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UTILIZING FIXED POINT METHODS IN MATHEMATICAL MODELLING

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Abstract. The use of mathematical modelling in the study of epidemiological disorders continues to grow substantially. In order to better support global policy initiatives and explain the possible consequence of an outbreak, mathematical models were constructed to forecast how epidemic illnesses spread. In this paper, fractional derivatives and $(\varpi - F_{\mathcal{C}})$-contractions are used to explore the existence and uniqueness solutions of the novel coronavirus-19 model.

1. Introduction

A strange virus known as Cov-19 has been waging a relentless attack on the globe for more than 24 months. Wuhan was the origin of the epidemic, which has since propagated to every nation on earth. A deadly contagious sickness has claimed the lives of millions of people all across the world. For elderly people and people with health concerns, the disorder has an extremely

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high mortality rate. Despite the fact that it is widely acknowledged that the transmission is much more vigorous at relatively low temperature, the data to support this claim has not yet been discovered. The etymology of this virus is not investigated, despite the fact that numerous scientists from various backgrounds have started some basic investigations on the dissemination, the surface life duration, and the tactics that may be employed to limit the spread. Many people insist that the virus originated in bats, pangolins, shellfish, etc., while others think it was created by humans.

Over the past few months, there have been few eradicative vaccinations for this virus available worldwide. Due to steps taken by humanity to stop the spread of the coronavirus, many cities throughout the world are now deserted. By waking up some invisible adversaries who may theoretically harm and paralyze everything that humans have created in previous decades, breaching the laws of nature is indirectly a strong indication that humanity should abide by them. Scientists can now comprehend that nature is dynamic and that human awareness is insufficient to endanger the natural world. It has extensive expertise in the creation of vaccines. Therefore, care must be made when interacting with the many beings that inhabit our environment.

The principles, life span, manner of propagation, impact of age, temperature affluence, and many other aspects of the human body have all been the subject of numerous studies. But utilizing the idea of differentiation and the observable facts, mathematicians build a mathematical model to better explain the distribution. These facts enable them to create a model in which the solution could describe the upcoming time-dependent activities in the practical case. With a collection of facts and constraints, mathematicians use this method to understand, control, and forecast how potential challenges will behave.

On the other hand, Wardowsky [21] defined and explored the idea of F-contraction. It has drawn numerous authors to publish numerous intriguing findings in this field. Topological concepts like Cauchy, completion, converges, and a contraction-type mapping of the form were sort of involved.

$$d(\mathscr{H}a, \mathscr{H}b) > 0 \Rightarrow \tau + F(d(\mathscr{H}a, \mathscr{H}b)) \leq F(d(a, b))$$

for all $a, b \in X$, where $\tau > 0$, $\mathscr{H} : X \to X$ and $F : (0, \infty) \to \mathbb{R}$ fulfills the subsequent conditions:

(F_1): F is strictly increasing;
(F_2): $\lim\limits_{n\to\infty} t_n = 0$ iff $\lim\limits_{n\to\infty} F(t_n) = -\infty$;
(F_3): There exists $k \in (0, 1)$ such that $\lim\limits_{t\to 0^+} t^k F(t) = 0$.

Definition 1.1. ([19]) A mapping $\mathscr{H} : X \to X$ is α-admissible if there exists a function $\alpha : X \times X \to \mathbb{R}^+$ such that $a, b \in X$, $\alpha(a, b) \geq 1 \Rightarrow \alpha(\mathscr{H}a, \mathscr{H}b) \geq 1$.

If we reckon the concepts of F-contraction and α-admissibility in the concept of fixed points, there is a wide growth in this concern. The ideas used in [21], [19] and/or generalizations of metric spaces have attracted several authors. For more clearly,

- for generalizations of metric space, the reader can refer [2], [3], [8], [10];
- for utilization of F-contractions and generalizations of F-contractions, the reader can refer [1], [6], [7], [9], [14], [15], [16], [18], [22], [24];
- applications of fixed point theorems, the reader can refer [12], [13], [17], [20], [23].

On the mirror side of this section, S. Czerwik [5] introduced the idea of a b-metric space in 1993.

