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ON THE q-GENOCCHI NUMBERS AND POLYNOMIALS WITH WEIGHT ZERO AND THEIR APPLICATIONS

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Abstract. In the paper, the authors discuss properties of the q-Genocchi numbers and polynomials with weight zero. They discover some interesting relations via the p-adic q-integral on $\mathbb{Z}_p$ and familiar basis Bernstein polynomials and show that the p-adic log gamma functions are associated with the q-Genocchi numbers and polynomials with weight zero.

1. Preliminaries

Let p be an odd prime number. Denote the ring of the p-adic integers by $\mathbb{Z}_p$, the field of rational numbers by $\mathbb{Q}$, the field of the p-adic rational numbers by $\mathbb{Q}_p$, and the completion of algebraic closure of $\mathbb{Q}_p$ by $\mathbb{C}_p$, respectively. Let $\mathbb{N}$ be the set of positive integers and $\mathbb{N}^* = \{0\} \cup \mathbb{N}$ the set of all non-negative integers. Let $|\cdot|_p$ be the p-adic norm on $\mathbb{Q}$ with $|p|_p = p^{-1}$.

When one talks of a q-extension, q can be variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assumes $|q| < 1$. If $q \in \mathbb{C}_p$, one normally assumes $|1 - q|_p < 1$.

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We use the notation $[x]_q = \frac{1-q^x}{1-q}$. Hence $\lim_{q\to1}[x]_q = x$ for any $x \in \mathbb{C}$ in the complex case and any x with $|x|_p \le 1$ in the present p-adic case. This is the hallmark of a q-analog: The limit as $q \to 1$ recovers the classical object.

A function f is said to be uniformly differentiable at a point $a \in \mathbb{Z}_p$ if the divided difference

$$F_f(x,y) = \frac{f(x)-f(y)}{x-y}$$

converges to $f'(a)$ as $(x,y) \to (a,a)$. The class of all the uniformly differentiable functions is denoted by $UD(\mathbb{Z}_p)$.

For $f \in UD(\mathbb{Z}_p)$, the p-adic q-analogue of Riemann sum for f is defined by

$$\frac{1}{[p^n]_q} \sum_{0\le\xi<p^n} f(\xi)q^\xi = \sum_{0\le\xi<p^n} f(\xi)\mu_q\big(\xi + p^n\mathbb{Z}_p\big) \tag{1.1}$$

in [7, 9], where $n \in \mathbb{N}$. The integral of f on $\mathbb{Z}_p$ is defined as the limit of (1.1) as n tends to ∞, if it exists, and represented by

$$I_q(f) = \int_{\mathbb{Z}_p} f(\xi)\,\mathrm{d}\mu_q(\xi). \tag{1.2}$$

The bosonic integral and the fermionic p-adic integral on $\mathbb{Z}_p$ are defined respectively by

$$I_1(f) = \lim_{q\to1} I_q(f) \tag{1.3}$$

and

$$I_{-q}(f) = \lim_{q\to-q} I_q(f). \tag{1.4}$$

For a prime p and a positive integer d with $(p,d)=1$, set

$$X = X_d = \varprojlim_n \mathbb{Z}/dp^n\mathbb{Z}, \quad X_1 = \mathbb{Z}_p,$$

$$X^* = \bigcup_{\substack{(a,p)=1\\ 0<a<dp}} a + dp\mathbb{Z}_p,$$

and

$$a + dp^n\mathbb{Z}_p = \big\{x \in X \mid x \equiv a \mod dp^n\big\},$$

where $a \in \mathbb{Z}$ satisfies $0 \le a < dp^n$ and $n \in \mathbb{N}$.

In this paper, we will discuss properties of the q-Genocchi numbers and polynomials with weight zero. Via the p-adic q-integral on $\mathbb{Z}_p$ and familiar basis Bernstein polynomials, we discover some interesting relations and show that the p-adic log gamma functions are associated with the q-Genocchi numbers and polynomials with weight zero.

2. Main results

Now we are in a position to state our main results.

