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DISKCYCLIC C_0-SEMIGROUPS AND DISKCYCLICITY CRITERIA

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Abstract. In this article, we prove that diskcyclic C_0-semigroups exist on any infinite-dimensional Banach space. We show that a C_0-semigroup $(T_t)_{t\geq 0}$ satisfies the diskcyclicity criterion if and only if any of T_t's satisfies the diskcyclicity criterion for operators. Moreover, we show that there are diskcyclic C_0-semigroups that do not satisfy the diskcyclicity criterion. Also, we state various criteria for diskcyclicity of C_0-semigroups based on dense sets and d-dense orbits.

1. Introduction

Assume that X is a complex Banach space with infinite-dimensional. An element $x \in X$ is called a hypercyclic vector for a boundea and linear operator T on X if $orb(T,x) = \{T^n x : n \geq 0\}$ is a dense set in X and it is called a supercyclic vector for T if $\overline{\mathbb{C}orb(T,x)} = \overline{\{\alpha T^n x : n \geq 0, \alpha \in \mathbb{C}\}} = X$. Also, it is named a diskcyclic vector for T if $\overline{\mathbb{D}orb(T,x)} = \overline{\{\alpha T^n x : n \geq 0, \alpha \in \mathbb{D}\}} = X$, where $\mathbb{D} = \{x \in X : ||x|| \leq 1\}$ is the closed unit disk.

By the above definitions, hypercyclic operators are diskcyclic and diskcyclic operators are supercyclic. Hypercyclicity and supercyclicity are studied for many years. We can see [10] and [6] to familiar more with these concepts. Diskcyclicity is a new concept in dynamical systems that is introduced by Jamil in [12]. As it proved in [4, Proposition 3.9] diskcyclic operators exists only on complex Banach spaces with $\dim(X) = 1$ or $\dim(X) = \infty$. The

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first diskcyclicity criterion is presented in [12] by Jamil. Authors in [4] stated another diskcyclicity criterion that is equivalent to to diskcyclicity criterion stated in [12] by Jamil. In [2], Abdulkareem and Jamil presented some other criteria for diskcyclicity and proved that the new criteria and the criteria that presented in [12] and [4] are equivalent.

In this paper, we use the following version of diskcyclicity criterion from [2, Theorem 1.5].

Definition 1.1. Assume that $T \in B(X)$, where $B(X)$ is the set of bounded and linear operators on X. Consider that X_0 and Y_0 are dense subsets of X. If increasing sequence $\{n_k\}$ of positive integers and a sequence $\{\alpha_{n_k}\} \subseteq \mathbb{D} \setminus \{0\}$ of positive real numbers and operators $S_{n_k} : Y_0 \to X$ can be found such that:

(i) $\alpha_{n_k} T^{n_k} x \to 0$ for any $x \in X_0$,
(ii) $\frac{1}{\alpha_{n_k}} S_{n_k}(y) \to 0$ for any $y \in Y_0$,
(iii) $T^{n_k} S_{n_k} y \to y$ for any $y \in Y_0$,

then we say that T satisfies in diskcyclicity criterion.

It is shown in [1, Proposition 2.5, Proposition 2.6] that if T satisfies the diskcyclicity criterion, then T^n for any $n \in \mathbb{N}$ and $\oplus_{i=1}^{\infty} T$ satisfies this criterion on X and $\oplus_{i=1}^{\infty} X$, respectively. Especially, they are diskcyclic [1, Theorem 2.2]. One can also see [5] and [11] for the other attractive facts about diskcyclicity.

An interesting structure in dynamical systems is C_0-semigroup. By a C_0-semigroup of operators on X, we mean a family $\{T_t\}_{t \geq 0}$ of operators on X such that $T_0 = I$ and

$$T_{t+s} = T_t T_s \quad \text{and} \quad \lim_{s \to t} T_s x = T_t x,$$

for any $s, t \geq 0$ and for any $x \in X$.

A C_0-semigroup $\{T_t\}_{t \geq 0}$ on X is said hypercyclic if $orb((T_t)_{t \geq 0}, x) = \{T_t x : t \geq 0\}$ is dense in X for a vector $x \in X$ and it is called supercyclic if $\{\alpha T_t x : \alpha \in \mathbb{C}, t \geq 0\}$ is dense in X for a vector $x \in X$ [10]. It is proved in [8, Theorem 2.3] that hypercyclicity of $\{T_t\}_{t \geq 0}$ implies that T_t is hypercyclic for any $t > 0$. Moreover, some criteria for subspace-hypercyclic C_0-semigroups exists in [15]. Also, in [13] we can find criteria and various examples of supercyclic C_0-semigroups. Remember that diskcyclic diskcyclic C_0-semigroups is defined in [3] as follows.

