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CORRIGENDUM OF THE PAPER ENTITLED: ON THE LOCATION OF ZEROS OF A CERTAIN CLASS OF POLYNOMIALS WITH RESTRICTED COEFFICIENTS

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Abstract. This paper is a corrigendum on a paper published in a recent volume of Nonlinear Functional Analysis and Applications: On the location of zeros of a certain class of polynomials with restricted coefficients, 23(2) (2018), 305-309. http://nfaa.kyungnam.ac.kr/journal-nfaa.

For a polynomial $P_\mu(z) = a_0 + \sum_{j=\mu}^{n} a_j z^j$ of degree n, where $a_0 \neq 0$ and for some reals $\rho_1, \rho_2 \geq 0$, $t > 0$ and $1 \leq \mu \leq \lambda \leq n$.

The conditions

$$\rho_1 + t^n a_n \leq t^{n-1} a_{n-1} \leq ... \leq t^{\lambda+1} a_{\lambda+1} \leq t^{\lambda} a_{\lambda} \geq t^{\lambda-1} a_{\lambda-1} \geq ... \geq t^{\mu} a_{\mu} \geq a_0 - \rho_2$$

imposed on the complex coefficients of the polynomial $P_\mu(z)$ given in Theorem 2.1, Corollary 2.2 and Corollary 2.3 [1] are not sufficients to prove the following results

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$$|z-\frac{\rho_1}{a_n}|\leq\frac{1}{|a_n|}(-a_nt+\rho_1-\frac{a_{\mu-1}}{t^{n-1}}+\frac{2\rho_2}{t^{n-1}}+\frac{|a_{\mu-1}|}{t^{n-1}}+\frac{2a_\lambda}{t^{n-\lambda-1}}).$$

The mistake appears in line 25 page 2 of the proof of Theorem 2.1 [1].

It is easy to see that the condition $\rho_1 \geq 0$ may not be possible because by taking ρ_1 to be very large, $\rho_1 + t^n a_n$ cannot be less than $t^{n-1}a_{n-1}$. Infact, the restrictions should read

$$t^na_n-\rho_1\leq t^{n-1}a_{n-1}\leq ...\leq t^{\lambda+1}a_{\lambda+1}\leq t^\lambda a_\lambda\geq t^{\lambda-1}a_{\lambda-1}\geq ...\geq t^\mu a_\mu\geq a_0-\rho_2.$$

Therefore, Theorem 2.1 [1] becomes:

Theorem 1. Let $P_\mu(z) = a_0 + \sum_{j=\mu}^{n} a_j z^j$ where $a_0 \neq 0$, and for some reals $\rho_1, \rho_2 \geq 0, t > 0$, $1 \leq \mu \leq \lambda \leq n$, and

$$t^na_n-\rho_1\leq t^{n-1}a_{n-1}\leq ...\leq t^{\lambda+1}a_{\lambda+1}\leq t^\lambda a_\lambda\geq t^{\lambda-1}a_{\lambda-1}\geq ...\geq t^\mu a_\mu\geq a_0-\rho_2.$$

Then all the zeros of $P_\mu(z)$ lie in the union of the disks $|z| \leq t$ and

$$|z-\frac{\rho_1}{a_n}|\leq\frac{1}{|a_n|}(-a_nt+\rho_1-\frac{a_{\mu-1}}{t^{n-1}}+\frac{2\rho_2}{t^{n-1}}+\frac{|a_{\mu-1}|}{t^{n-1}}+\frac{2a_\lambda}{t^{n-\lambda-1}}).$$

Proof. Consider the polynomial

$$\begin{aligned}
F(z) &= (t-z)P_\mu(z)\\
&= a_0(t-z)+\sum_{j=\mu}^{n} a_jtz^j-\sum_{j=\mu+1}^{n+1} a_{j-1}z^j\\
&= a_0t+(a_{\mu-1}-a_0-\rho_2)z+\rho_2z-a_{\mu-1}z+a_\mu tz^\mu\\
&\quad +(a_nt-a_{n-1})z^n+\sum_{j=\mu+1}^{n-1}(a_jt-a_{j-1})z^j-a_nz^{n+1}\\
&= -a_nz^{n+1}+(-\rho_1-a_nt)z^n+a_\mu tz^\mu+(a_nt+\rho_1-a_{n-1})z^n\\
&\quad +a_ntz^n+a_0t+(a_{\mu-1}-a_0-\rho_2)z+\rho_2z-a_{\mu-1}z\\
&\quad +(a_{\mu+1}t-a_\mu)z^{\mu+1}+\sum_{j=\mu+1}^{n-1}(a_jt-a_{j-1})z^j.
\end{aligned}$$

