Nonlinear Functional Analysis and Applications
Vol. 26, No. 5 (2021), pp. 1105-1114
ISSN: 1229-1595(print), 2466-0973(online)

https://doi.org/10.22771/nfaa.2021.26.05.18
http://nfaa.kyungnam.ac.kr/journal-nfaa

A FIXED POINT THEOREM FOR NON-SELF G-CONTRACTIVE TYPE MAPPINGS IN CONE METRIC SPACE ENDOWED WITH A GRAPH

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Abstract In this paper, we prove a fixed point theorem for G-contractive type non-self mapping in cone metric space endowed with a graph. Our result generalizes many results in the literature and provide a new pavement for solving nonlinear functional equations.

1. Introduction and preliminaries

Continuity and convergence of functions have been dealt in many branches of Mathematics. The study of metric space and its generalizations showed a new way for many mathematicians to put this concept of continuity and convergence in a more elaborative setting. Recently, Huang and Zhang [13] defined the concept of cone metric space by replacing the set of real numbers by

[0]Received September 3, 2020. Revised November 13, 2020. Accepted April 11, 2021.
[0]2010 Mathematics Subject Classification: 47H10, 54H25.
[0]Keywords: G-contractive type mapping, symmetric graph, subgraph, non-self mapping.
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an ordered Banach space and established some fixed point theorems for mappings satisfying variety of contraction conditions along with much celebrated Banach contraction mapping in the setting of cone metric space in which the normality of cone is an essential ingredient. Several authors [5, 7, 8, 11] analyzed Kannan type non-self contraction mappings and Chatterjea type non-self contractive mappings in Banach space endowed with graph. Also, Imdad and Kumar [14] proved Rhoades-type Fixed point theorems for a pair of non-self mappings for Banach space.

The aim of this paper is to prove a fixed point theorem for G-contractive type non-self mapping in cone metric space endowed with a graph. Our result generalizes many results in the literature.

Definition 1.1. ([13]) Let E be a real Banach space. A subset P of E is called a cone if the following conditions are hold:

(a) P is closed, nonempty and $P \neq \{0\}$;
(b) $a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P$ implies $ax + by \in P$;
(c) $x \in P$ and $-x \in P$ implies $x = 0$.

Definition 1.2. ([13]) Let P be a cone in Banach space E, define partial ordering '$\leq$' with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in Int\ P$, where $Int\ P$ denotes the interior of P. The cone P is called normal if there is a number $k > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\|x\| \leq k\|y\|$. The least positive number satisfying this inequality is called the normal constant of P.

In the following, suppose E is a Banach space, P is a cone in E with $Int\ P \neq \emptyset$ and $\leq$ is a partial ordering with respect to P.

Definition 1.3. ([13]) Let X be a nonempty set and E be a real Banach space. Suppose that the mapping $d : X \times X \to E$ satisfy:

(d1) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
(d2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(d3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X, d) is called a cone metric space.

It is clear that the concept of a cone metric space is more general than that of a metric space.

Example 1.4. ([13]) Let $E = \mathbb{R}^2$, $P = \{(x, y) \in E \text{ such that} : x, y \geq 0\} \subset \mathbb{R}^2$, $X = \mathbb{R}$ and $d : X \times X \to E$ defined by $d(x, y) = (|x-y|, \alpha|x-y|)$ where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space.

Definition 1.5. ([13]) Let (X, d) be a cone metric space. We say that the sequence $\{x_n\}$ is

(1) a Cauchy sequence if, for every c in E with $0 \ll c$, there is an N such that for all $n, m > N$, $d(x_n, x_m) \ll c$;

(2) a convergent sequence, if for every c in E with $0 \ll c$, there is an N such that for all $n > N$, $d(x_n, x) \ll c$ for some fixed x in X.

Remark 1.6. ([13]) If $c \in IntP$, $0 \leq a_n$ and $a_n \to 0$, then there exists an n_0 such that for all $n > n_0$, we have $a_n \ll c$.

Remark 1.7. ([13]) If $0 \leq d(x_n, x) \leq b_n$ and $b_n \to 0$, then $d(x_n, x) \ll c$ where $\{x_n\}$ is a sequence and x is a given point in the cone metric space X.

Remark 1.8. ([13]) If $0 \leq u \ll c$ for each $c \in int\ P$, then $u = 0$.

