



## DECOMPOSITION FOR CARTAN'S SECOND CURVATURE TENSOR OF DIFFERENT ORDER IN FINSLER SPACES

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**Abstract.** The Cartan's second curvature tensor  $P_{jkh}^i$  is a positively homogeneous of degree-1 in  $y^i$ , where  $y^i$  represent a directional coordinate for the line element in Finsler space. In this paper, we discuss the decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in two spaces, a generalized  $\mathfrak{B}P$ -recurrent space and generalized  $\mathfrak{B}P$ -birecurrent space. We obtain different tensors which satisfy the recurrence and birecurrence property under the decomposition. Also, we prove the decomposition for different tensors are non-vanishing. As an illustration of the applicability of the obtained results, we finish this work with some illustrative examples.

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## 1. INTRODUCTION

Finsler geometry has many uses in relative physics and many of mathematicians contributed in this study and improved it. The decomposition of curvature tensor of recurrent manifold discussed initially by Takano [19], Sinha and Singh [17] and others. The decomposition of Berwald curvature tensor  $H_{jkh}^i$  and Cartan's fourth curvature tensor  $K_{jkh}^i$  for some spaces in sense of Berwald and Cartan discussed by Pandey [12]. The decomposition of Cartan's third curvature tensor  $R_{jkh}^i$  equipped with non-symmetric connection studied by Mishra et al. [9]. The decomposition of Riemannian curvature tensor field discussed by Gicheru and Ngari [6]. The decomposition of normal projective curvature tensor studied by Qasem [13]. Hit [7] introduced Berwald curvature tensor which be decomposable in the form  $H_{jkh}^i = X^i Y_{jkh}$  and obtained several results, Pande and Khan [10] discussed Berwald curvature tensor which be decomposable in the form  $H_{jkh}^i = X_j^i Y_{kh}$ . Rawat and Chauhan [15] studied the decomposition of curvature tensor fields  $R_{jkh}^i$  in terms of two non-zero vectors and a tensor field in some spaces. Pande and Shukla [11] discussed the decomposition of curvature tensor field  $K_{jkh}^i$  and  $H_{jkh}^i$  which satisfy the recurrence property.

Assallal [4] studied the decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in generalized  $P^h$ -birecurrent Finsler space, Sinha and Tripathi [18] discussed the birecurrent Finsler space whose curvature tensor be decomposition. Recently, Bisht and Neg [5] studied decomposition of normal projective curvature tensor fields in Finsler manifolds.

The aim of this paper is to study some decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in various spaces. Additionally, several theorems have been established and proved. Finally, some examples have been discussed under the decomposition in  $G(\mathfrak{B}P) - RF_n$  and  $G(\mathfrak{B}P) - BRF_n$ .

## 2. PRELIMINARIES

In this section, some conditions and definitions will be provided for the purpose of this paper. The line element in Finsler geometry is  $(x, y)$ ,  $x$  and  $y$  are called positional and directional coordinate, respectively [12, 14].

An  $n$ - dimensional space  $X_n$  equipped with a function  $F(x, y)$  which denoted by  $F_n = (X_n, F(x, y))$  called a Finsler space if the function  $F(x, y)$  satisfies the following three conditions [9, 16]:

- (i) The function  $F(x, y)$  is positively homogeneous of degree one in  $y^i$ , that is,

$$F(x, ky) = kF(x, y),$$

where  $k$  is some positive scalar.

- (ii) The function  $F(x, y)$  is positive unless all  $y^i$  vanish simultaneously, that is,  $F(x, y) > 0$ , with  $\sum_i (y^i)^2 \neq 0$ .
- (iii) The quadratic form

$$\left\{ \dot{\partial}_i \dot{\partial}_j F^2(x, y) \right\} \xi^i \xi^j, \quad \dot{\partial}_i := \frac{\partial}{\partial y^i}$$

is assumed to be positive definite for all variable  $\xi^i$ .

The vector  $y_i$  is defined by

$$y_i = g_{ij}(x, y)y^j, \tag{2.1}$$

where  $g_{ij}$  is a metric tensor of the space  $F_n$ . The vectors  $y^j, y_i$  are given by

$$\delta_j^i y^j = y^i \text{ and } \delta_j^i y_i = y_j. \tag{2.2}$$

Matsumoto [8] introduced a tensor  $C_{ijk}$  called it  $(h)hv$ -torsion tensor which is positively homogeneous of degree -1 in  $y^i$  and symmetric in all its indices and defined by

$$C_{ijk} = \frac{1}{2} \dot{\partial}_i g_{jk} = \frac{1}{4} \dot{\partial}_i \dot{\partial}_j \dot{\partial}_k F^2,$$

which satisfies the following

$$C_{ijk}y^i = C_{kij}y^i = C_{jki}y^i = 0. \tag{2.3}$$

Berwald covariant derivative  $\mathfrak{B}_k T_j^i$  of an arbitrary tensor field  $T_j^i$  with respect to  $x^k$  is given by [15]

$$\mathfrak{B}_k T_j^i := \partial_k T_j^i - (\dot{\partial}_r T_j^i)G_k^r + T_j^r G_{rk}^i - T_r^i G_{jk}^r.$$

Berwald covariant derivative of the vector  $y^i$  vanish identically, that is,

$$\mathfrak{B}_k y^i = 0 \text{ and } \mathfrak{B}_k y_i = 0. \tag{2.4}$$

But, in general, Berwald covariant derivative of the metric tensor  $g_{ij}$  does not vanish and given by

$$\mathfrak{B}_k g_{ij} = -2C_{ijk|h}y^h = -2y^h \mathfrak{B}_h C_{ijk}. \tag{2.5}$$

**Example 2.1.** Let us consider the functions:

- (1)  $F(x, y) = \frac{|y|+|xy|}{1+|x|}$ ,
- (2)  $\vartheta(u, v) = \frac{\sqrt{|v|^2+(|u|^2|v|^2-|uv|^2)}}{1+|u|^2}$ .