Definition 1.2. A C_0-semigroup $\{T_t\}_{t \geq 0}$ on X is called diskcyclic if

$$\mathbb{D} orb((T_t)_{t \geq 0}, x) = \{\alpha T^n x : n \geq 0, \alpha \in \mathbb{D}\}$$

is dense in X for a vector $x \in X$.

By definition, hypercyclic C_0-semigroups are diskcyclic and diskcyclic C_0-semigroups are supercyclic. So, we can give the following example.

Example 1.3. The translation C_0-semigroup on a weighted function space $L^{\rho}_{p}(I)$ or $C_{0,\rho}(I)$ is a hypercyclic C_0-semigroup, where ρ is an admissible weight function and $I = [-\infty, \infty]$ or $I = (0, \infty)$[14, p. 361]. So, it is diskcyclic. Recall that $\rho : I \to \mathbb{R}$ is called an admissible weight function on I when $\rho(x) > 0$ and $M \geq 1$ and $k \in \mathbb{R}$ exist such that $\rho(x) \leq Me^{kt}\rho(t + x)$ for any $t > 0$ and any $x \in I$. Also,

$$L^{p}_{\rho}(I, \mathbb{C}) = \{u : I \to \mathbb{C} : u \text{ is measurable and } \int_I |u(t)|^p \rho(t)dt < \infty\}$$

and

$$C_{0,\rho}(I, \mathbb{C}) = \{u : I \to \mathbb{C} : u \text{ is continuous and } \lim_{t\to\pm\infty} \rho(t)u(t) = 0\}.$$

It is interasting that if $2 \leq \dim X < \infty$, then this space does not support diskcyclic C_0-semigroups [3, Theorem 4.3]. Surprisingly, we show that infinite-dimensional Banach spaces support diskcyclic C_0-semigroups.

Proposition 1.4. *Diskcyclic C_0-semigroups exist on any separable and infinite-dimensional Banach spaces.*

Proof. By [7, Theorem 2.4], hypercyclic C_0-semigroups exist on such spaces. Thus we can conclude that diskcyclic operators exist on them. □

So, like diskcyclic operators, diskcyclic C_0-semigroups only exist on Banach spaces with $\dim X = 1$ or $\dim X = \infty$. In the next theorem, we illustrate that diskcyclicity of one operator of a C_0-semigroup implies diskcyclicity of the C_0-semigroup.

Theorem 1.5. *Consider that $\{T_t\}_{t\geq 0}$ is a C_0-semigroup on X. If T_{t_0} is diskcyclic for some $t_0 > 0$, then $\{T_t\}_{t\geq 0}$ is diskcyclic.*

Proof. Assume that

$$\overline{\{\alpha T^n_{t_0}x : n \in \mathbb{N}, \alpha \in \mathbb{D}\}} = X \tag{1.1}$$

for a vector $x \in X$. Moreover, $T^n_{t_0} = T_{nt_0}$, by properties of a C_0-semigroup. Also,

$$\{\alpha T_{nt_0}x : n \in \mathbb{N}, \alpha \in \mathbb{D}\} \subseteq \{\alpha T_t x : t \geq 0, \alpha \in \mathbb{D}\}. \tag{1.2}$$

Now, by (1.1) and (1.2) we can conclude that x is a diskcyclic vector for $(T_t)_{t\geq 0}$. □

It is illustrated in Section 2 that if a C_0-semigroup $(T_t)_{t\geq 0}$ fulfills the diskcyclicity criterion, then any T_t satisfies the diskcyclicity criterion for operators and vice versa. Moreover, we show there are diskcyclic C_0-semigroups that do not satisfy the diskcyclicity criterion. Also, we prove various criteria for the diskcyclicity of C_0-semigroups based on dense sets and d-dense orbits in Section 3.

2. Diskcyclicity criterion for C_0-semigroups and diskcyclicity criterion for operators

In this section, we investigate the effect of satisfying a C_0-semigroup in diskcyclicity criterion to diskcyclicity of its operators. We begin by recalling a fact stated in [3, Theorem 4.7].