Theorem 2.1. *For $n \in \mathbb{N}$, we have*

$$\frac{\widetilde{G}_{n+1,q}(x)}{n+1} = H_n\big(-q^{-1}, x\big). \tag{2.1}$$

Proof. In [2, 3], Ararí, Acikgoz, and Seo considered the q-Genocchi polynomials with weight α in the form

$$\frac{\widetilde{G}^{(\alpha)}_{n+1,q}(x)}{n+1} = \int_{\mathbb{Z}_p} [x+\xi]^n_{q^\alpha} \,\mathrm{d}\mu_{-q}(\xi), \tag{2.2}$$

where $\widetilde{G}^{(\alpha)}_{n+1,q} = \widetilde{G}^{(\alpha)}_{n+1,q}(0)$ is called the q-Genocchi numbers with weight α. Taking $\alpha = 0$ in (2.2), we easily see that

$$\frac{\widetilde{G}_{n+1,q}}{n+1} \triangleq \frac{\widetilde{G}^{(0)}_{n+1,q}}{n+1} = \int_{\mathbb{Z}_p} \xi^n \,\mathrm{d}\mu_{-q}(\xi), \tag{2.3}$$

where $\widetilde{G}_{n,q}$ are called the q-Genocchi numbers and polynomials with weight 0. From (2.3), it is simple to see

$$\sum_{n=0}^{\infty} \widetilde{G}_{n,q}\frac{t^n}{n!} = t\int_{\mathbb{Z}_p} e^{\xi t} \,\mathrm{d}\mu_{-q}(\xi). \tag{2.4}$$

By (1.4), we have

$$q^n I_{-q}(f_n) + (-1)^{n-1} I_{-q}(f) = [2]_q \sum_{0\le \ell<n} q^\ell (-1)^{n-1-\ell} f(\ell), \tag{2.5}$$

where $f_n(x) = f(x+n)$ and $n \in \mathbb{N}$(see, [6, 8, 10]). Taking $n = 1$ in (2.5) leads to the well-known equality

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0). \tag{2.6}$$

When setting $f(x) = e^{xt}$ in (2.6), we find

$$\sum_{n=0}^{\infty} \widetilde{G}_{n,q}\frac{t^n}{n!} = \frac{[2]_q t}{qe^t+1}. \tag{2.7}$$

By (2.7), we obtain the q-Genocchi polynomials with weight 0 as follows

$$\sum_{n=0}^{\infty} \widetilde{G}_{n,q}(x)\frac{t^n}{n!} = \frac{[2]_q t}{qe^t+1} e^{xt}. \tag{2.8}$$

By (2.8), we see that

$$\sum_{n\ge 0} \widetilde{G}_{n,q}(x)\frac{t^n}{n!} = t\frac{1-\big(-q^{-1}\big)}{e^t-\big(-q^{-1}\big)}e^{xt} = t\sum_{n\ge 0} H_n\big(-q^{-1}, x\big)\frac{t^n}{n!},$$

where $H_n\bigl(-q^{-1}, x\bigr)$ are the n-th Frobenius-Euler polynomials defined by

$$\sum_{n=0}^{\infty} H_n(\lambda, x)\frac{t^n}{n!} = \frac{1-\lambda}{e^t-\lambda}, \quad \lambda \in \mathbb{C} \setminus \{1\}.$$

Equating coefficients of t^n on both sides of the above equality leads to the identity (2.1). □

Theorem 2.2. *For $n \in \mathbb{N}$, the identity*

$$qH_n\bigl(-q^{-1}, x+1\bigr) + H_n\bigl(-q^{-1}, x\bigr) = [2]_q x^n \tag{2.9}$$

is valid.