Definition 1.9. ([15]) Let Δ denote the diagonal of the Cartesian product $X \times X$. Let $G = (V(G), E(G))$ be simple directed graph, where $V(G)$ is the set of vertices coincides with X and $E(G)$ is the set of its edges containing all loops, that is, $\Delta \subset E(G)$. G^{-1} is called the converse graph of G, defined as

$$E(G^{-1}) = \{(y, x) \in X \times X : (x, y) \in E(G)\}.$$

If x and y are vertices in the graph G, then a *path* from x to y of length N is a sequence $\{x_i\}_{i=0}^{N}$ of $N + 1$ vertices of G such that

$x_0 = x, x_N = y$ and $\{x_{i-1}, x_i\} \in E(G), i = 1, 2, ..., N$.

A Graph is called connected if there is at least a path between any two vertices.

Definition 1.10. ([15]) If $G = \{V(G), E(G)\}$ is a graph and $H \subset V(G)$. Then the graph $\{H, E(H)\}$ with $E(H) = E(G) \cap (H \times H)$ is known as the subgraph of G determined by H. It is mentioned as G_H. $\tilde{G}$ is called a symmetric graph by uniting G and G^{-1}, that is, $E(\tilde{G}) = E(G) \cup E(G^{-1})$.

Definition 1.11. ([15]) A mapping $T : X \to X$ is said to be well defined on a metric space endowed with a graph G if it has the following property:

$$(x, y) \in E(G) \quad \Rightarrow \quad (Tx, Ty) \in E(G). \tag{1.1}$$

Definition 1.12. ([15]) A well defined non-self mapping $T : K \to X$ on a metric space endowed with a graph G is said to be

(i) a G-contraction if there is a constant $\alpha \in (0, 1)$ such that $d(Tx, Ty) \leq \alpha d(x, y)$ for all $x, y \in E(G_K)$,

(ii) a G-contractive type (or generalized G-contractive) mapping, if the following inequality holds:

$$d(Tx,Ty) \leq \alpha \max\left\{\frac{1}{2}d(x,y), d(Tx,x), d(Ty,y), \frac{1}{q}[d(Tx,y)+d(Ty,x)]\right\} \tag{1.2}$$

for all $x, y \in E(G_K)$, and $0 < \alpha < 1$, $q \geq 1 + 2\alpha$.

The following important concept used in [9] is needed in the sequel:

Definition 1.13. ([9]) Let X be a Banach space, K be a nonempty closed subset of X and $T : K \to X$ be a non-self mapping. Let $x \in K$ and $Tx \notin K$. Let $y \in \partial K$ be the corresponding element such that $y = (1-\lambda)x + \lambda Tx$ $(0 < \lambda < 1)$ which in turn express the fact that $d(x, Tx) = d(x, y) + d(y, Tx)$, $y \in \partial K$. If for any such element x, we have

$$d(y, Ty) \leq d(x, Tx) \tag{1.3}$$

for all corresponding $y \in Y$, then we say that T has property (M).

Definition 1.14. ([5]) Let (X, d, G) be a Banach space endowed with a simple, directed and weakly connected graph G is said to hold the property (L), if for any sequence $\{x_n\}_{n=1}^{\infty} \subset X$ with $x_n \to x$ as $n \to \infty$ and $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$, there exists a subsequence $\{x_{k_n}\}_{n=1}^{\infty}$ satisfying

$$(x_{k_n}, x) \in E(G), \ \forall n \in \mathbb{N}. \tag{1.4}$$

2. Main result

Theorem 2.1. *Let (X, d, G) be a cone metric space endowed with a simple, directed and weakly connected graph G with property (L). Also, let K be a nonempty closed subset of X such that $(x, y) \in E(G_K)$ where G_K is the sub graph of G confined by the nonempty subset K. Suppose that $T : K \to X$ is a G-contractive type mapping having property (M). If $K_T := \{x \in \partial K : (x, Tx) \in E(G)\} \neq \emptyset$ and T satisfies Rothe's boundary condition*

$$T(\partial K) \subset K, \tag{2.1}$$

then

(a) *$Fix(T) \neq \emptyset$, and*
(b) *Picard iteration $\{x_n = T^n x_0\}_{n=1}^{\infty}$ converges to $w \in Fix(T)$, for all $x_0 \in K_T$.*

