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LOCAL APPROXIMATE SOLUTIONS OF A CLASS OF NONLINEAR DIFFUSION POPULATION MODELS

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Abstract. This paper studies approximate solutions for a class of nonlinear diffusion population models. Our methods are to use the fundamental solution of heat equations to construct integral forms of the models and the well-known Banach compression map theorem to prove the existence of positive solutions of integral equations. Non-steady-state local approximate solutions for suitable harvest functions are obtained by utilizing the approximation theorem of multivariate continuous functions.

1. Introduction

The following equation governed by reaction-diffusion equations

$$w_t = d\Delta w + rw(1 - \frac{w}{K}) - h(X, w, t), \quad (X, t) \in \Omega \times R_+ \tag{1.1}$$

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subject to the suitable boundary conditions (such as Dirichlet boundary conditions $w(X,t) = 0, X \in \Omega$) or initial value conditions $w(X,0) = w_0(X)$ has been used to describe the temporal behavior of population of one species which inhabits a suitable set $\Omega \subset R^n$, where Δw is the Laplace operator, $w_t = \dfrac{\partial w}{\partial t}$, the parameter $r > 0$ is the intrinsic growth rate of the species, $d > 0$ is the diffusion coefficient, $K > 0$ is the environmental carrying capacity, $w(X,t)$ is the population number of a species at time t and location X in Ω, $h(X,w,t)$ is a harvest function.

When $h(X,w,t) \equiv 0$ and $K = 1$, (1.1) is often called Fisher's equation, it was introduced by Fisher to model the advance of a mutant gene in an infinite one-dimensional habitat[4]. Since then, such model has been widely studied by many authors. Here, we only mention a few. In 1979, Ludwig, Aronson and Weinberger [6] used (1.1) to investigate the critical size of the spruce budworm survival in a patch of forest and the width of an effective barrier that prevent spruce budworm transmission. In 2003, Neubert [9] studied the the optimal capture of a Marine protected area by using a proportional harvest function and

$$\begin{cases} w_t = rw(1 - \dfrac{w}{K}) + D\Delta w - qE(X)w, \\ w(T,0) = w(T,L) = 0, \end{cases}$$

where $0 < X < L$ is the size of habitat patch.

In 2007, Roques and Chekroun [10] considered the quasi-constant-yield harvest rate $\delta h(X)\rho_\epsilon(w)$, that is, they studied the steady-state solutions (w is independent of t, that is, $\frac{\partial w}{\partial t} \equiv 0$) of the following equation

$$w_t = D\Delta w + w(\mu(X) - \upsilon(X)w) - \delta h(X)\rho_\epsilon(w), \quad (X,t) \in \Omega \times R_+,$$

subject to Neumann boundary conditions and a more general setting $\Omega \subset R^n$.

In 2017, by studying the existence of positive solutions of semi-positone Hammerstein integral equations, Lan and Lin [7] proved that in one-dimensional habitat

$$w_t = rw(1 - \frac{w}{K}) + d\Delta w - h(X,w,t),$$

with the Dirichlet boundary conditions

$$w(T,0) = w(T,L) = 0,$$

has steady-state positive solutions for a harvest function $h(X,w,t) = \sigma$.

Up to now, to the best of our knowledge, existing study is limited basically to the steady-state solutions, there is very little study on non-steady-state solutions (that is, $\frac{\partial w}{\partial t} \neq 0$).

The work of this paper is to study non-steady-state local approximate solutions of the initial value problem in higher dimensions

$$\begin{cases} w_t = d\Delta w + rw(1-\dfrac{w}{K}) - h(X,w,t), & (X,t) \in R^n \times (0,\infty), \\ w(X,0) = w_0(X) \geq 0, w_0 \neq 0, & X \in R^n, \end{cases} \tag{1.2}$$

where $h(X,w,t) = \sigma w$ is a proportional harvest function, $0 < \sigma < r$.

2. Preliminaries

The following result is the approximation theorem for multivariate continuous functions ([2], Proposition 1.2, page 6).

