), G_\omega(x_1(\cdot))),$$
$$\|y_0 - y_1\| \le (1 + \frac{1}{1})\hat{H}(P_\omega(x_0(\cdot)), P_\omega(x_1(\cdot))),$$

Step 2. Iterative*:*

By induction, we can define a measurable sequences x_n, z_n, and $y_n : \Omega \to X$ inductively satisfying

$$\begin{aligned} x_{n+1} &= (1-\varpi)x_n + \varpi[x_n - y_n + R^{H_\omega,\eta_\omega}_{\rho_\omega k_\omega, M_\omega}(H_\omega(y_n) \\ &\quad + \rho_\omega g_\omega - \rho_\omega F_\omega(x_n, z_n))] + e_n \end{aligned} \tag{3.10}$$

for $n = 0, 1, 2, \cdots$, where $e_n = e_n(\omega) \in X(n \geq 0)$ is an random error to take into account a possible inexact computation of the proximal point.

Step 3. Iterative procedure*:*
Choose $z_n \in G_\omega(x_n), y_n \in P_\omega(x_n)$ such that

$$\|z_n - z_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(G_\omega(x_n), G_\omega(x_{n+1})), \tag{3.11}$$

$$\|y_n - y_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(P_\omega(x_n), P_\omega(x_{n+1})) \tag{3.12}$$

for $n = 0, 1, 2, \cdots$.

Step 4. Condition for stopping algorithm*: If x_{n+1}, z_{n+1}, and y_{n+1} satisfy (3.10)-(3.12) sufficiently accurate, stop; otherwise, set $n := n + 1$ and return to Step 2.*

Remark 3.3. *If we choose suitable η, H, F, G, P, f and M, then the Algorithm 3.2 reduce to a number of known algorithms for solving some classes of variational inequalities and variational inclusions (see, for example,* [14], [17], [16], [25]*).*

3.3. **Existence and convergence theorem of the problem (2.5).**
From Theorem 3.1, we have the following theorem.

Theorem 3.4. *Let $g_\omega, \eta_\omega, F_\omega, M_\omega, f_\omega, X$ be the same as in Theorem 3.1, $A_\omega = H_\omega$, and $G_\omega, P_\omega : \Omega \times X \to CB(X)$ be D-Lipschitz continuous with random variables $\zeta_\omega, \chi_\omega$, respectively, and let P_ω be β_ω-strongly monotone random in the second argument of $P_\omega(\cdot)$. Let $F_\omega : \Omega \times X \times X \to X$ be Lipschitz continuous with random variables (μ_ω, ν_ω), and F_ω be $\psi(\omega))$-P_ω-strongly monotone with respect to A_ω in the first argument of $F_\omega(\cdot, \cdot)$. If conditions (3.1)-(3.2) of Theorem 3.1 hold, then the random variable iterative sequences $\{x_n\}$, $\{z_n\}$ and $\{y_n\} : \Omega \to X$ generated by Algorithm 3.2 converge strongly to random variables x^*, z^* and $y^* : \Omega \to X$, respectively, (x^*, z^*, y^*) is a solution of the problem (2.5), and (x_n, y_n, z_n) is a random approximation solution of the problem(2.5).*

Remark 3.5. *For a suitable choice of the mappings $A_\omega, g_\omega, \eta_\omega, F_\omega, f_\omega, M_\omega, G_\omega, P_\omega$ and X_ω, we can obtain several known results of* [5], [9]-[11], *and* [17] *as special cases of Theorems 3.1 and 3.4.*

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