Proof. By (2.6), we discover that

$$\begin{aligned}
[2]_q \sum_{n=0}^{\infty} x^n \frac{t^n}{n!} &= q\int_{\mathbb{Z}_p} e^{(x+\xi+1)t} \,\mathrm{d}\mu_{-q}(\xi) + \int_{\mathbb{Z}_p} e^{(x+\xi)t} \,\mathrm{d}\mu_{-q}(\xi) \\
&= \sum_{n=0}^{\infty} \left[q\int_{\mathbb{Z}_p} (x+\xi+1)^n \,\mathrm{d}\mu_{-q}(\xi) + \int_{\mathbb{Z}_p} (x+\xi)^n \,\mathrm{d}\mu_{-q}(\xi) \right] \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \bigl[qH_n\bigl(-q^{-1}, x+1\bigr) + H_n\bigl(-q^{-1}, x\bigr) \bigr] \frac{t^n}{n!}.
\end{aligned}$$

Equating coefficients of $\frac{t^n}{n!}$ on both sides of the above equation leads to the identity (2.9). □

Theorem 2.3. *The identities*

$$G_n(x+1) + G_n(x) = 2nx^{n-1}, \quad n \ge 1 \tag{2.10}$$

and

$$q\widetilde{G}_{n,q}(1) + \widetilde{G}_{n,q} = \begin{cases} [2]_q, & n = 1 \\ 0, & n \ne 1 \end{cases} \tag{2.11}$$

are true, where $G_n(x)$ are called the Genocchi polynomials.

Proof. These follow from respectively letting $q = 1$ and $x = 0$ into the identity (2.9). □

Theorem 2.4. *The following identity holds*

$$\widetilde{G}_{n,q^{-1}}(1-x) = (-1)^{n+1}\widetilde{G}_{n,q}(x). \tag{2.12}$$

Proof. When we substitute x by $1-x$ and q by q^{-1} in (2.8), it follows that

$$\sum_{n=0}^{\infty} \widetilde{G}_{n,q^{-1}}(1-x)\frac{t^n}{n!} = t\frac{1+q^{-1}}{q^{-1}e^t+1}e^{(1-x)t} = \frac{1+q}{e^t+q}e^t e^{xt}$$
$$= -\frac{[2]_q(-t)}{qe^{-t}+1}e^{(-t)x} = \sum_{n=0}^{\infty}(-1)^{n+1}\widetilde{G}_{n,q}(x)\frac{t^n}{n!}.$$

From this, we procure the equality (2.12), the symmetric property of this type polynomials. □

Theorem 2.5. *The identity*

$$\widetilde{G}_{n,q}(x) = \sum_{k=0}^{n}\binom{n}{k}\widetilde{G}_{k,q}x^{n-k} \tag{2.13}$$

is true.

Proof. By using (2.2) for $\alpha = 0$ and the binomial theorem, we readily obtain that

$$\frac{\widetilde{G}_{n+1,q}(x)}{n+1} = \int_{\mathbb{Z}_p}(x+\xi)^n \, \mathrm{d}\mu_{-q}(\xi)$$
$$= \sum_{k=0}^{n}\binom{n}{k}\left[\int_{\mathbb{Z}_p}\xi^k \, \mathrm{d}\mu_{-q}(\xi)\right]x^{n-k} = \sum_{k=0}^{n}\binom{n}{k}\frac{\widetilde{G}_{k+1,q}}{k+1}x^{n-k}.$$

Further using

$$\frac{n+1}{k+1}\binom{n}{k} = \binom{n+1}{k+1},$$

we obtain

$$\widetilde{G}_{n+1,q}(x) = \sum_{k=0}^{n}\binom{n+1}{k+1}\widetilde{G}_{k+1,q}x^{n-k} = \sum_{k=1}^{n+1}\binom{n+1}{k}\widetilde{G}_{k,q}x^{n+1-k}.$$

Thus, the equality (2.13) follows. □

Proposition 2.1. *The identities*

$$\widetilde{G}_{0,q} = 0 \quad \text{and} \quad q\big(\widetilde{G}_q+1\big)^n + \widetilde{G}_{n,q} = \begin{cases} [2]_q, & n=1 \\ 0, & n \neq 1 \end{cases} \tag{2.14}$$

are true, where the usual convention of replacing $\big(\widetilde{G}_q\big)^n$ *by* $\widetilde{G}_{n,q}$ *is used.*