Theorem 2.1. *Let $\Omega \subset R^n$ be bounded, $f \in C(\overline{\Omega})$. Then, for any $\epsilon > 0$, there exists $g \in C^\infty(R^n)$ such that $|f(x) - g(x)| < \epsilon$ on $\overline{\Omega}$, where g is defined as*

$$g(x) = f_\alpha(x) = \int_{R^n} \underline{f}(y)\psi_\alpha(y-x)dy, \quad x \in R^n, \alpha > 0,$$

$\underline{f}$ is the continuous expansion of f from $\overline{\Omega}$ to R^n,

$$\psi_1(x) := \begin{cases} c \cdot exp(-\dfrac{1}{1-|x|^2}), & |x| < 1, \\ 0, & |x| \geq 1, \end{cases}$$

$c > 0$ such that $\int_{R^n} \psi_1(x)dx = 1$, $\psi_\alpha(x) = \alpha^{-n}\psi_1(\dfrac{x}{\alpha})$.

Remark 2.2. We can choose g in Theorem 2.1 to have a compact support set (that is, there is a compact set N of R^n such that f is only non-zero on N). In fact, letting $R > \max\{|x| : x \in \overline{\Omega}\}$, $B_R(0) = \{x \in R^n : |x| \leq R\}$, $\partial B_R(0) = \{x \in R^n : |x| = R\}$, h is the continuous expansion of f from $\overline{\Omega}$ to $B_R(0)$,

$$\underline{f}(x) := \begin{cases} \dfrac{d(x, \partial B_R(0))}{d(x,\overline{\Omega}) + d(x,\partial B_R(0))} h(x), & x \in B_R(0), \\ 0, & x \in R^n \backslash B_R(0), \end{cases}$$

where $d(x,D)$ is the distance from x to the set D. It is easy to verify that $\underline{f}$ is continuous and when $\|x\| > R + \alpha$, $g(x) = 0$. Hence g has a compact support set.

Remark 2.3. In Theorem 2.1, if $f(x) \geq 0 (x \in \overline{\Omega})$, we can take g satisfying $g(x) \geq 0 (x \in R^n)$. In fact, according to the expansion theorem of continuous functions, we can take a non-negative continuous expansion h of f in Remark 2.2 from $\overline{\Omega}$ to $B_R(0)$ and from this obtain $g(x) \geq 0 (x \in R^n)$.

Next, we introduce the fundamental solution of heat equations [3]

$$u_t - \Delta u = 0 \tag{2.1}$$

and use it to construct the solutions to the initial value problems (2.1) and the nonhomogeneous

$$u_t - \Delta u = f, \tag{2.2}$$

where $t \geq 0$, $x \in R^n$, $u : R^n \times [0, \infty) \to R$, $f : R^n \times [0, \infty) \to R$ and Δu is the Laplace operator of u defined by

$$\Delta u = \frac{{}^2u}{x_1^2} + \frac{{}^2u}{x_2^2} + \cdots + \frac{{}^2u}{x_n^2} = \sum_{i=1}^{n} \frac{{}^2u}{x_i^2}.$$

The function

$$\Phi(x,t) := \begin{cases} \dfrac{1}{(4\pi t)^{n/2}} e^{-\frac{|x|^2}{4t}}, & x \in R^n,\ t > 0, \\ 0, & x \in R^n,\ t < 0, \end{cases}$$

satisfies (2.1) for $(x,t) \in R^n \times (0, \infty)$ and is called to be the fundamental solution of (2.1).

Lemma 2.4. ([3]) *For each time* $t > 0$, $\int_{R^n} \Phi(x,t)dx = 1$.

Assume that $g \in C(R^n) \bigcap L^\infty(R^n)$, we define

$$\begin{aligned} u(x,t) &= \int_{R^n} \Phi(x-y,t)g(y)dy \\ &= \frac{1}{(4\pi t)^{n/2}} \int_{R^n} e^{-\frac{|x-y|^2}{4t}} g(y)dy, (x \in R^n, t > 0). \end{aligned} \tag{2.3}$$

Theorem 2.5. ([3]) *Let* u *be defined in* (2.3). *Then*

(1) $u \in C^\infty(R^n \times (0, \infty))$,
(2) $u_t - \Delta u = 0 (x \in R^n, t > 0)$,
(3) $\lim_{(x,t)\to(x^0,0)} u(x,t) = g(x^0)(x \in R^n, t > 0)$ *for each point* $x^0 \in R^n$.