Proof. If $T(K) \subset K$, then T is actually a self mapping of the closed set K and the conclusion follows by Rhoades fixed point theorem [22] with $X = K$. Therefore, in the following, we consider only the case $T(K)$ is not a subset of K. Choose $x_0 \in K_T$, which in turn imply that $(x_0, Tx_0) \in E(G_K)$ and by repeated performance of (1.1),

$$(T^n x_0, T^{n+1} x_0) \in E(G),\ \forall\, n \in \mathbb{N}. \tag{2.2}$$

Denote $y_n := T^n x_0$ for all $n \in \mathbb{N}$. By (2.2), it follows that $Tx_0 \in K$. Denote $x_1 := y_1 = Tx_0$. Now, if $Tx_1 \in K$, set $x_2 := y_2 = Tx_1$. If Tx_1 is not in K, we can choose an element x_2 on the segment $[x_1, Tx_1]$ which also belong to ∂K, that is

$$x_2 = (1-\lambda)x_1 + \lambda T x_1 \ \ (0 < \lambda < 1). \tag{2.3}$$

Continuing in this way, we form two sequences $\{x_n\}$ and $\{y_n\}$.

(i) $\{x_n\} = \{y_n\} = Tx_{n-1}$, if Tx_{n-1} is in K,
(ii) $x_n = (1-\lambda)x_{n-1} + \lambda T x_{n-1} \in \partial K (0 < \lambda < 1)$, if Tx_{n-1} is not in K.

Next, denote

$$P = \{x_k \in \{x_n\} : x_k = y_k = Tx_{k-1}\}, \tag{2.4}$$

$$Q = \{x_k \in \{x_n\} : x_k \neq Tx_{k-1}\}. \tag{2.5}$$

Note that $\{x_n\} \subset K$ for all $n \in \mathbb{N}$ and that if $x_k \in Q$, then both x_{k-1} and x_{k+1} belong to the set P. By (2.2), we cannot have two consecutive terms of $\{x_n\}$ in the set P.

Continuing this, we get three cases, to prove $\{x_n\}$ is Cauchy.

Case (1): Let $x_n, x_{n+1} \in P$. In this case, $x_n = y_n = Tx_{n-1} \in K$ and $x_{n+1} = y_{n+1} = Tx_n \in K$. But x_{n-1} need not be equal to y_{n-1}.

Since $\{x_n\} \subset K$ for all $n \in \mathbb{N}$, by (2.3), $(x_n, x_{n+1}) \in E(G_K)$ and so by contraction condition (2.1),

$$\begin{aligned} d(x_n, x_{n+1}) &= d(y_n, y_{n+1}) = d(Tx_{n-1}, Tx_n) \\ &\leq \alpha \max\left\{\frac{d(x_{n-1}, x_n)}{2}, d(Tx_{n-1}, x_{n-1}), d(Tx_n, x_n), \right. \\ &\qquad \left. \frac{d(Tx_{n-1}, x_n) + d(Tx_n, x_{n-1})}{q}\right\} \\ &= \alpha \max\left\{\frac{d(x_{n-1}, x_n)}{2}, d(y_n, x_{n-1}), d(y_{n+1}, x_n), \frac{d(y_n, x_n) + d(y_{n+1}, x_{n-1})}{q}\right\}. \end{aligned} \tag{2.6}$$

Since there are infinitely many n values, we obtain

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \alpha \frac{d(x_{n-1}, x_{n+1})}{q} \\ &\leq \alpha \frac{1}{q}[d((x_{n-1}, x_n) + d(x_n, x_{n+1})], \end{aligned}$$

that is,

$$\left(1 - \frac{\alpha}{q}\right) d(x_n, x_{n+1}) \leq \frac{\alpha}{q} d(x_{n-1}, x_n).$$

Hence, we have

$$d(x_n, x_{n+1}) \leq \frac{\alpha}{q - \alpha} d(x_{n-1}, x_n).$$

Case (2): Let $x_n \in P$, $x_{n+1} \in Q$. Then, $x_n = y_n = Tx_{n-1} \in K$ and $x_{n+1} \neq y_{n+1} = Tx_n \in K$. Then $x_n \in \partial K$ and

$$d(x_n, x_{n+1}) + d(x_{n+1}, Tx_n) = d(x_n, Tx_n).$$

Since $d(x_{n+1}, Tx_n) \neq 0$,

$$d(x_n, x_{n+1}) = d(x_n, Tx_n) - d(x_{n+1}, Tx_n) < d(x_n, Tx_n). \tag{2.7}$$

Now by a similar argument as in Case 1, $(x_n, x_n - 1) \in E(G_K)$ and hence by contraction condition (2.1),