Proof. These can be deduced from combining (2.11) with (2.13). □

Proposition 2.2. *For* $n > 1$,

$$\widetilde{G}_{n+1,q}(2) = \frac{(n+1)}{q}[2]_q + \frac{1}{q^2}\widetilde{G}_{n+1,q}. \tag{2.15}$$

Proof. From (2.13), it follows that

$$\begin{aligned} q^2\widetilde{G}_{n+1,q}(2) &= q^2\big(\widetilde{G}_q+1+1\big)^{n+1} = q^2\sum_{k=0}^{n+1}\binom{n+1}{k}\big(\widetilde{G}_q+1\big)^k \\ &= (n+1)q^2\big(\widetilde{G}_q+1\big) + q\sum_{k=2}^{n+1}\binom{n+1}{k}q\big(\widetilde{G}_q+1\big)^k \\ &= (n+1)q\big([2]_q-\widetilde{G}_{1,q}\big) - q\sum_{k=2}^{n+1}\binom{n+1}{k}\widetilde{G}_{k,q} \\ &= (n+1)q[2]_q - \left[q\sum_{k=2}^{n+1}\binom{n+1}{k}\widetilde{G}_{k,q} + (n+1)q\widetilde{G}_{1,q}\right] \\ &= (n+1)q[2]_q - q\sum_{k=0}^{n+1}\binom{n+1}{k}\widetilde{G}_{k,q} \\ &= (n+1)q[2]_q - q\big(\widetilde{G}_q+1\big)^{n+1} = (n+1)q[2]_q + \widetilde{G}_{n+1,q} \end{aligned}$$

for $n > 1$. Therefore, we deduce (2.15). □

Theorem 2.6. *The identity*

$$\int_{\mathbb{Z}_p}(1-\xi)^n \,\mathrm{d}\mu_{-q}(\xi) = [2]_q + q^2\frac{\widetilde{G}_{n+1,q^{-1}}}{n+1} \tag{2.16}$$

is valid.

Proof. By virtue of (1.4), (2.12), and (2.15), we find

$$\begin{gathered} (n+1)\int_{\mathbb{Z}_p}(1-\xi)^n \,\mathrm{d}\mu_{-q}(\xi) = (n+1)(-1)^n\int_{\mathbb{Z}_p}(\xi-1)^n \,\mathrm{d}\mu_{-q}(\xi) \\ = (-1)^n\widetilde{G}_{n+1,q}(-1) = \widetilde{G}_{n+1,q^{-1}}(2) = (n+1)[2]_q + q^2\widetilde{G}_{n+1,q^{-1}}. \end{gathered}$$

As a result, we conclude Theorem 2.6. □

Theorem 2.7. *The following identity holds:*

$$\begin{aligned}
&\sum_{\ell=0}^{n-k}\binom{n-k}{\ell}(-1)^{\ell}\frac{\widetilde{G}_{\ell+k+1,q}}{\ell+k+1}\\
&=\begin{cases}[2]_q+q^2\dfrac{\widetilde{G}_{n+1,q^{-1}}}{n+1}, & k=0,\\ \displaystyle\sum_{s=0}^{k}\binom{k}{s}(-1)^{k-s}\left([2]_q+q^2\frac{\widetilde{G}_{n-s+1,q^{-1}}}{n-s+1}\right), & k\neq 0.\end{cases}
\end{aligned}$$