Let (see [3])

$$C_1^2(R^n \times [0,\infty)) = \{f : R^n \times [0,\infty) \to R | f, D_x f, D_x^2 f, f_t \in C(R^n \times [0,\infty))\}.$$

Assume that $f \in C_1^2(R^n \times [0,\infty))$ and f has a compact support set (that is, there is a compact set N of $R^n \times [0,\infty)$ such that f is only non-zero on

N), we define

$$u(x,t)=\int_0^t\int_{R^n}\Phi(x-y,t-s)f(y,s)dyds$$
$$=\int_0^t\frac{1}{(4\pi(t-s))^{n/2}}\int_{R^n}e^{-\frac{|x-y|^2}{4(t-s)}}f(y,s)dyds, \qquad (2.4)$$

where $x\in R^n, t>0$.

Theorem 2.6. ([3]) *Let u be defined in* (2.4). *Then*

(1) $u\in C_1^2(R^n\times(0,\infty))$,
(2) $u_t-\Delta u=f(x,t)(x\in R^n,t>0)$,
(3) $\lim_{(x,t)\to(x^0,0)}u(x,t)=0(x\in R^n,t>0)$ *for each point* $x^0\in R^n$.

Combining Theorem 2.5 and Theorem 2.6, we have

Theorem 2.7. *Let $f\in C_1^2(R^n\times[0,\infty))$ and it has a compact support set, $g\in C(R^n)\bigcap L^\infty(R^n)$. If $u\in C_1^2(R^n\times[0,\infty))$ satisfies the equation*

$$u(x,t)=\int_{R^n}\Phi(x-y,t)g(y)dy+\int_0^t\int_{R^n}\Phi(x-y,t-s)f(y,s)dyds, \qquad (2.5)$$

then u satisfies

$$\begin{cases} u_t-\Delta u=f, & (x,t)\in R^n\times(0,\infty),\\ \lim\limits_{(x,t)\to(x^0,0)}u(x,t)=g(x^0), & (x\in R^n,t>0)\ \text{for each point}\ x^0\in R^n.\end{cases}$$

Proof. Let

$$\nu(x,t)=\int_{R^n}\Phi(x-y,t)g(y)dy$$

and

$$\omega(x,t)=\int_0^t\int_{R^n}\Phi(x,y,t-s)f(y,s)dyds.$$

Then

$$u=\nu+\omega.$$

By Theorem 2.5, we have

$$\begin{cases} \nu_t-\Delta\nu=0, & (x,t)\in R^n\times(0,\infty),\\ \lim\limits_{(x,t)\to(x^0,0)}\nu(x,t)=0, & (x\in R^n,t>0)\ \text{for each point}\ x^0\in R^n.\end{cases} \qquad (2.6)$$

By Theorem 2.6, we have

$$\begin{cases} \omega_t-\Delta\omega=f, & (x,t)\in R^n\times(0,\infty),\\ \lim\limits_{(x,t)\to(x^0,0)}\omega(x,t)=g(x^0), & (x\in R^n,t>0)\ \text{for each point}\ x^0\in R^n.\end{cases}$$

Since $u_t=\nu_t+\omega_t$, $\Delta u=\Delta\nu+\Delta\omega$, we have the desired result. □

3. A fixed point of compression map

Let $x = \frac{X}{\sqrt{d}}$ and $u(x,t) = w(X,t)$. Then $d\Delta w_X = \Delta u_x$. The initial-value problem (1.2) is transformed into the diffusion equation of the form

$$\begin{cases} u_t = \Delta u + ru(1 - \dfrac{u}{K}) - h(\sqrt{d}x, u, t), & (X,t) \in R^n \times (0,\infty), \\ u(x,0) = w_0(X,0) = w_0(\sqrt{d}x, 0) = g(x) \geq 0, g \neq 0, & X \in R^n, \end{cases} \tag{3.1}$$

where $(x,t) \in R^n \times (0,\infty)$, which allows us to study (1.2) by studying (3.1). Based on the relevant properties and conclusions of heat equations, integral forms of non-steady-state solutions of (3.1) is constructed, and the existence of integral equations is proved by applying the well-known Banach compression theorem.