$$\begin{aligned} &d(y_n, y_{n+1}) = d(x_n, Tx_n) = d(Tx_{n-1}, Tx_n) \\ &\leq \alpha \max\left\{\frac{d(x_{n-1}, x_n)}{2}, d(Tx_{n-1}, x_{n-1}), d(Tx_n, x_n), \right. \\ &\qquad \left. \frac{d(Tx_{n-1}, x_n) + d(Tx_n, x_{n-1})}{q}\right\} \\ &= \alpha \max\left\{\frac{d(x_{n-1}, x_n)}{2}, d(x_n, x_{n-1}), d(y_n, y_{n+1}), \frac{d(y_{n+1}, x_{n-1})}{q}\right\}. \end{aligned} \tag{2.8}$$

From the above, we conclude that

$$d(x_n, Tx_n) \leq \delta d(x_{n-1}, x_n), \text{ where } \delta = \max\left\{\frac{\alpha}{2}, \alpha, \frac{\alpha}{q - \alpha}\right\} = \alpha$$

and hence by (2.8), we have

$$d(x_{n+1}, Tx_n) < d(x_n, Tx_n) \leq \alpha d(x_{n-1}, x_n).$$

Case (3): Let $x_n \in Q$, $x_{n+1} \in P$. Then $x_n \neq y_n = Tx_{n-1}$, $x_{n+1} = y_{n+1} = Tx_n$ and

$$d(x_{n-1}, x_n) + d(x_n, Tx_{n-1}) = d(x_{n-1}, Tx_{n-1}). \tag{2.9}$$

By property (M),

$$d(x_n, x_{n+1}) = d(x_n, Tx_n) \leq d(x_{n-1}, Tx_{n-1}) \leq d(Tx_{n-2}, Tx_{n-1}). \tag{2.10}$$

By (2.3) $(y_{n-1}, y_n) \in E(G)$ and by using contraction condition (2.1), we get

$$\begin{aligned} &d(Tx_{n-2}, Tx_{n-1}) \\ &\leq \alpha \max\Big\{\frac{d(x_{n-2}, x_{n-1})}{2}, d(Tx_{n-2}, x_{n-2}), d(Tx_{n-1}, x_{n-1}), \\ &\qquad \frac{d(Tx_{n-2}, x_{n-1}) + d(Tx_{n-1}, x_{n-2})}{q}\Big\} \\ &= \alpha \max\left\{d(x_{n-1}, x_{n-2}), d(x_n, x_{n-1}), \frac{d(Tx_{n-1}, x_{n-2})}{q}\right\}. \end{aligned} \tag{2.11}$$

Hence we have

$$\begin{aligned} \frac{d(Tx_{n-1}, x_{n-2})}{q} &\leq \frac{d(Tx_{n-1}, x_{n-1}) + d(x_{n-1}, x_{n-2})}{q} \\ &\leq \frac{d(Tx_{n-2}, Tx_{n-1}) + d(x_{n-1}, x_{n-2})}{q}. \end{aligned}$$

Therefore, we have

$$\begin{aligned} &d(Tx_{n-2}, Tx_{n-1}) \\ &\leq \alpha \max\left\{d(x_{n-1}, x_{n-2}), d(x_n, x_{n-1}), \frac{d(Tx_{n-2}, Tx_{n-1}) + d(x_{n-1}, x_{n-2})}{q}\right\}. \end{aligned} \tag{2.12}$$

Here also we have three cases,

(iii) $d(Tx_{n-2}, Tx_{n-1}) \leq \alpha d(x_{n-1}, x_{n-2})$,
(iv) $d(Tx_{n-2}, Tx_{n-1}) \leq \alpha d(x_n, x_{n-1})$,
(v) $d(Tx_{n-2}, Tx_{n-1}) \leq \alpha \frac{d(Tx_{n-2}, Tx_{n-1}) + d(x_{n-1}, x_{n-2})}{q}$ and so, $d(Tx_{n-2}, Tx_{n-1}) \leq \frac{\alpha}{q-\alpha} d(x_{n-1}, x_{n-2})$.