Proof. Let $UD(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f\in UD(\mathbb{Z}_p)$, the p-adic analogue of Bernstein operator for f is defined by

$$B_n(f,x)=\sum_{k=0}^{n}f\left(\frac{k}{n}\right)B_{k,n}(x)=\sum_{k=0}^{n}f\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k},$$

where $n,k\in\mathbb{N}^*$ and the p-adic Bernstein polynomials of degree n is defined by

$$B_{k,n}(x)=\binom{n}{k}x^k(1-x)^{n-k},\quad x\in\mathbb{Z}_p, \tag{2.17}$$

see [4, 11, 12, 13]. Via the p-adic q-integral on $\mathbb{Z}_p$ and Bernstein polynomials in (2.17), we can obtain that

$$\begin{aligned}
I_1&=\int_{\mathbb{Z}_p}B_{k,n}(\xi)\,\mathrm{d}\mu_{-q}(\xi)\\
&=\binom{n}{k}\int_{\mathbb{Z}_p}\xi^k(1-\xi)^{n-k}\,\mathrm{d}\mu_{-q}(\xi)\\
&=\binom{n}{k}\sum_{\ell=0}^{n-k}\binom{n-k}{\ell}(-1)^{\ell}\left[\int_{\mathbb{Z}_p}\xi^{\ell+k}\,\mathrm{d}\mu_{-q}(\xi)\right]\\
&=\binom{n}{k}\sum_{\ell=0}^{n-k}\binom{n-k}{\ell}(-1)^{\ell}\frac{\widetilde{G}_{\ell+k+1,q}}{\ell+k+1}.
\end{aligned}$$

On the other hand, by symmetric properties of Bernstein polynomials, we have

$$
\begin{aligned}
I_2 &= \int_{\mathbb{Z}_p} B_{n-k,n}(1-\xi)\,\mathrm{d}\mu_{-q}(\xi)\\
&= \binom{n}{k}\sum_{s=0}^{k}\binom{k}{s}(-1)^{k-s}\int_{\mathbb{Z}_p}(1-\xi)^{n-s}\,\mathrm{d}\mu_{-q}(x)\\
&= \binom{n}{k}\sum_{s=0}^{k}\binom{k}{s}(-1)^{k-s}\left([2]_q+q^2\frac{\widetilde{G}_{n-s+1,q^{-1}}}{n-s+1}\right)\\
&= \begin{cases} [2]_q+q^2\dfrac{\widetilde{G}_{n+1,q^{-1}}}{n+1}, & k=0,\\ \dbinom{n}{k}\displaystyle\sum_{s=0}^{k}\dbinom{k}{s}(-1)^{k-s}\left([2]_q+q^2\dfrac{\widetilde{G}_{n-s+1,q^{-1}}}{n-s+1}\right), & k\ne 0.\end{cases}
\end{aligned}
$$

Equating I_1 and I_2 yields Theorem 2.7. □

Theorem 2.8. *The identity*

$$
\begin{aligned}
&\sum_{\ell=0}^{n_1+\cdots+n_m-mk}\binom{n_1+\cdots+n_m-mk}{\ell}(-1)^{\ell}\frac{\widetilde{G}_{\ell+mk+1,q}}{\ell+mk+1}\\
&= \begin{cases} [2]_q+q^2\dfrac{\widetilde{G}_{n_1+\cdots+n_m+1,q^{-1}}}{n_1+\cdots+n_m+1}, & k=0\\ \displaystyle\sum_{\ell=0}^{mk}\dbinom{mk}{\ell}(-1)^{mk+\ell}\left([2]_q+q^2\dfrac{\widetilde{G}_{n_1+\cdots+n_m+\ell+1,q^{-1}}}{n_1+\cdots+n_m+\ell+1}\right), & k\ne 0\end{cases}
\end{aligned}
\tag{2.18}
$$

is true.

Proof. The p-adic q-integral on $\mathbb{Z}_p$ of the product of several Bernstein polynomials can be calculated as