Definition 1.2. ([5]) Assume that X is a nonempty set and $s \geq 1$. A function $d : X \times X \longrightarrow [0, \infty)$ is called a b-metric space provided that the subsequent properties are fulfilled:

(i) $d(a, b) = 0$ iff $a = b$;
(ii) $d(a, b) = d(b, a)$, $\forall\ a, b \in X$;
(iii) $d(a, b) \leq s[d(a, c) + d(c, b)]$, $\forall\ a, b, c \in X$.

Definition 1.3. ([10]) Assume that X is a non-empty set and $p : X \times X \to [1, \infty)$. A function $\mathcal{C} : X \times X \to [0, \infty)$ is said to be a controlled metric type if for all $a, b, c \in X$, it the subsequent properties are fulfilled:

(i) $\mathcal{C}(a, b) = 0$ iff $a = b$;
(ii) $\mathcal{C}(a, b) = \mathcal{C}(b, a)$;
(iii) $\mathcal{C}(a, b) \leq p(a, c)\mathcal{C}(a, c) + p(c, b)\mathcal{C}(c, b)$.

The pair $(X, \mathcal{C}_b)$ is said to be a controlled metric type space.

Now, we present the idea of controlled b-metric space by summing up the above concepts, that is, a b-metric space and a controlled metric type space in the following direction:

Definition 1.4. Assume that X is a nonempty set and $s \geq 1$. Given $p : X \times X \to [1, \infty)$. A function $\mathcal{C}_b : X \times X \to [0, \infty)$ is said to be a controlled b-metric (simply, a $\mathcal{C}_b$-metric) provided that for all $a, b, c \in X$,

(i) $\mathcal{C}_b(a, b) = 0$ iff $a = b$;
(ii) $\mathcal{C}_b(a, b) = \mathcal{C}_b(b, a)$;
(iii) $\mathcal{C}_b(a, b) \leq s[p(a, c)\mathcal{C}_b(a, c) + p(c, b)\mathcal{C}_b(c, b)]$.

Example 1.5. Let $X = \{0, 1, 2\}$. Define $p : X \times X \to [1, \infty)$ and $\mathcal{C}_b : X \times X \to [0, \infty)$ as $p(a, b) = 1 + ab$ and

$$\mathcal{C}_b(2, 2) = \mathcal{C}_b(0, 0) = \mathcal{C}_b(1, 1) = 0;$$

$$\mathcal{C}_b(2,0) = \mathcal{C}_b(0,2) = 5; \mathcal{C}_b(0,1) = \mathcal{C}_b(1,0) = 10;$$
$$\mathcal{C}_b(2,1) = \mathcal{C}_b(1,2) = 30 \text{ and } \; s = 2.$$

Note that (i) and (ii) trivially hold. For (iii), we obtain

$$\mathcal{C}_b(2,0) = 5; \quad 2[p(2,1)\mathcal{C}_b(2,1) + p(1,0)\mathcal{C}_b(1,0)] = 200;$$
$$\mathcal{C}_b(1,2) = 30; \quad 2[p(1,0)\mathcal{C}_b(1,0) + p(0,2)\mathcal{C}_b(0,2)] = 30;$$
$$\mathcal{C}_b(0,1) = 10; \quad 2[p(0,2)\mathcal{C}_b(0,2) + p(2,1)\mathcal{C}_b(2,1)] = 190.$$

Hence, for all $a, b, c \in X$, $\mathcal{C}_b(a,b) \leq s[p(a,c)\mathcal{C}_b(a,c) + p(c,b)\mathcal{C}_b(c,b)]$. So $(X, \mathcal{C}_b)$ is a controlled b-metric space.

Remark 1.6. A controlled b-metric space is not a controlled metric type space since from above example

$$\mathcal{C}_b(1,2) \nleq 2[p(1,0)\mathcal{C}_b(1,0) + p(0,2)\mathcal{C}_b(0,2)].$$

Definition 1.7. Let $(X, \mathcal{C}_b)$ be a $\mathcal{C}_b$-metric space and $\{a_n\}$ be a sequence of points of X. Then

(1) $\{a_n\}$ converges to $a \in X$ if for all $\epsilon > 0$, there exist $N = N(\epsilon) \in \mathbb{N}$ such that $\mathcal{C}_b(a_n, a) < \epsilon$ for all $n \geq N$. In this situation, we define $\lim\limits_{n\to\infty} a_n = a$;

(2) $\{a_n\}$ is a Cauchy sequence if $\lim\limits_{m,n\to\infty} \mathcal{C}_b(a_m, a_n) = 0$;

(3) $(X, \mathcal{C}_b)$ is complete if every Cauchy sequence $\{a_n\}$ is convergent in X.