Using (2.11), the above cases imply

$$d(x_n, x_{n+1}) \leq \lambda \beta_n,$$

where $\beta_n \in \{d(x_{n-2}, x_{n-1}), d(x_{n-1}, x_n)\}$ and $\lambda := \frac{\alpha}{q-\alpha}$.

For $n > 1$, $d(x_n, x_{n+1}) \le \lambda^{\frac{n-1}{2}}\beta_2$, $\beta_2 \in \{d(x_0, x_1), d(x_1, x_2)\}$.
Using triangle inequality, for $n > m$, we have

$$\begin{aligned} d(x_n, x_m) &\le d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + \cdots + d(x_{m+1}, x_m) \\ &\le \left(\lambda^{\frac{n-1}{2}} + \lambda^{\frac{n-2}{2}} + \cdots + \lambda^{\frac{m-1}{2}}\right)\beta_2 \\ &\le \frac{\sqrt{\lambda}^{m-1}}{1-\sqrt{\lambda}}\beta_2 \to 0 \quad \text{as} \quad m \to \infty. \end{aligned}$$

By Remarks 1.6 and 1.7, we have $d(x_n, x_m) \ll k$. Therefore $\{x_n\}$ is a Cauchy sequence in K. Since K is complete, there is some point $w \in K$ such that $x_n \to w$. There exists a subsequence $\{x_{n_m}\}$ such that $x_{n_m} = y_{n_m} = Tx_{n_{m-1}}$ and so $Tx_{n_{m-1}} \to w$.

Next we prove that $Tw = w$.

$$\begin{aligned} &d(Tw, w) \\ &\le d(Tw, Tx_{n_{m-1}}) + d(Tx_{n_{m-1}}, w) \\ &\le \alpha \max\left\{\frac{d(x_{n_{m-1}}, w)}{2}, d(Tx_{n_{m-1}}, x_{n_{m-1}}), d(Tw, w),\right. \\ &\qquad \left.\frac{d(Tx_{n_{m-1}}, w) + d(Tw, x_{n_{m-1}})}{q}\right\} + d(Tx_{n_{m-1}}, w). \end{aligned}$$

Using $x_{n_m} = y_{n_m} = Tx_{n_{m-1}} \to w$, as $m \to \infty$, we get, the following cases,

(1) $d(Tw, w) \le \alpha\frac{d(x_{n_{m-1}},w)}{2} + d(Tx_{n_{m-1}}, w) \ll \alpha\frac{k}{2\alpha} + \frac{k}{2} = k$,
(2) $d(Tw, w) \le \alpha d(Tx_{n_{m-1}}, x_{n_{m-1}}) + d(Tx_{n_{m-1}}, w)$
$\le \alpha(d(Tx_{n_{m-1}}, w) + d(w, x_{n_{m-1}}) + d(Tx_{n_{m-1}}, w)$
$= \alpha\frac{k}{2\alpha} + \frac{k}{2(\alpha+1)}(\alpha + 1) = k$,
(3) $d(Tw, w) \le \alpha d(Tw, x) + d(Tx_{n_{m-1}}, w)$ implies
$d(Tw, w) \le \frac{1}{1-\alpha}d(Tx_{n_{m-1}}, w) \ll \frac{1}{1-\alpha}(1-\alpha)k = k$,
(4)

$$\begin{aligned} d(Tw, w) &\le \alpha\frac{d(Tx_{n_{m-1}}, w) + d(Tw, x_{n_{m-1}})}{q} + d(Tx_{n_{m-1}}, w) \\ &\le \alpha\frac{d(Tx_{n_{m-1}}, w) + d(Tw, w) + d(w, x_{n_{m-1}})}{q} + d(Tx_{n_{m-1}}, w) \\ &\le \frac{\alpha}{q-\alpha}d(Tx_{n_{m-1}}, w) + \frac{\alpha}{q-\alpha}d(w, x_{n_{m-1}}) \\ &\ll \frac{\alpha}{q-\alpha}.\frac{k}{2\frac{\alpha}{q-\alpha}} + \frac{\alpha}{q-\alpha}.\frac{k}{2\frac{\alpha}{q-\alpha}} = k. \end{aligned}$$

Thus in all the above cases, $d(Tw, w) \ll k$ for each $k \in int\ P$. Using Remark 1.8, we get $d(Tw, w) = 0$ implies $Tw = w$. Hence, w is the fixed point of T. This completes the proof. □

Acknowledgments: The authors would like to thank the reviewers for giving valuable suggestions and useful comments.

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