Theorem 2.1. ([3]) *A C_0-semigroup $\{T_t\}_{t\geq 0}$ on X is diskcyclic if and only if for any nonempty open sets U and V, there is some $t > 0$ and some $\alpha \in \mathbb{D}$ with $\alpha \neq 0$ such that $\alpha T_t(U) \cap V \neq \phi$.*

A diskcyclicity criterion for a set of operators is defined in [3, Definition 3.21] by authors. Now, we define a new diskcyclicity criterion for C_0-semigroups as follows. The idea of the following criterion is from [3, Definition 3.21] and from [16, Theorem 3.1].

Theorem 2.2. (Diskcyclicity criterion for C_0-semigroups) *Assume that $\{T_t\}_{t\geq 0}$ is a C_0-semigroup on X and assume that X_0 and Y_0 is dense subsets of X. Suppose that $\{t_n\}$ is an increasing sequence of positive real numbers and $\{\alpha_{t_n}\} \subseteq \mathbb{D} \setminus \{0\}$. If for any n, there is an operator $S_{t_n} : Y_0 \to Y_0$ such that:*

(i) $\alpha_{t_n} T_{t_n} x \to 0$ *for any* $x \in X_0$,
(ii) $\frac{1}{\alpha_{t_n}} S_{t_n} y \to 0$ *for any* $y \in Y_0$,
(iii) $T_{t_n} S_{t_n} y \to y$ *for any* $y \in Y_0$,

then $\{T_t\}_{t\geq 0}$ is diskcyclic.

Proof. By hypothesis, X_0 and Y_0 are dense in X. Thus, if U and V are open sets, then $x_0 \in U \cap X_0$ and $y_0 \in V \cap Y_0$ can be chosen. Therefore, there exists $\varepsilon > 0$ such that

$$B(x_0, \varepsilon) \subseteq U \quad \text{and} \quad B(y_0, \varepsilon) \subseteq V.$$

Now, by conditions (i), (ii) and (iii),

$$\alpha_{t_n} T_{t_n} x_0 \to 0, \quad \frac{1}{\alpha_{t_n}} S_{t_n} y_0 \to 0 \quad \text{and} \quad T_{t_n} S_{t_n} y_0 \to y_0. \tag{2.1}$$

If we consider $z_n = x_0 + \frac{1}{\alpha_{t_n}} S_{t_n} y_0$, then by (2.1), $z_n \to x_0$ and

$$\alpha_{t_n} T_{t_n} z_n = \alpha_{t_n} T_{t_n} x_0 + T_{t_n} S_{t_n} y_0 \to y_0. \tag{2.2}$$

Thus, by (2.2), N can be found large enough such that for any $n \geq N$,

$$\alpha_{t_n} T_{t_n}(U) \cap V \neq \phi.$$

Therefore, $\{T_t\}_{t\geq 0}$ is a diskcyclic by Theorem 2.1. □

It is proved in [13] that satisfying $\{T_t\}_{t\geq 0}$ in supercyclicity criterion is equivalent to satisfying T_t in supercyclicity criterion for any $t > 0$ [13]. In the next theorem, we prove that the diskcyclicity criterion for $\{T_t\}_{t\geq 0}$ is equivalent to the diskcyclicity of each of T_t.

Theorem 2.3. *For a C_0-semigroup $\{T_t\}_{t\geq 0}$ on X, $\{T_t\}_{t\geq 0}$ satisfies the diskcyclicity criterion if and only if T_t satisfies the diskcyclicity criterion for any $t > 0$.*

Proof. Obviously, if T_t for any $t > 0$ satisfies the conditions of the diskcyclicity criterion, then $\{T_t\}_{t\geq 0}$ satisfies the diskcyclicity criterion (one can see the proof of Theorem 3 in [13].

Now, let $\{T_t\}_{t\geq 0}$ fulfills the conditions of diskcyclicity criterion. So, dense sets X_0 and Y_0 of X, an increasing sequence $\{t_k\}$ of positive numbers and $\{\alpha_{t_k}\} \subseteq \mathbb{D} \setminus \{0\}$ can be chosen so that for any $x \in X_0$ and $y \in Y_0$:

$$\alpha_{t_k} T_{t_k} x \to 0, \frac{1}{\alpha_{t_k}} S_{t_k} y \to 0 \quad \text{ and } \quad T_{t_k} S_{t_k} y \to y.$$