Then, it is obvious that the functions  $F(x, y)$  and  $\vartheta(u, v)$  satisfy the condition (i), (ii) and (iii). These functions are called the fundamental function or the metric function of the Finsler space  $F_n$ .

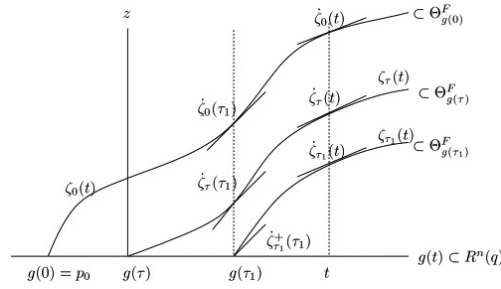


FIGURE 1. Relation Between Metric Spaces and Finsler Spaces

**Definition 2.2.** Let the current coordinates in the tangent space at the point  $x_0$  be  $x^i$ . Then the indicatrix  $I_{n-1}$  is a hypersurface defined by [15]  $F(x_0, x^i) = 1$  or by the parametric form defined by  $x^i = x^i(u^a)$ ,  $a = 1, 2, \dots, n - 1$ .

**Definition 2.3.** The projection of any tensor  $T_j^i$  on indicatrix  $I_{n-1}$  given by [1, 15]

$$p.T_j^i = T_b^a h_a^i h_j^b, \tag{2.6}$$

where

$$h_c^i = \delta_c^i - l^i l_c. \tag{2.7}$$

The projection of the vector  $y^i$ , the unit vector  $l^i$  and the metric tensor  $g_{ij}$  on the indicatrix are given by  $p.y^i = 0$ ,  $p.l^i = 0$  and  $p.g_{ij} = h_{ij}$ , where  $h_{ij} = g_{ij} - l_i l_j$ .

Abdallah et al. [1, 2, 3] introduced the generalized  $\mathfrak{B}P$ -recurrent space and generalized  $\mathfrak{B}P$ -birecurrent space which are characterized by the conditions:

$$\mathfrak{B}_m P_{jkh}^i = \lambda_m P_{jkh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}) \tag{2.8}$$

and

$$\mathfrak{B}_l \mathfrak{B}_m P_{jkh}^i = a_{lm} P_{jkh}^i + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}), \tag{2.9}$$

respectively. These spaces are denoted by  $G(\mathfrak{B}P) - RF_n$  and  $G(\mathfrak{B}P) - BRF_n$ .

Let us consider a Finsler space which Cartan's second curvature tensor  $P_{jkh}^i$  is decomposition. Since the curvature tensor is a mixed tensor of the type (1, 3), that is, rank 4, it may be written as product of contravariant (or covariant) vector and tensor of rank 3, that is, covariant tensor of the type (0,3) (or mixed tensor of the type (1,2)) as following [13, 14]:

$$P_{jkh}^i = X^i Y_{jkh}, \tag{2.10}$$

$$P_{jkh}^i = X_j Y_{kh}^i, \tag{2.11}$$

$$P_{jkh}^i = X_k Y_{jh}^i \quad (2.12)$$

and

$$P_{jkh}^i = X_h Y_{jk}^i, \quad (2.13)$$

as first case. Or in second case as product of two tensors each them of rank 2, that is, mixed tensors of the type (1, 1) and covariant tensor of the type (0, 2) as following [13, 14]

$$P_{jkh}^i = T_j^i \psi_{kh}, \quad (2.14)$$

$$P_{jkh}^i = T_k^i \psi_{jh} \quad (2.15)$$

and

$$P_{jkh}^i = T_h^i \psi_{jk}. \quad (2.16)$$

In next sections, we will discuss the possible forms in three decomposable of the tensor, two decompositions for the first case (the other are similar) and one decomposition for the second case (the other are similar). Obviously, from all several possibilities, we will study the possibilities which given by (2.10), (2.11) and (2.14).

### 3. DECOMPOSITION OF CARTAN'S SECOND CURVATURE TENSOR IN $G(\mathfrak{B}P) - RF_n$

In this section, we will discuss the decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in generalized  $\mathfrak{B}P$ -recurrent space. Let us consider Cartan's second curvature tensor  $P_{jkh}^i$  is decomposable as (2.10), where  $Y_{jkh}$  is non-zero covariant tensor field and homogeneous of degree-1 in its directional argument which called decomposition tensor field and  $X^i$  is independent of  $x^m$ .

In next theorem we will discuss the decomposition (2.10) for Cartan's second curvature tensor  $P_{jkh}^i$  which is generalized recurrent.