Let $T > 0$ be constant, $C_T(R^n \times [0,T])$ be a set of all real-valued bounded continuous functions on $R^n \times [0,T]$. For $u \in C_T(R^n \times [0,T])$, we define the norm

$$||u|| = \sup\{|u(x,t)| : (x,t) \in (R^n \times [0,T])\}.$$

A standard augment shows that $C_T(R^n \times [0,T])$ is a Banach space and the details are omitted.

To obtain local approximate solutions of (1.2), we define a map A and prove that A has a fixed point.

Let $\tilde{f} \in C(R^n \times R^1 \times [0,T])$ and it has a compact support set and $g \in C(R^n) \bigcap L^\infty(R^n)$. For $u \in C_T(R^n \times [0,T])$, we define a map A by

$$Au(x,t) := \begin{cases} B(x,t) + C(x, u(x), t) & x \in R^n, t \in (0,T], \\ g(x) & x \in R^n, t = 0, \end{cases} \tag{3.2}$$

where

$$B(x,t) := \begin{cases} \int_{R^n} \Phi(x-y,t) g(y) dy & x \in R^n, t \in (0,T], \\ g(x) & x \in R^n, t = 0, \end{cases}$$

$$C(x, u(x), t) := \begin{cases} \int_0^t \int_{R^n} \Phi(x-y, t-s) \tilde{f}(y, u(y), s) dy ds & x \in R^n, t \in (0,T], \\ 0 & x \in R^n, t = 0. \end{cases}$$

Then by Theorem 2.5 and Theorem 2.6, we have $B(x,t), C(x,u(x),t) \in C_T(R^n \times [0,T])$ and A maps $C_T(R^n \times [0,T])$ into $C_T(R^n \times [0,T])$.

Theorem 3.1. *Let A be defined by (3.2). Assume that $\tilde{f}$ with respect to the second variable satisfies the Lipschitz condition*

$$|\tilde{f}(y,u,t) - \tilde{f}(y,v,t)| \leq L|u - v|,$$

where L is a Lipschitz constant. If $LT < 1$, then A has a unique fixed point u in $C_T(R^n \times [0,T])$. Further, if $\tilde{f} \geq 0$ on $R^n \times R^1 \times [0,T]$, then $u(x,t) > 0$ for $(x,t) \in R^n \times (0,T]$.

Proof. For $u, v \in C_T(R^n \times [0,T])$, we get

$$\begin{aligned} |Au - Av| &\le \int_0^t \int_{R^n} \Phi(x-y,t-s)|\tilde{f}(y,u(y,s),s) - \tilde{f}(y,v(y,s),s)|dyds \\ &\le \int_0^t \int_{R^n} \Phi(x-y,t-s) \cdot L|u(y,s)-v(y,s)|dyds \\ &\le L\|u-v\| \int_0^t \int_{R^n} \Phi(x-y,t-s)dyds. \end{aligned}$$

According to Lemma 2.4, we have

$$\int_{R^n} \Phi(x-y,t-s)dy = \int_{R^n} \Phi(y,t-s)dy = 1, \quad t > s$$

and so

$$\|Au - Av\| \le L\|u-v\| \int_0^t \int_{R^n} \Phi(x-y,t-s)dyds \le LT\|u-v\|.$$

Since $LT < 1$, by the well-known Banach compression theorem, there exists a unique $u \in C_T(R^n \times [0,T])$ such that $Au = u$, and for any $u_0 \in C_T(R^n \times [0,T])$, $A^n u_0 \to u$, where

$$u_n = A^n u_0, ||u_n - u|| \le \alpha^{n-1}||u_1 - u_0||, \alpha = LT < 1.$$

Let $\tilde{f} \ge 0$ on $R^n \times R^1 \times [0,T]$. If there exists $(x_0,t_0) \in (R^n \times (0,T])$ such that $u(x_0,t_0) = 0$, by (3.2), we have $\int_{R^n} \Phi(x_0 - y, t_0)g(y)dy = 0$ and then $g(y) \equiv 0$, which contradicts $g \ne 0$. □