2. Banach contraction principle

Now we state and prove the first result.

Theorem 2.1. *Assume that $(X, \mathcal{C}_b)$ is a complete controlled b-metric space. Let $\mathscr{H} : X \to X$ satisfy*

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) \leq k\mathcal{C}_b(a,b), \tag{2.1}$$

for all $a, b \in X$ and $k \in (0,1)$. For $a_0 \in X$, take $a_n = \mathscr{H}^n a_0 = \mathscr{H}a_{n-1}$. Suppose that

$$\sup_{m\geq 1} \lim_{i\to\infty} s\frac{p(a_{i+1}, a_{i+2})}{p(a_i, a_{i+1})} p(a_{i+1}, a_m) < \frac{1}{k}. \tag{2.2}$$

In addition, let us consider that for every $a \in X$, we obtain

$$\lim_{n\to\infty} p(a_n, a) \;\; \textit{and} \;\; \lim_{n\to\infty} p(a, a_n) \;\; \textit{exist and are finite.} \tag{2.3}$$

Then $\mathscr{H}$ has a unique fixed point.

Proof. Initially, we have to verify the uniqueness. On contrary, let $\mathscr{H}$ has two fixed points, say u and v. Then

$$\mathcal{C}_b(u,v) = \mathcal{C}_b(\mathscr{H}u, \mathscr{H}v) \leq k\mathcal{C}_b(u,v),$$

so

$$(1-k)\mathcal{C}_b(u,v) \leq 0,$$

this implies that, $1-k=0$ which gives a contradiction. Therefore, $\mathscr{H}$ has a unique fixed point. Assume that $a_0 \in X$ be arbitrary. Define the iterative sequence $\{a_n\}$ by $a_n = \mathscr{H}^n a_0$. By using (2.1), we get

$$\mathcal{C}_b(a_n, a_{n+1}) \leq k^n \mathcal{C}_b(a_0, a_1), \ \ \forall\ n \geq 0.$$

For all $n, m \in \mathbb{N}(n<m)$, we have

$$\begin{aligned}
&\mathcal{C}_b(a_n, a_m)\\
&\leq s[p(a_n,a_{n+1})\mathcal{C}_b(a_n,a_{n+1}) + p(a_{n+1},a_m)\mathcal{C}_b(a_{n+1},a_m)]\\
&\leq sp(a_n,a_{n+1})\mathcal{C}_b(a_n,a_{n+1})\\
&\ \ + s^2p(a_{n+1},a_m)p(a_{n+1},a_{n+2})\mathcal{C}_b(a_{n+1},a_{n+2})\\
&\ \ + s^2p(a_{n+1},a_m)p(a_{n+2},a_m)\mathcal{C}_b(a_{n+2},a_m)\\
&\leq sp(a_n,a_{n+1})\mathcal{C}_b(a_n,x_{n+1})\\
&\ \ + s^2p(a_{n+1},a_m)p(a_{n+1},a_{n+2})\mathcal{C}_b(a_{n+1},a_{n+2})\\
&\ \ + s^3p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+2},a_{n+3})\mathcal{C}_b(a_{n+2},a_{n+3})\\
&\ \ + s^3p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+3},a_m)\mathcal{C}_b(a_{n+3},a_m)\\
&\leq sp(a_n,a_{n+1})\mathcal{C}_b(a_n,a_{n+1})\\
&\ \ + s^2p(a_{n+1},a_m)p(a_{n+1},a_{n+2})\mathcal{C}_b(a_{n+1},a_{n+2})\\
&\ \ + s^3p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+2},a_{n+3})\mathcal{C}_b(a_{n+2},a_{n+3})\\
&\ \ + s^4p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+3},a_m)p(a_{n+3},a_{n+4})\mathcal{C}_b(a_{n+3},a_{n+4})\\
&\ \ + s^4p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+3},a_m)p(a_{n+4},a_m)\mathcal{C}_b(a_{n+4},a_m)\\
&\ \vdots\\
&\leq sp(a_n,a_{n+1})\mathcal{C}_b(a_n,a_{n+1})\\
&\ \ + s^2p(a_{n+1},a_m)p(a_{n+1},a_{n+2})\mathcal{C}_b(a_{n+1},a_{n+2})\\
&\ \ + s^3p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+2},a_{n+3})\mathcal{C}_b(a_{n+2},a_{n+3})\\
&\ \ + s^4p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+3},a_m)p(a_{n+3},a_{n+4})\mathcal{C}_b(a_{n+3},a_{n+4})\\
&\ \ + s^4p(a_{n+1},a_m)p(a_{n+2},a_m)p(a_{n+3},a_m)\\
&\ \vdots
\end{aligned}$$