**Theorem 3.1.** *In  $G(\mathfrak{B}P) - RF_n$ , under the decomposition (2.10) and if  $X^i$  is covariant constant, then the decomposition tensor  $(X^i Y_{jkh})$  satisfies the generalized recurrence property.*

*Proof.* Assume that  $X^i$  is covariant constant. Taking  $\mathfrak{B}$ -covariant derivative for equation (2.10) with respect to  $x^m$ , we get

$$\mathfrak{B}_m P_{jkh}^i = (\mathfrak{B}_m X^i) Y_{jkh} + X^i \mathfrak{B}_m Y_{jkh}. \quad (3.1)$$

Since the decomposition vector field  $X^i$  is covariant constant, that is,  $(\mathfrak{B}_m X^i = 0)$ , therefore equation (3.1) can be written as

$$\mathfrak{B}_m P_{jkh}^i = X^i \mathfrak{B}_m Y_{jkh}.$$

By using the condition (2.8) in above equation and in view of (2.10), we get

$$X^i \mathfrak{B}_m Y_{jkh} = \lambda_m X^i Y_{jkh} + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \quad (3.2)$$

Since  $X^i$  is independent of  $x^m$ , equation (3.2) can be written as

$$\mathfrak{B}_m (X^i Y_{jkh}) = \lambda_m (X^i Y_{jkh}) + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}).$$

The last equation refers that the decomposition tensor ( $X^i Y_{jkh}$ ) is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed.  $\square$

Now, from the Theorem 3.1, we can get the following corollary.

**Corollary 3.2.** *Under the decomposition (2.10) and if the tensor field  $\emptyset_{m_jkh}$  is skewsymmetric in second and third indicator, then the decomposition tensor  $Y_{jkh}$  is non-vanishing.*

*Proof.* In view of equation (3.2), we get

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + \alpha_{mi} (\delta_j^i g_{kh} - \delta_k^i g_{jh}),$$

where  $\alpha_{mi} = \mu_m / X^i$ . Above equation can be written as

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + (\emptyset_{m_jkh} - \emptyset_{mkjh}),$$

where  $\emptyset_{m_jkh} = \alpha_{mj} g_{kh}$  and  $\emptyset_{mkjh} = \alpha_{mk} g_{jh}$ .

Now, if the tensor field  $\emptyset_{m_jkh}$  is skew-symmetric in second and third indicator, then above equation can be written as

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + 2\emptyset_{m_jkh}. \quad (3.3)$$

The equation (3.3) refers that the decomposition tensor  $Y_{jkh}$  is non-vanishing in  $G(\mathfrak{B}P) - RF_n$ . The proof for this corollary is completed.  $\square$

Let us consider a Finsler space which Cartan's second curvature tensor  $P_{jkh}^i$  is decomposition (2.11), where  $X_j$  is non-zero covariant vector field and  $Y_{kh}^i$  decomposition tensor field.

In next theorem we will discuss the decomposition (2.11) for Cartan's second curvature tensor  $P_{jkh}^i$  which be generalized recurrent.

**Theorem 3.3.** *In  $G(\mathfrak{B}P) - RF_n$ , under the decomposition (2.11) and if the covariant vector field  $\lambda_m$  is not equal the covariant vector field  $v_m$ , then the decomposition tensor ( $X_j Y_{kh}^i$ ) satisfies the generalized recurrence property.*

*Proof.* Taking  $\mathfrak{B}$ -covariant derivative for equation (2.11) with respect to  $x^m$ , we get

$$\mathfrak{B}_m P_{jkh}^i = (\mathfrak{B}_m X_j) Y_{kh}^i + X_j \mathfrak{B}_m Y_{kh}^i \quad (3.4)$$

or

$$\mathfrak{B}_m P_{jkh}^i = v_m X_j Y_{kh}^i + X_j \mathfrak{B}_m Y_{kh}^i, \tag{3.5}$$

where  $\mathfrak{B}_m X_j = v_m X_j$ . By using the condition (2.8) in equation (3.5), we obtain

$$v_m X_j Y_{kh}^i + X_j \mathfrak{B}_m Y_{kh}^i = \lambda_m P_{jkh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \tag{3.6}$$

In view of (2.11), then equation (3.6) can be written as

$$X_j \mathfrak{B}_m Y_{kh}^i = (\lambda_m - v_m) X_j Y_{kh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \tag{3.7}$$

Now, assume that the vector field  $\lambda_m$  is not equal to the vector field  $v_m$ , we get

$$X_j \mathfrak{B}_m Y_{kh}^i = \chi_m X_j Y_{kh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}), \tag{3.8}$$

where  $\chi_m = \lambda_m - v_m$ . Since  $X_j$  is independent of  $x^m$ , so equation (3.8) can be written as

$$\mathfrak{B}_m (X_j Y_{kh}^i) = \chi_m (X_j Y_{kh}^i) + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}).$$

The last equation refers that the decomposition tensor  $(X_j Y_{kh}^i)$  is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed. □

Now, from the Theorem 3.3, we can get the following corollary.

**Corollary 3.4.** *Under the decomposition (2.11), if  $X_j$  is covariant constant and  $X$  is constant, then the behavior of decomposition tensors  $Y_{kh}^i, XY_h^i, Y_h^i$  and  $XY$  is recurrent.*

*Proof.* By using the condition (2.8) in equation (3.4) and in view of (2.11), we get

$$(\mathfrak{B}_m X_j) Y_{kh}^i + X_j \mathfrak{B}_m Y_{kh}^i = \lambda_m X_j Y_{kh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \tag{3.9}$$

Since  $X_j$  is covariant constant, equation (3.9) can be written as

$$\mathfrak{B}_m Y_{kh}^i = \lambda_m Y_{kh}^i + \omega_m^j (\delta_j^i g_{kh} - \delta_k^i g_{jh}),$$

where  $\omega_m^j = \mu_m X_j$ . Above equation can be written as

$$\mathfrak{B}_m Y_{kh}^i = \lambda_m Y_{kh}^i, \tag{3.10}$$

where  $\theta_{mjh}^i = \omega_m^j \delta_j^i g_{kh}$  and  $\theta_{mjh}^i = \omega_m^j \delta_k^i g_{jh}$ .