Then we have

$$\begin{aligned}
|F(z)| &= |-a_nz^{n+1}+(-\rho_1-a_nt)z^n+a_\mu tz^\mu+(a_nt+\rho_1-a_{n-1})z^n\\
&\quad +a_ntz^n+a_0t+(a_{\mu-1}-a_0+\rho_2)z-\rho_2z-a_{\mu-1}z\\
&\quad +(a_{\mu+1}t-a_\mu)z^{\mu+1}+\sum_{j=\mu+2}^{\lambda}(a_jt-a_{j-1})z^j\\
&\quad +\sum_{j=1+\lambda}^{n-1}(a_jt-a_{j-1})z^j|\\
&= |z^n(-a_nz+\rho_1+(a_nt-\rho_1-a_{n-1})+\frac{(a_{\mu+1}t-a_\mu)z^{\mu+1}}{z^n}+\frac{a_\mu tz^\mu}{z^n}\\
&\quad +\frac{a_0t+(a_{\mu-1}-a_0+\rho_2)z-\rho_2z-a_{\mu-1}z}{z^n}+\sum_{j=\mu+2}^{\lambda}\frac{(a_jt-a_{j-1})z^j}{z^n}\\
&\quad +\sum_{j=1+\lambda}^{n-1}\frac{(a_jt-a_{j-1})z^j}{z^n})|\\
&\geq |z|^n|a_nz-\rho_1|-|z|^n[|a_nt-\rho_1-a_{n-1}|+\frac{|a_{\mu+1}t-a_\mu|}{|z|^{n-\mu-1}}+\frac{|a_\mu|}{|z|^{n-\mu}}\\
&\quad +\frac{|a_0|t}{|z|^n}+\frac{|a_{\mu-1}-a_0+\rho_2|}{|z|^{n-1}}+\frac{\rho_2}{|z|^{n-1}}\\
&\quad +\frac{|a_{\mu-1}|}{|z|^{n-1}}+\sum_{j=\mu+2}^{\lambda}\frac{|a_jt-a_{j-1}|}{|z|^{n-j}}+\sum_{j=1+\lambda}^{n-1}\frac{|a_jt-a_{j-1}|}{|z|^{n-j}}].
\end{aligned}$$

Now, let $|z| \geq t$, so that $\frac{1}{|z|^{n-j}} \leq \frac{1}{|t|^{n-j}}$ for $1 \leq \mu \leq j \leq n$. Then, we have

$$\begin{aligned} |F(z)| &\geq |z|^n\Big[|a_n z - \rho_1| - |(-a_n t + \rho_1 + a_{n-1} + \tfrac{a_\mu}{t^{n-\mu-1}} \\ &\quad + \tfrac{a_0}{t^{n-1}} - \tfrac{a_{\mu-1}}{t^{n-1}} - \tfrac{a_0}{t^{n-1}} + \tfrac{\rho_2}{t^{n-1}} + \tfrac{\rho_2}{t^{n-1}} \\ &\quad + \tfrac{|a_{\mu-1}|}{t^{n-1}} + \tfrac{a_\lambda}{t^{n-\lambda-1}} - \textstyle\sum_{j=\mu+2}^{\lambda-1} - \tfrac{a_\mu}{t^{n-\mu-1}} - \textstyle\sum_{j=\mu+2}^{\lambda-1} \tfrac{a_j}{t^{n-j-1}} \\ &\quad + \tfrac{a_\lambda}{t^{n-\lambda-1}} + \textstyle\sum_{j=\lambda+1}^{n-2} \tfrac{a_j}{t^{n-\lambda-1}} - a_{n-1} - \textstyle\sum_{j=\lambda+1}^{n-2} \tfrac{a_j}{t^{n-\lambda-1}})|\Big] \\ &> 0. \end{aligned}$$

It is easy to see that

$$|a_n z - \rho_1| > (-a_n t + \rho_1 - \tfrac{a_{\mu-1}}{t^{n-1}} + \tfrac{2\rho_2}{t^{n-1}} + \tfrac{|a_{\mu-1}|}{t^{n-1}} + \tfrac{2a_\lambda}{t^{n-\lambda-1}}),$$

that is,

$$|z - \tfrac{\rho_1}{a_n}| > \tfrac{1}{|a_n|}(-a_n t + \rho_1 - \tfrac{a_{\mu-1}}{t^{n-1}} + \tfrac{2\rho_2}{t^{n-1}} + \tfrac{|a_{\mu-1}|}{t^{n-1}} + \tfrac{2a_\lambda}{t^{n-\lambda-1}}),$$

then all the zeros of $F(z)$ whose modulus is greater than or equal to t lie in

$$|z - \tfrac{\rho_1}{a_n}| \leq \tfrac{1}{|a_n|}(-a_n t + \rho_1 - \tfrac{a_{\mu-1}}{t^{n-1}} + \tfrac{2\rho_2}{t^{n-1}} + \tfrac{|a_{\mu-1}|}{t^{n-1}} + \tfrac{2a_\lambda}{t^{n-\lambda-1}}).$$

But observe that all the zeros of $P_\mu(z)$ are also the zeros of $F(z)$. Hence it follows that all the zeros of $F(z)$ and hence of $P_\mu(z)$ lie in the union of the disks $|z| \leq t$ and

$$|z - \tfrac{\rho_1}{a_n}| \leq \tfrac{1}{|a_n|}(-a_n t + \rho_1 - \tfrac{a_{\mu-1}}{t^{n-1}} + \tfrac{2\rho_2}{t^{n-1}} + \tfrac{|a_{\mu-1}|}{t^{n-1}} + \tfrac{2a_\lambda}{t^{n-\lambda-1}}).$$

This completes the proof of Theorem 1. □

References

[1] A.A. Mogbademu and J.A. Adepoju, *On the location of zeros of a certain class of polynomials with restricted coefficients*, Nonlinear Funct. Anal. Appl., **23**(2) (2018), 305-309.