4. Local approximate solutions of (1.2)

Local approximate solutions of (1.2) mean that there exist some $T > 0$, for any $M > 0$ and $\epsilon > 0$, there is $w^{(\epsilon)} \in C_1^2(B_M(0) \times (0,T])$ with $w^{(\epsilon)} > 0$ on $B_M(0) \times (0,T]$ satisfying

$$\begin{cases} \sup\{|w_t^{(\epsilon)} - d\Delta w^{(\epsilon)} - \tilde{h}| : (X,t) \in B_M(0) \times (0,T]\} \to 0, \\ \lim\limits_{(x,t)\to(x^0,0)} w^{(\epsilon)}(X,t) = g(x^0)(x \in R^n, t > 0) \text{ for each point } x^0 \in B_M(0) \end{cases} \tag{4.1}$$

as $\epsilon \to 0$, where $B_M(0) = \{X : x \in R^n, |X| \le M\}$ and $\tilde{h}(w) = rw(1 - \frac{w}{K}) - \sigma w$.

Let $a = r - \sigma$, $b = \frac{r}{K}$ and

$$f_0(z) := \begin{cases} 0, & z < 0, \\ \tilde{h}(z), & 0 \le z \le \frac{a}{b}, \\ 0, & z > \frac{a}{b}. \end{cases}$$

Notice that $f_0'(z) = 0$ $(z \in (-\infty, 0) \cup (\frac{a}{b}, \infty))$, $f_0'(z) = a - 2bz$ $(0 \leq z \leq \frac{a}{b})$. It is easy to know that f_0 is non-negative on R^1 and satisfies Lipschitz condition with the constant $L = a$.

Theorem 4.1. *Assume $g \in C_+(R^n) \bigcap L^\infty(R^n) \backslash \{0\}$ and $T > 0$ satisfies $\|g\|_{L^\infty} + \frac{Ta^2}{4b} = \frac{a}{b}$. If $Ta < 1$, then (1.2) has local approximate solutions.*

Proof. Setting $\tilde{f}(y, z, t) = f_0(z)$. By $Ta < 1$ and Theorem 3.1, there is a unique $u \in C_T(R^n \times [0, T])$, $u(x, t) > 0$, $x \in R^n$, $0 < t \leq T$ satisfying

$$u(x,t) = \int_{R^n} \Phi(x-y,t)g(y)dy + \int_0^t \int_{R^n} \Phi(x-y,t-s) f_0(u(y,s))dyds. \quad (4.2)$$

Notice that $0 \leq f_0(z) \leq \frac{a^2}{4b}$, we obtain

$$\begin{aligned} |u(x,t)| &\leq \|g\|_{L^\infty} \int_{R^n} \Phi(x-y,t)dy + \frac{a^2}{4b} \int_0^T \int_{R^n} \Phi(x-y,t-s)dyds \\ &= \|g\|_{L^\infty} + \frac{Ta^2}{4b} = \frac{a}{b} \end{aligned}$$

and $f_0(u(x,t)) = \tilde{h}(u(x,t)) \in C(R^n \times [0,T])$. Hence

$$u(x,t) = \int_{R^n} \Phi(x-y,t)g(y)dy + \int_0^t \int_{R^n} \Phi(x-y,t-s)\tilde{h}(u(y,s))dyds.$$

For any $M > 0$ and $\epsilon > 0$, by Theorem 2.1 and the Remarks, there is $h^{(\epsilon)}$ with a compact support set satisfying

$$h^{(\epsilon)} \in C^{(\infty)}(R^n \times [0,T]), \quad h^{(\epsilon)} \geq 0, \quad (x,t) \in R^n \times [0,T]$$

and so $|\tilde{h}(u(x,t)) - h^{(\epsilon)}(x,t)| < \epsilon$ on $B_{\frac{M}{\sqrt{d}}}(0) \times [0,T]$. Let

$$u^{(\epsilon)}(x,t) = \int_{R^n} \Phi(x-y,t)g(y)dy + \int_0^t \int_{R^n} \Phi(x-y,t-s)h^{(\epsilon)}(y,s)dyds. \quad (4.3)$$