Transvecting equation (3.9) by  $y^j$ , using (2.1), (2.2) and (2.4), we get

$$(\mathfrak{B}_m X) Y_{kh}^i + X \mathfrak{B}_m Y_{kh}^i = \lambda_m X Y_{kh}^i + \mu_m (y^j g_{kh} - \delta_k^i y_h),$$

where  $X = X_j y^j$ . If  $X$  is constant, that is,  $(\mathfrak{B}_m X = 0)$ , then above equation can be written

$$\mathfrak{B}_m(XY_{kh}^i) = \lambda_m(XY_{kh}^i) + \mu_m(y^i g_{kh} - \delta_k^i y_h). \quad (3.11)$$

Transvecting equation (3.11) by  $y^k$ , using (2.1), (2.2) and (2.4), we get

$$\mathfrak{B}_m(XY_h^i) = \lambda_m(XY_h^i), \quad (3.12)$$

where  $Y_h^i = Y_{kh}^i y^k$ . Since  $X$  is constant, equation(3.12) can be written as

$$\mathfrak{B}_m Y_h^i = \lambda_m Y_h^i. \quad (3.13)$$

Contracting the indices  $i$  and  $h$  in eq. (3.12), we get

$$\mathfrak{B}_m(XY) = \lambda_m(XY), \quad (3.14)$$

where  $Y = Y_i^i$ . The equations (3.10), (3.12), (3.13) and (3.14) refer that the tensors  $Y_{kh}^i$ ,  $XY_h^i$ ,  $Y_h^i$  and  $XY$  satisfy the recurrence property in  $G(\mathfrak{B}P) - RF_n$ . The proof for this corollary is completed.  $\square$

Let us consider a Finsler space which Cartan's second curvature tensor  $P_{jkh}^i$  is decomposition (2.14), where  $T_j^i$  and  $\psi_{kh}$  are the decomposition tensors field.

In next theorem we will discuss the decomposition (2.14) for Cartan's second curvature tensor  $P_{jkh}^i$  which is generalized recurrent.

**Theorem 3.5.** *In  $G(\mathfrak{B}P) - RF_n$ , under the decomposition (2.14) and if the covariant vector field  $\lambda_m$  is not equal the covariant vector field  $v_m$ , then the decomposition tensor  $(T_j^i \psi_{kh})$  satisfies the generalized recurrence property.*

*Proof.* Taking  $\mathfrak{B}$ -covariant derivative for equation (2.14) with respect to  $x^m$ , we get

$$\mathfrak{B}_m P_{jkh}^i = (\mathfrak{B}_m T_j^i) \psi_{kh} + T_j^i \mathfrak{B}_m \psi_{kh} \quad (3.15)$$

or

$$\mathfrak{B}_m P_{jkh}^i = v_m T_j^i \psi_{kh} + T_j^i \mathfrak{B}_m \psi_{kh}. \quad (3.16)$$

where  $\mathfrak{B}_m T_j^i = v_m T_j^i$ .

Using the condition (2.8) in above equation, we obtain

$$v_m T_j^i \psi_{kh} + T_j^i \mathfrak{B}_m \psi_{kh} = \lambda_m P_{jkh}^i + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \quad (3.17)$$

In view of (2.14), equation (3.17) can be written as

$$T_j^i \mathfrak{B}_m \psi_{kh} = (\lambda_m - v_m) T_j^i \psi_{kh} + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}). \quad (3.18)$$

Now, assume that the vector field  $\lambda_m$  is not equal the vector field  $v_m$ , we get

$$T_j^i \mathfrak{B}_m \psi_{kh} = \chi_m T_j^i \psi_{kh} + \mu_m (\delta_j^i g_{kh} - \delta_k^i g_{jh}), \quad (3.19)$$



where  $\chi_m = \lambda_m - v_m$ . Since  $T_j^i$  is independent of  $x^m$ , equation (3.19) can be written as

$$\mathfrak{B}_m(T_j^i \psi_{kh}) = \chi_m(T_j^i \psi_{kh}) + \mu_m(\delta_j^i g_{kh} - \delta_k^i g_{jh}).$$

The last equation refers that the decomposition tensor  $(T_j^i \psi_{kh})$  is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed.  $\square$

Now, from the Theorem 3.5, we can get the following corollary.

**Corollary 3.6.** *Under the decomposition (2.14) and if the tensor field  $\varphi_{mkh}$  is skew symmetric in first and second indices, then the decomposition tensor  $\psi_{kh}$  is non-vanishing.*

*Proof.* By using the condition (2.8) in equation (3.15), we get

$$(\mathfrak{B}_m T_j^i) \psi_{kh} + T_j^i \mathfrak{B}_m \psi_{kh} = \lambda_m P_{jkh}^i + \mu_m(\delta_j^i g_{kh} - \delta_k^i g_{jh}).$$

In view of (2.14) and if the decomposition tensor field  $T_j^i$  is covariant constant, the above equation can be written as

$$\mathfrak{B}_m \psi_{kh} = \lambda_m \psi_{kh} + \chi_{im}^j (\delta_j^i g_{kh} - \delta_k^i g_{jh}),$$

where  $\chi_{im}^j = \mu_m T_j^i$ . Also above equation can be written as

$$\mathfrak{B}_m \psi_{kh} = \lambda_m \psi_{kh} + (\varphi_{mkh} - \varphi_{kmh}),$$

where  $\varphi_{mkh} = \chi_{im}^j \delta_j^i g_{kh}$  and  $\varphi_{kmh} = \chi_{im}^j \delta_k^i g_{jh}$ .