Let $p > 0$. Accordingly, $t_k = n_k p + m_k$ for any $k \in \mathbb{N}$ where $n_k \geq 1$ is a natural number and $m_k \in [0, p)$. By [10, p. 182],

$$\sup_{s\in[0,p]} \|T_s\| < \infty.$$

Now, let $t \in [0, p]$. For any $x \in X_0$,

$$\begin{aligned} \lim_{k\to\infty} \alpha_{t_k} T_{n_k p} T_p x &= \lim_{k\to\infty} \alpha_{t_k} T_{n_k p + m_k} T_{p - m_k} x \\ &= \lim_{k\to\infty} \alpha_{t_k} T_{p-m_k} T_{n_k p + m_k} x \\ &= \lim_{k\to\infty} \alpha_{t_k} T_{p-m_k} T_{t_k} x = 0. \end{aligned}$$

Hence,

$$\lim_{k\to\infty} \alpha_{t_k} (T_p)^{n_k} x = 0, \text{ for any } x \in T_p(X_0). \tag{2.3}$$

But X_0 is dense in X and T_p is continuous. So, $T_p(X_0)$ is a dense subset of X. If we consider $S'_{t_k} = T_{m_k} S_{t_k}$, then for any $y \in Y_0$,

$$\frac{1}{\alpha_{t_k}} S'_{t_k} y = \frac{1}{\alpha_{t_k}} T_{m_k} S_{t_k} y = T_{m_k}(\frac{1}{\alpha_{t_k}} S_{t_k} y) \to 0. \tag{2.4}$$

Also,

$$T_{n_k p} S'_{t_k} y = T_{n_k p} T_{m_k} S_{t_k} y = T_{t_k} S_{t_k} y = y.$$

But $T_{n_k p} = T_p^{n_k}$. Hence,

$$T_p^{n_k} S'_{t_k} y = y, \text{ for any } y \in Y_0. \tag{2.5}$$

So by (2.3), (2.4), and (2.5), T_p satisfies the conditions of diskcyclicity criterion for operators with respect to $\{\alpha_{t_k}\}$, $\{t_k\}$, $\{S'_{t_k}\}$, and dense subsets $T_p(X_0)$ and Y_0 of X. □

Now the following question arises:

Question: Does any diskcyclic C_0-semigroup satisfy the diskcyclicity criterion?

The answer is negative. It is established in [4, Example 3.8] that if we consider $T = 2I$, then T is diskcyclic on $\mathbb{C}$ but T does not satisfy the diskcyclicity criterion. Let $\{T_t\}_{t\geq 0}$ be a C_0-semigroup such that $T_{t_0} = T$ for some $t_0 > 0$. Then $\{T_t\}_{t\geq 0}$ is diskcyclic by Theorem 1.5, since T_{t_0} is diskcyclic. But $\{T_t\}_{t\geq 0}$ does not satisfy the diskcyclicity criterion. Since if this occurs, then all T_t's must be satisfied the diskcyclicity criterion. Especially, T_{t_0} must be satisfied the diskcyclicity criterion. But it is impossible since as mentioned above, T_{t_0} is not satisfied the diskcyclicity criterion.

3. Some criteria for diskcyclicity of C_0-semigroups

Let $0 < d < \infty$. We say that $x \in X$ has a d-dense orbit under $T \in B(X)$ if for any $y \in X$, there is $n > 0$ such that $\|T^n x - y\| < d$. Feldman proved that operators with d-dense orbits are supercyclic [9, Proposition 2.5]. Note that as it is mentioned in [9], d-dense orbits are not necessarily dense. Similarly, we can define d-dense orbit for C_0-semigroups as follows.

Definition 3.1. We say a vector $x \in X$ has a d-dense orbit under C_0-semigroup $\{T_t\}_{t\geq 0}$ if for any $y \in X$ there is $t > 0$ such that $\|T_t x - y\| < d$.

We prove that having d-dense orbits leads to the diskcyclicity as follows.

Theorem 3.2. *Suppose that $\{T_t\}_{t\geq 0}$ is a C_0-semigroup on X. If $x \in X$ has a d-dense orbit under $\{T_t\}_{t\geq 0}$, then it is a diskcyclic C_0-semigroup.*

Proof. Let $y \in X$ and let $\varepsilon > 0$. So, there is $n > 1$ such that $\frac{d}{n} < \varepsilon$. If we consider $\alpha = \frac{1}{n}$, then $\alpha \in \mathbb{D} \setminus \{0\}$. Also, by hypothesis, there is $t > 0$ such that $\|T_t x - ny\| < d$. Hence, $\|\frac{1}{n} T_t x - y\| < \frac{d}{n}$ and therefore $\|\alpha T_t x - y\| < \varepsilon$. This shows x is a diskcyclic vector for $\{T_t\}_{t\geq 0}$. □

The idea of the next criterion is from [9, Theorem 3.1].