Since $g(x) \geq 0$ and $g(x) \neq 0$, then $u^{(\epsilon)}(x,t) > 0$ on $B_{\frac{M}{\sqrt{d}}}(0) \times (0,T]$. By Theorem 2.7, we have

(1) $u^{(\epsilon)} \in C^{(\infty)}(B_{\frac{M}{\sqrt{d}}}(0) \times (0,T])$,

(2) $u_t^{(\epsilon)} - \Delta u^{(\epsilon)} = h^{(\epsilon)}, (x,t) \in B_{\frac{M}{\sqrt{d}}}(0) \times (0,T]$,

(3) $\lim\limits_{(x,t) \to (x^0,0)} u^{(\epsilon)}(x,t) = g(x^0) (x \in B_{\frac{M}{\sqrt{d}}}(0), 0 < t < T)$.

By

$$|u(x,t) - u^{(\epsilon)}(x,t)| \leq \int_0^t \int_{R^n} \Phi(x-y,t-s)|\tilde{h}(u(y,s)) - h^{(\epsilon)}(y,s)|dyds$$

for $(x,t) \in B_{\frac{M}{\sqrt{d}}}(0) \times [0,T]$ and Lemma 2.4, we obtain

$$|u(x,t) - u^{(\epsilon)}(x,t)| \le \epsilon \int_0^t \int_{R^n} \Phi(x-y,t-s)dyds \le \epsilon T$$

and

$$|\tilde{h}(u(x,t)) - h(u^{(\epsilon)}(x,t))| \le a|u(x,t) - u^{(\epsilon)}(x,t)| \le a\epsilon T$$

for $(x,t) \in B_{\frac{M}{\sqrt{d}}}(0) \times (0,T]$. Let

$$\begin{aligned} \Sigma_1 &= u_t^{(\epsilon)}(x,t) - \Delta u^{(\epsilon)}(x,t) - h^{(\epsilon)}(x,t), \\ \Sigma_2 &= h^{(\epsilon)}(x,t) - \tilde{h}(u(x,t)), \\ \Sigma_3 &= \tilde{h}(u(x,t)) - h(u^{(\epsilon)}(x,t)). \end{aligned}$$

Then for $(x,t) \in B_{\frac{M}{\sqrt{d}}}(0) \times (0,T]$,

$$\begin{aligned} |u_t^{(\epsilon)}(x,t) - \Delta u^{(\epsilon)}(x,t) - \tilde{h}(u^{(\epsilon)}(x,t))| &= |\Sigma_1 + \Sigma_2 + \Sigma_3| = |\Sigma_2 + \Sigma_3| \\ &\le |\Sigma_2| + |\Sigma_3| \le (aT+1)\epsilon \to 0 \end{aligned}$$

and $\lim\limits_{(x,t)\to(x^0,0)} u^{(\epsilon)}(x,t) = g(x^0)$ as $\epsilon \to 0$ for each point $x^0 \in B_{\frac{M}{\sqrt{d}}}(0)$ and $t > 0$.

Let $X = \sqrt{d}x$ and $w^{(\epsilon)}(X,t) = u^{(\epsilon)}(\sqrt{d}x,t) \in C_1^2(B_M(0) \times [0,T])$. By (4.1), (1.2) has local approximate solutions. This completes the proof. □

5. Discussion

In this paper, local approximate solutions of the initial value problem (1.2) are obtained for $h = rw(1-w) - \sigma w$. Since the function $\Phi(x,t)$ appears in the integral equation (3.2), it brings great difficulties to the calculation of approximate solutions. How to calculate approximate solutions is our future work.

Theorem 2.7 plays a key role in the study of approximate solutions of (1.2). If f in Theorem 2.7 does not satisfy Lipschitz condition, then the study will be difficult and we need to use the theory of partial differential equations [1, 5] and other methods such as topological or variational methods [1, 2, 8].

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