Now, if the tensor field  $\varphi_{mkh}$  is skew symmetric in first and second indicator, then above equation can be written as

$$\mathfrak{B}_m \psi_{kh} = \lambda_m \psi_{kh} + 2\varphi_{mkh}. \quad (3.20)$$

The equation(3.20) refers that the decomposition tensor  $\psi_{kh}$  is non-vanishing in  $G(\mathfrak{B}P) - RF_n$ . The proof for this corollary is completed.  $\square$

In view the Theorems 3.1, 3.3 and 3.5, we can conclude that if  $\delta_j^i g_{kh} = \delta_k^i g_{jh}$ , then the decomposition tensors  $(X^i Y_{jkh})$ ,  $(X_j Y_{kh}^i)$  and  $(T_j^i \psi_{kh})$  behave as recurrent, clearly, satisfy the following conditions:

$$\mathfrak{B}_m(X^i Y_{jkh}) = \lambda_m(X^i Y_{jkh}), \quad (3.21)$$

$$\mathfrak{B}_m(X_j Y_{kh}^i) = \chi_m(X_j Y_{kh}^i) \quad (3.22)$$

and

$$\mathfrak{B}_m(T_j^i \psi_{kh}) = \chi_m(T_j^i \psi_{kh}), \quad (3.23)$$

respectively.

#### 4. DECOMPOSITION OF CARTAN'S SECOND CURVATURE TENSOR IN $G(\mathfrak{B}P) - BRF_n$

In this section, we will discuss the decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in generalized  $\mathfrak{B}P$ - birecurrent space.

In next theorem we will discuss the decomposition (2.10) for Cartan's second curvature tensor  $P_{jkh}^i$  which be generalized birecurrent.

**Theorem 4.1.** *In  $G(\mathfrak{B}P) - BRF_n$ , under the decomposition (2.10) and if  $X^i$  is covariant constant, then the decomposition tensor  $(X^i Y_{jkh})$  satisfies the generalized birecurrence property.*

*Proof.* Assume that  $X^i$  is covariant constant. Taking  $\mathfrak{B}$ - covariant derivative for equation (2.10) twice with respect to  $x^m$  and  $x^l$ , respectively, we get

$$\mathfrak{B}_l \mathfrak{B}_m P_{jkh}^i = X^i \mathfrak{B}_l \mathfrak{B}_m Y_{jkh}. \quad (4.1)$$

Using the condition (2.9) in equation (4.1) and in view of (2.10), we get

$$\begin{aligned} X^i \mathfrak{B}_l \mathfrak{B}_m Y_{jkh} &= a_{lm} X^i Y_{jkh} + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) \\ &\quad - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}). \end{aligned} \quad (4.2)$$

Since  $X^i$  is covariant constant, equation (4.2) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m (X^i Y_{jkh}) = a_{lm} (X^i Y_{jkh}) + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}).$$

The last equation refers that the decomposition tensor  $(X^i Y_{jkh})$  is generalized birecurrent, that is, satisfies the condition (2.9). The proof for this theorem is completed.  $\square$

Now, from the Theorem 4.1, we can obtain the following corollary.

**Corollary 4.2.** *Under the decomposition (2.10) if the tensor field  $\Phi_{lmjkh}$  is skew symmetric in third and fourth indicator, then the decomposition tensor field  $Y_{jkh}$  is non-vanishing.*

*Proof.* In view of equation (4.2), we get

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + \gamma_{lmi} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \alpha_{mi} \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}),$$

where  $\gamma_{lmi} = b_{lm} X^i$  and  $\alpha_{mi} = \mu_m X^i$ . The above equation can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + (\Phi_{lmjkh} - \Phi_{lmkjh}) - 2y^t \alpha_{mi} \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}),$$

where  $\Phi_{lmjkh} = \gamma_{lmi} \delta_j^i g_{kh}$  and  $\Phi_{lmkjh} = \gamma_{lmi} \delta_k^i g_{jh}$ .

Now, if the tensor field  $\Phi_{lmjkh}$  is skew symmetric in third and fourth indicator, above equation can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + 2\Phi_{lmjkh} - 2y^t \alpha_{mi} \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}). \quad (4.3)$$

The equation (4.3) refers that the decomposition tensor  $Y_{jkh}$  is non-vanishing in  $G(\mathfrak{B}P) - BRF_n$ . The proof for this corollary is completed.  $\square$

In next theorem we will discuss the decomposition (2.11) for Cartan's second curvature tensor  $P_{jkh}^i$  which be generalized birecurrent.

**Theorem 4.3.** *In  $G(\mathfrak{B}P) - BRF_n$ , under the decomposition (2.11) and if  $X_j$  is covariant constant, then the decomposition tensor  $(X_j Y_{kh}^i)$  satisfies the generalized birecurrence property.*

*Proof.* Assume that  $X_j$  is covariant constant. Taking  $\mathfrak{B}$ -covariant derivative for equation (2.11) twice with respect to  $x^m$  and  $x^l$ , respectively, we get

$$\mathfrak{B}_l \mathfrak{B}_m P_{jkh}^i = X_j \mathfrak{B}_l \mathfrak{B}_m Y_{kh}^i. \tag{4.4}$$

Using the condition (2.9) in equation (4.4) and in view of (2.11), we get

$$\begin{aligned} X_j \mathfrak{B}_l \mathfrak{B}_m Y_{kh}^i &= a_{lm} X_j Y_{kh}^i + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) \\ &\quad - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}). \end{aligned} \tag{4.5}$$

Since  $X_j$  is covariant constant, equation (4.5) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m (X_j Y_{kh}^i) = a_{lm} (X_j Y_{kh}^i) + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}).$$

The last equation refers that the decomposition tensor  $(X_j Y_{kh}^i)$  is generalized birecurrent, that is, satisfies the condition (2.9). The proof for this theorem is completed.  $\square$

Now, from the Theorem 4.3, we can obtain the following corollary.