$$
\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p}\prod_{s=1}^{m}B_{k,n_s}(\xi)\,\mathrm{d}\mu_{-q}(\xi)\\
&= \prod_{s=1}^{m}\binom{n_s}{k}\int_{\mathbb{Z}_p}\xi^{mk}(1-\xi)^{n_1+\cdots+n_m-mk}\,\mathrm{d}\mu_{-q}(\xi)\\
&= \prod_{s=1}^{m}\binom{n_s}{k}\sum_{\ell=0}^{n_1+\cdots+n_m-mk}\binom{n_1+\cdots+n_m-mk}{\ell}(-1)^{\ell}\left[\int_{\mathbb{Z}_p}\xi^{\ell+mk}\,\mathrm{d}\mu_{-q}(\xi)\right]\\
&= \prod_{s=1}^{m}\binom{n_s}{k}\sum_{\ell=0}^{n_1+\cdots+n_m-mk}\binom{n_1+\cdots+n_m-mk}{\ell}(-1)^{\ell}\frac{\widetilde{G}_{\ell+mk+1,q}}{\ell+mk+1}.
\end{aligned}
$$

On the other hand, by symmetric properties of Bernstein polynomials and the equality (2.16), we have

$$\begin{aligned}
I_4 &= \int_{\mathbb{Z}_p} \prod_{s=1}^{m} B_{n_s-k,n_s}(1-\xi)\,\mathrm{d}\mu_{-q}(\xi) \\
&= \prod_{s=1}^{m}\binom{n_s}{k}\sum_{\ell=0}^{mk}\binom{mk}{\ell}(-1)^{mk-\ell}\int_{\mathbb{Z}_p}(1-\xi)^{n_1+\cdots+n_m-\ell}\,\mathrm{d}\mu_{-q}(\xi) \\
&= \prod_{s=1}^{m}\binom{n_s}{k}\sum_{\ell=0}^{mk}\binom{mk}{\ell}(-1)^{mk-\ell}\left([2]_q + q^2\frac{\widetilde{G}_{n_1+\cdots+n_m-\ell+1,q^{-1}}}{n_1+\cdots+n_m-\ell+1}\right) \\
&= \begin{cases} [2]_q + q^2\dfrac{\widetilde{G}_{n_1+\cdots+n_m+1,q^{-1}}}{n_1+\cdots+n_m+1}, & k=0, \\ \displaystyle\prod_{s=1}^{m}\binom{n_s}{k}\sum_{\ell=0}^{mk}\binom{mk}{\ell}(-1)^{mk-\ell}\left([2]_q + q^2\frac{\widetilde{G}_{n_1+\cdots+n_m-\ell+1,q^{-1}}}{n_1+\cdots+n_m-\ell+1}\right), & k\neq 0. \end{cases}
\end{aligned}$$

Equating I_3 and I_4 results in an interesting identity (2.18) for the q-analogue of Genocchi polynomials with weight 0. □

3. An identity on p-adic locally analytic functions

In this section, we consider Kim's p-adic q-log gamma functions related to the q-analogue of Genocchi polynomials.

Definition 3.1. ([5, 7]) For $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$(1+x)\log(1+x) = x + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+1)}x^{n+1}.$$

Kim's p-adic locally analytic function on $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$ can be defined as follows.

Definition 3.2. ([5, 7]) For $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$G_{p,q}(x) = \int_{\mathbb{Z}_p}[x+\xi]_q(\log[x+\xi]_q - 1)\,\mathrm{d}\mu_{-q}(\xi).$$

By considering Kim's p-adic q-log gamma function, we introduce the following p-adic locally analytic function

$$G_{p,1}(x) \triangleq G_p(x) = \int_{\mathbb{Z}_p}(x+\xi)[\log(x+\xi)-1]\,\mathrm{d}\mu_{-q}(\xi). \tag{3.1}$$

Theorem 3.1. *For* $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$G_p(x) = \left(x + \frac{\widetilde{G}_{2,q}}{2}\right)\log x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)(n+2)} \frac{\widetilde{G}_{n+2,q}}{x^n} - x. \tag{3.2}$$

Proof. Replacing x by $\frac{\xi}{x}$ in (3.1) leads to

$$(x+\xi)[\log(x+\xi) - 1] = (x+\xi)\log x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)} \frac{\xi^{n+1}}{x^n} - x. \tag{3.3}$$

From (3.1) and (3.3), we can establish an interesting formula (3.2). □

Remark 3.1. This is a revised version of the preprint [1].

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