Theorem 3.3. *Let $\{T_t\}_{t\geq 0}$ be a C_0-semigroup on X. Let X_0 and Y_0 be dense subsets of X. Assume that there is a sequence $\{\alpha_{t_n}\} \subseteq \mathbb{D} \setminus \{0\}$ and there is an increasing sequence of positive real numbers $\{t_n\}$ so that*

(i) *$\alpha_{t_n} T_{t_n} x \to 0$ for any $x \in X_0$,*
(ii) *for any $y \in Y_0$ there is $(x_{t_n}) \subseteq X$ such that $x_{t_n} \to 0$ and $\alpha_{t_n} T_{t_n} x_{t_n} \to y$.*

Then $\{T_t\}_{t\geq 0}$ is a diskcyclic C_0-semigroup.

Proof. Let U and V be open subsets of X. By hypothesis, X_0 and Y_0 are dense in X. Thus $x_0 \in U \cap X_0$ and $y_0 \in V \cap Y_0$ can be chosen such that for some $\varepsilon > 0$,

$$B(x_0, \varepsilon) \subseteq U \quad \text{and} \quad B(y_0, \varepsilon) \subseteq V.$$

A natural number N_1 can be found such that for any $n \geq N_1$,

$$\|\alpha_{t_n} T_{t_n} x_0\| < \frac{\varepsilon}{3}, \|x_{t_n}\| < \frac{\varepsilon}{3} \quad \text{and} \quad \|\alpha_{t_n} T_{t_n} x_{t_n} - y_0\| < \frac{\varepsilon}{3}. \tag{3.1}$$

But $x_0 + x_{t_n} \to x_0$. So, a natural number N_2 can be found such that

$$x_0 + x_{t_n} \in U \quad \text{for any } n \geq N_2. \tag{3.2}$$

On the other hand, by (3.1),

$$\begin{aligned} \|\alpha_{t_n} T_{t_n}(x_0 + x_{t_n}) - y_0\| &\leq \|\alpha_{t_n} T_{t_n} x_0\| + \|\alpha_{t_n} T_{t_n} x_{t_n} - y_0\| \\ &< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} < \varepsilon. \end{aligned} \tag{3.3}$$

Thus if we consider $N := \max\{N_1, N_2\}$, then by (3.2) and (3.3) for any $n \geq N$,

$$x_0 + x_{t_n} \in U \quad \text{and} \quad \alpha_{t_n} T_{t_n}(x_0 + x_{t_n}) \in V.$$

Hence $\alpha_{t_n} T_{t_n}(U) \cap V \neq \phi$ for any $n \geq N$. Theorem 2.1 implies $\{T_t\}_{t\geq 0}$ is diskcyclic. □

Feldman in his criterion for hypercyclicity [9, Theorem 3.2], used limit infimum and limit supremum instead of limit. Now, we extend his idea to diskcyclicity with using the concept of d-dense orbits.

Theorem 3.4. *Let $\{T_t\}_{t\geq 0}$ be a C_0-semigroup on X. Let Y and Z be dense subsets of X. Suppose that $d > 0$ and $\{t_n\}$ is an increasing sequence of positive real numbers. Let $S_{t_n} : Z \to X$ for any n can be found such that*

(i) $\limsup_n \|T_{t_n} y\| \leq d$ *for any $y \in Y$,*
(ii) $\|S_{t_n} z\| \to 0$ *for any $z \in Z$,*
(iii) $\liminf_n \|T_{t_n} S_{t_n} z - z\| \leq d$ *for any $z \in Z$.*

Then $\{T_t\}_{t\geq 0}$ is a diskcyclic C_0-semigroup.

Proof. We prove that for any open set U and any $x \in X$, there is some $t > 0$ such that $T_t(U) \cap B(x, 3d) \neq \phi$. Let U be an open set and $V = B(x, d)$, where $x \in X$ is arbitrary. By hypothesis, there is $y_0 \in U \cap Y$ and $z_0 \in V \cap Z$ and there is $\varepsilon > 0$ such that $B(y_0, \varepsilon) \subseteq U$. Therefore, we can find $N_1 \in \mathbb{N}$ such that for any $n \geq N_1$,

$$\|T_{t_n} y_0\| < d + \frac{\varepsilon}{2}, \|S_{t_n} z_0\| < \frac{\varepsilon}{2} \quad \text{and} \quad \|T_{t_n} S_{t_n} z_0 - z_0\| < d - \frac{\varepsilon}{2}. \tag{3.4}$$