**Corollary 4.4.** *Under the decomposition (2.11) and if  $X$  is constant, then the tensors  $XY_h^i$ ,  $Y_h^i$  and  $XY$  behave as birecurrent.*

*Proof.* Transvecting equation (4.5) by  $y^j$ , using (2.1), (2.2), (2.3) and (2.4), we get

$$X \mathfrak{B}_l \mathfrak{B}_m Y_{kh}^i = a_{lm} X Y_{kh}^i + b_{lm} (y^i g_{kh} - \delta_k^i y_h) - 2y^t \mu_m \mathfrak{B}_t (y^i C_{khl}),$$

where  $X = X_j y^j$ . If  $X$  is constant, that is,  $(\mathfrak{B}_m X = 0)$ , then above equation reduces to

$$\mathfrak{B}_l \mathfrak{B}_m (X Y_{kh}^i) = a_{lm} (X Y_{kh}^i) + b_{lm} (y^i g_{kh} - \delta_k^i y_h) - 2y^t \mu_m \mathfrak{B}_t (y^i C_{khl}). \tag{4.6}$$

Transvecting equation (4.6) by  $y^k$ , using (2.1), (2.2), (2.3) and (2.4), we get

$$\mathfrak{B}_l \mathfrak{B}_m (X Y_h^i) = a_{lm} (X Y_h^i), \tag{4.7}$$

where  $Y_h^i = Y_{kh}^i y^k$ . Since  $X$  is constant, equation (4.7) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y_h^i = a_{lm} Y_h^i. \tag{4.8}$$

Contracting the indices  $i$  and  $h$  in equation (4.7), we get

$$\mathfrak{B}_l \mathfrak{B}_m (XY) = a_{lm} (XY), \quad (4.9)$$

where  $Y = Y_h^i$ . The equations (4.7), (4.8) and (4.9) refer that the tensors  $XY_h^i$ ,  $Y_h^i$  and  $XY$  satisfy the birecurrent property in  $G(\mathfrak{B}P) - BRF_n$ . The proof for this corollary is completed.  $\square$

In next theorem we will discuss the decomposition (2.14) for Cartan's second curvature tensor  $P_{jkh}^i$  which be generalized birecurrent.

**Theorem 4.5.** *In  $G(\mathfrak{B}P) - BRF_n$ , under the decomposition (2.14) and if  $T_j^i$  is covariant constant, then the decomposition tensor  $(T_j^i \psi_{kh})$  satisfies the generalized birecurrence property.*

*Proof.* Assume that  $T_j^i$  is covariant constant. Taking  $\mathfrak{B}$ -covariant derivative for equation (2.14) twice with respect to  $x^m$  and  $x^l$ , respectively, we get

$$\mathfrak{B}_l \mathfrak{B}_m P_{jkh}^i = T_j^i \mathfrak{B}_l \mathfrak{B}_m \psi_{kh}. \quad (4.10)$$

Using the condition (2.9) in equation (4.10) and in view of (2.14), we get

$$T_j^i \mathfrak{B}_l \mathfrak{B}_m \psi_{kh} = a_{lm} T_j^i \psi_{kh} + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}). \quad (4.11)$$

Since  $T_j^i$  is covariant constant, therefore equation (4.11) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m (T_j^i \psi_{kh}) = a_{lm} (T_j^i \psi_{kh}) + b_{lm} (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}).$$

The last equation refers that the decomposition tensor  $(T_j^i \psi_{kh})$  is generalized birecurrent, that is, satisfies the condition (2.9). The proof for this theorem is completed.  $\square$

Now, from the Theorem 4.5, we can obtain the following corollary.

**Corollary 4.6.** *Under the decomposition (2.14) and if the decomposition tensor field  $T_j^i$  is covariant constant, then the decomposition  $\psi_{kh}$  behaves as birecurrent if and only if  $2y^t \chi_{im}^j \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}) = 0$ .*

*Proof.* In view of equation (4.11) and the decomposition tensor field  $T_j^i$  is covariant constant, we get

$$\mathfrak{B}_l \mathfrak{B}_m \psi_{kh} = a_{lm} \psi_{kh} + \alpha_{ilm}^j (\delta_j^i g_{kh} - \delta_k^i g_{jh}) - 2y^t \chi_{im}^j \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}),$$

where  $\alpha_{ilm}^j = b_{lm} T_j^i$  and  $\chi_{im}^j = \mu_m T_j^i$ . Above equation can be written as

$$\mathfrak{B}_l \mathfrak{B}_m \psi_{kh} = a_{lm} \psi_{kh} - 2y^t \chi_{im}^j \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}),$$

where  $\alpha_{ilm}^j \delta_j^i g_{kh} = \alpha_{ilm}^j \delta_k^i g_{jh}$ . This shows that  $\mathfrak{B}_l \mathfrak{B}_m \psi_{kh} = a_{lm} \psi_{kh}$  if and only if

$$2y^t \chi_{im}^j \mathfrak{B}_t (\delta_j^i C_{khl} - \delta_k^i C_{jhl}) = 0. \quad (4.12)$$

Then, the decomposition tensor  $\psi_{kh}$  is birecurrent in  $G(\mathfrak{B}P) - BRF_n$  if and only if equation (4.12) hold. The proof for this corollary is completed.  $\square$

In view the theorems 4.1, 4.3 and 4.5, we can conclude that if  $\delta_j^i g_{kh} = \delta_k^i g_{jh}$  and  $\delta_j^i C_{khl} = \delta_k^i C_{jhl}$ , then the decomposition tensors  $(X^i Y_{jkh})$ ,  $(X_j Y_{kh}^i)$  and  $(T_j^i \psi_{kh})$  behave as birecurrent, clearly, satisfy the following conditions

$$\mathfrak{B}_l \mathfrak{B}_m (X^i Y_{jkh}) = a_{lm} (X^i Y_{jkh}), \tag{4.13}$$

$$\mathfrak{B}_l \mathfrak{B}_m (X_j Y_{kh}^i) = a_{lm} (X_j Y_{kh}^i) \tag{4.14}$$

and

$$\mathfrak{B}_l \mathfrak{B}_m (T_j^i \psi_{kh}) = a_{lm} (T_j^i \psi_{kh}), \tag{4.15}$$

respectively.

### 5. EXAMPLES

In order to illustrate the effectiveness of the proposed findings, we consider some examples of the recurrence and birecurrence properties.

**Example 5.1.** The decomposition tensor  $(X^i Y_{jkh})$  is recurrent if and only if it satisfies

$$\mathfrak{B}_m [p.(X^i Y_{jkh})] = \lambda_m [p.(X^i Y_{jkh})].$$

Firstly, since the decomposition tensor  $(X^i Y_{jkh})$  is recurrent, the condition (3.21) is satisfied. In view of (2.6), the decomposition tensor  $(X^i Y_{jkh})$  on indicatrix given by

$$p.(X^i Y_{jkh}) = X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d. \tag{5.1}$$

By using  $\mathfrak{B}$ -covariant derivative for eq. (5.1) with respect to  $x^m$ , using equation (3.21) and the fact that  $h_b^a$  is covariant constant in above equation, we get

$$\mathfrak{B}_m [p.(X^i Y_{jkh})] = \lambda_m X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d.$$

Using equation (5.1) in above equation, we get

$$\mathfrak{B}_m [p.(X^i Y_{jkh})] = \lambda_m [p.(X^i Y_{jkh})]. \tag{5.2}$$

Above equation means the projection on indicatrix for the decomposition tensor  $(X^i Y_{jkh})$  behaves as recurrent.

Secondly, let the projection on indicatrix for the decomposition tensor  $(X^i Y_{jkh})$  is recurrent, that is, it satisfies equation (5.2). By using (2.6) in equation (5.2), we get

$$\mathfrak{B}_m \left( X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d \right) = \lambda_m X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d.$$

Using (2.7) in above equation, we get

$$\begin{aligned}
& \mathfrak{B}_m \left[ (X^i Y_{jkh}) - (X^i Y_{jkd}) l^d l_h - (X^i Y_{jch}) l^c l_k + (X^i Y_{jcd}) l^c l_k l^d l_h \right. \\
& \quad - (X^i Y_{bkh}) l^b l_j + (X^i Y_{bkd}) l^b l_j l^d l_h + (X^i Y_{bch}) l^b l_j l^c l_k - (X^i Y_{bcd}) l^b l_j l^c l_k l^d l_h \\
& \quad - (X^a Y_{jkh}) l^i l_a + (X^a Y_{jkd}) l^i l_a l^d l_h + (X^a Y_{jch}) l^i l_a l^c l_k - (X^a Y_{jcd}) l^i l_a l^c l_k l^d l_h \\
& \quad + (X^a Y_{bkh}) l^i l_a l^b l_j - (X^a Y_{bkd}) l^i l_a l^b l_j l^d l_h - (X^a Y_{bch}) l^i l_a l^b l_j l^c l_k \\
& \quad \left. + (X^a Y_{bcd}) l^i l_a l^b l_j l^c l_k l^d l_h \right] \\
& = \lambda_m \left[ (X^i Y_{jkh}) - (X^i Y_{jkd}) l^d l_h - (X^i Y_{jch}) l^c l_k + (X^i Y_{jcd}) l^c l_k l^d l_h \right. \\
& \quad - (X^i Y_{bkh}) l^b l_j + (X^i Y_{bkd}) l^b l_j l^d l_h + (X^i Y_{bch}) l^b l_j l^c l_k - (X^i Y_{bcd}) l^b l_j l^c l_k l^d l_h \\
& \quad - (X^a Y_{jkh}) l^i l_a + (X^a Y_{jkd}) l^i l_a l^d l_h + (X^a Y_{jch}) l^i l_a l^c l_k - (X^a Y_{jcd}) l^i l_a l^c l_k l^d l_h \\
& \quad + (X^a Y_{bkh}) l^i l_a l^b l_j - (X^a Y_{bkd}) l^i l_a l^b l_j l^d l_h - (X^a Y_{bch}) l^i l_a l^b l_j l^c l_k \\
& \quad \left. + (X^a Y_{bcd}) l^i l_a l^b l_j l^c l_k l^d l_h \right].
\end{aligned}$$

From,  $l^i = \frac{y^i}{F}$  and  $l_i = \frac{y_i}{F}$ , if  $(X^a Y_{bcd}) y_a = (X^a Y_{bcd}) y^b = (X^a Y_{bcd}) y^c = (X^a Y_{bcd}) y^d = 0$ , then above equation can be written as

$$\mathfrak{B}_m(X^i Y_{jkh}) = \lambda_m(X^i Y_{jkh}).$$

Above equation means the decomposition tensor  $(X^i Y_{jkh})$  behaves as recurrent.

Also, we can use same technique for showing the decomposition tensors  $(X_j Y_{kh}^i)$  and  $(T_j^i \psi_{kh})$  are recurrent if and only if the projection on indicatrix for them behave as recurrent.