If we consider $z_n = y_0 + S_{t_n} z_0$, then $z_n \to y_0$. Thus, there is $N_2 \in \mathbb{N}$ such that $z_n \in B(y_0, \varepsilon) \subseteq U$ for any $n \geq N_2$. Also, by (3.4),

$$\begin{aligned} \|T_{t_n}(z_n) - z_0\| &= \|T_{t_n} y_0 + T_{t_n} S_{t_n} z_0 - z_0\| \\ &\leq \|T_{t_n} y_0\| + \|T_{t_n} S_{t_n} z_0 - z_0\| \\ &< d + \frac{\varepsilon}{2} + d - \frac{\varepsilon}{2} = 2d. \end{aligned} \tag{3.5}$$

Hence, by (3.5),

$$\begin{aligned} \|T_{t_n}(z_n) - x\| &\leq \|T_{t_n}(z_n) - z_0\| + \|z_0 - x\| \\ &< 2d + d = 3d. \end{aligned}$$

Therefore, $T_{t_n}(U) \cap B(x, 3d)$ is nonempty for large enough n. Now, by Theorem 3.2, $\{T_t)_{t \geq 0}$ is diskcyclic. □

Remark 3.5. *It is remarkable that if C_0-semigroup $\{T_t\}_{t \geq 0}$ on X satisfies the criteria presented in this section, then $\{T_t \oplus T_t\}_{t \geq 0}$ satisfies them on space $X \oplus X$ and hence $\{T_t \oplus T_t\}_{t \geq 0}$ is diskcyclic.*

References

[1] R. Abdulkareem and Z.Z. Jamil, *Characterization of diskcyclic criterion*, J. Phys.: Conf. Ser., **1530** (2020), 012097.

[2] R. Abdulkareem and Z.Z. Jamil, *On the diskcyclic criterions*, Iraqi J. Sci., **61**(10) (2020), 2651–2654.

[3] M. Amouch and O. Benchiheb, *Diskcyclicity of sets of operators and applications*, Acta. Math. Sin., **36** (2020), 1203–1220.

[4] N. Bamerni, A. Klicman, and M.S.M. Noorani, *A Review of some works in the theory of diskcyclic operators*, Bull. Malays. Math. Sci. Soc., **39**(2) (2016), 723–739.

[5] N. Bamerni and A. Klicman, *Operators with diskcyclic vectors subspaces*, J. Taibah Univ. Sci., **9**(3) (2015), 414–419.

[6] F. Bayart and E. Matheron, *Dynamics of linear operators*, Cambridge University Press, 2009.

[7] T. Bermudez, A. Bonilla and A. Martinon, *On the existence of chaotic and hypercyclic semigroups on Banach spaces*, Proc. Amer. Math. Soc., **131**(8) (2002), 2435–2441.

[8] J.A. Conejero, V. Muller and A. Peris, *Hypercyclic behaviour of operators in a hypercyclic C_0-semigroup*, J. Funct. Anal., **244**(1) (2007), 342–348.

[9] N.S. Feldman, *Perturbations of hypercyclic vectors*, J. Math. Anal. Appl., **273**(1) (2002), 67–74.
[10] K.G. Grosse-Erdmann and A. Peris Manguillot, *Linear chaos*, Lodon, Springer-Verlag, 2011.
[11] Z.Z. Jamil and M. Helal, *Equivalent between the criterion and the three open set's conditions in diskcyclicity*, Int. J. Contemp. Math. Sci., **8**(6) (2013), 257–261.
[12] Z.Z. Jamil, *Cyclic phenomena of operators on Hilbert space(Ph. D Thesis)*, University of Baghdad, 2002.
[13] A. Kumar and S. Srivastava, *Supercyclicity criteria for C_0-semigroups*, Adv. Oper. Theory., **5** (2020), 1646–1666.
[14] Y.X. Liang and Z.H. Zhou, *Supercyclic translation C_0-semigroups on complex sectors*, Discrete Contin. Dyn. Syst., **36** (2016), 361–370.
[15] M. Moosapoor, *Some criteria for subspace-hypercyclicity of C_0-semigroups*, Aust. J. Math. Anal. Appl., **18**(1) (2021), Article 12, 8 pp.
[16] M. Moosapoor, *On subspace-supercyclic operators*, Aust. J. Math. Anal. Appl., **17**(2) (2020), Article 4, 8 pp.