**Example 5.2.** The decomposition tensor  $(X^i Y_{jkh})$  is birecurrent if and only if it satisfies

$$\mathfrak{B}_l \mathfrak{B}_m [p.(X^i Y_{jkh})] = a_{lm} [p.(X^i Y_{jkh})].$$

Firstly, since the decomposition tensor  $(X^i Y_{jkh})$  is birecurrent, that, the condition (4.13) is satisfied. By using  $\mathfrak{B}$ -covariant derivative for equation (5.1) with respect to  $x^m$  and  $x^l$ , using equation (4.13) and the fact that  $h_b^a$  is covariant constant, we get

$$\mathfrak{B}_l \mathfrak{B}_m [p.(X^i Y_{jkh})] = a_{lm} X^a Y_{bcd}^a h_a^i h_j^b h_k^c h_h^d.$$

Using equation (5.1) in above equation, we get

$$\mathfrak{B}_l \mathfrak{B}_m [p.(X^i Y_{jkh})] = a_{lm} [p.(X^i Y_{jkh})]. \quad (5.3)$$

Equation (5.3) means the projection on indicatrix for the decomposition tensor  $(X^i Y_{jkh})$  behaves as birecurrent.

Secondly, let the projection on indicatrix for the decomposition tensor  $(X^i Y_{jkh})$  is birecurrent, that is, it satisfies equation (5.3). By using (2.6) in equation (5.3), we get

$$\mathfrak{B}_l \mathfrak{B}_m \left( X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d \right) = a_{lm} X^a Y_{bcd} h_a^i h_j^b h_k^c h_h^d.$$

Using (2.7) in above equation, we get

$$\begin{aligned} & \mathfrak{B}_l \mathfrak{B}_m \left[ (X^i Y_{jkh}) - (X^i Y_{jkd}) l^d l_h - (X^i Y_{jch}) l^c l_k + (X^i Y_{jcd}) l^c l_k l^d l_h \right. \\ & \quad - (X^i Y_{bkh}) l^b l_j + (X^i Y_{bkd}) l^b l_j l^d l_h + (X^i Y_{bch}) l^b l_j l^c l_k - (X^i Y_{bcd}) l^b l_j l^c l_k l^d l_h \\ & \quad - (X^a Y_{jkh}) l^i l_a + (X^a Y_{jkd}) l^i l_a l^d l_h + (X^a Y_{jch}) l^i l_a l^c l_k - (X^a Y_{jcd}) l^i l_a l^c l_k l^d l_h \\ & \quad + (X^a Y_{bkh}) l^i l_a l^b l_j - (X^a Y_{bkd}) l^i l_a l^b l_j l^d l_h - (X^a Y_{bch}) l^i l_a l^b l_j l^c l_k \\ & \quad \left. + (X^a Y_{bcd}) l^i l_a l^b l_j l^c l_k l^d l_h \right] \\ & = a_{lm} \left[ (X^i Y_{jkh}) - (X^i Y_{jkd}) l^d l_h - (X^i Y_{jch}) l^c l_k + (X^i Y_{jcd}) l^c l_k l^d l_h \right. \\ & \quad - (X^i Y_{bkh}) l^b l_j + (X^i Y_{bkd}) l^b l_j l^d l_h + (X^i Y_{bch}) l^b l_j l^c l_k - (X^i Y_{bcd}) l^b l_j l^c l_k l^d l_h \\ & \quad - (X^a Y_{jkh}) l^i l_a + (X^a Y_{jkd}) l^i l_a l^d l_h + (X^a Y_{jch}) l^i l_a l^c l_k - (X^a Y_{jcd}) l^i l_a l^c l_k l^d l_h \\ & \quad + (X^a Y_{bkh}) l^i l_a l^b l_j - (X^a Y_{bkd}) l^i l_a l^b l_j l^d l_h - (X^a Y_{bch}) l^i l_a l^b l_j l^c l_k \\ & \quad \left. + (X^a Y_{bcd}) l^i l_a l^b l_j l^c l_k l^d l_h \right]. \end{aligned}$$

From  $l^i = \frac{y^i}{F}$  and  $l_i = \frac{y_i}{F}$ , if  $(X^a Y_{bcd}) y_a = (X^a Y_{bcd}) y^b = (X^a Y_{bcd}) y^c = (X^a Y_{bcd}) y^d = 0$ , then above equation can be written as  $\mathfrak{B}_l \mathfrak{B}_m (X^i Y_{jkh}) = a_{lm} (X^i Y_{jkh})$ . Last equation means the decomposition tensor  $(X^i Y_{jkh})$  behaves as birecurrent.

Also, we can use same technique for showing the decomposition tensors  $(X_j Y_{kh}^i)$  and  $(T_j^i \psi_{kh})$  are birecurrent if and only if the projection on indicatrix for them behave as birecurrent.

## 6. CONCLUSION

This article contributed in particular to the growth of the decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in the generalized  $\mathfrak{B}P$ -recurrent space and generalized  $\mathfrak{B}P$ -birecurrent space. We obtained some tensors satisfy the recurrence and birecurrence property under the decomposition. Also, different identities and several theorems have been discussed under the decomposition in  $G(\mathfrak{B}P) - RF_n$  and  $G(\mathfrak{B}P) - BRF_n$ . The topic examined in this manuscript can be expanded to a greater extent by the use of decomposition of Cartan's second curvature tensor  $P_{jkh}^i$  in generalized  $\mathfrak{B}P$ -trirecurrent space and we find some theorems under the decomposition in  $G(\mathfrak{B}P) - TRF_n$ .

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