$$+ s^i p(a_{n+1}, a_m) p(a_{n+2}, a_m) p(a_{n+3}, a_m) .... p(a_{n+i}, a_m) \mathcal{C}_b(a_{n+i}, a_m).$$

Therefore, we have

$$\begin{aligned}
\mathcal{C}_b(a_n, a_m) \leq\ & sp(a_n, a_{n+1}) \mathcal{C}_b(a_n, a_{n+1}) \\
& + \prod_{j=n+1}^{n+1} sp(a_j, a_m) sp(a_{n+1}, a_{n+2}) \mathcal{C}_b(a_{n+1}, a_{n+2}) \\
& + \prod_{j=n+1}^{n+2} (sp(a_j, a_m)) sp(a_{n+2}, a_{n+3}) \mathcal{C}_b(a_{n+2}, a_{n+3}) \\
& + \prod_{j=n+1}^{n+3} (sp(a_j, a_m)) sp(a_{n+3}, a_{n+4}) \mathcal{C}_b(a_{n+3}, a_{n+4}) \\
& + \prod_{j=n+1}^{n+4} (sp(a_j, a_m)) sp(a_{n+4}, a_{n+5}) \mathcal{C}_b(a_{n+4}, a_{n+5}) \\
& \vdots \\
& + \prod_{j=n+1}^{m-2} (sp(a_j, a_m)) sp(a_{m-2}, a_{m-1}) \mathcal{C}_b(a_{m-2}, a_{m-1}) \\
& + \prod_{i=n+1}^{m-1} (sp(a_i, a_m)) \mathcal{C}_b(a_{m-1}, a_m)) \\
\leq\ & sp(a_n, a_{n+1}) \mathcal{C}_b(a_n, a_{n+1}) \\
& + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) \mathcal{C}_b(a_i, a_{i+1}) \\
& + \prod_{i=n+1}^{m-1} (sp(a_i, a_m)) \mathcal{C}_b(a_{m-1}, a_m)) \\
\leq\ & sp(a_n, a_{n+1}) k^n \mathcal{C}_b(a_0, a_1) \\
& + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) k^i \mathcal{C}_b(a_0, a_1) \\
& + \prod_{i=n+1}^{m-1} (sp(a_i, a_m)) k^{m-1} \mathcal{C}_b(a_0, a_1)) \\
\leq\ & sp(a_n, a_{n+1}) k^n \mathcal{C}_b(a_0, a_1)
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) k^i \mathcal{C}_b(a_0, a_1) \\
&+ \left(\prod_{i=n+1}^{m-1} (sp(a_i, a_m)) \right) sk^{m-1} p(a_{m-1}, a_m) \mathcal{C}_b(a_0, a_1)) \\
\leq{}& sp(a_n, a_{n+1}) k^n \mathcal{C}_b(a_0, a_1) \\
&+ \sum_{i=n+1}^{m-1} \left(\prod_{j=n+1}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) k^i \mathcal{C}_b(a_0, a_1) \\
\leq{}& sp(a_n, a_{n+1}) k^n \mathcal{C}_b(a_0, a_1) \\
&+ \sum_{i=n+1}^{m-1} \left(\prod_{j=0}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) k^i \mathcal{C}_b(a_0, a_1). \qquad (2.4)
\end{aligned}
$$

Let us consider the sum

$$\mathcal{S}_l = \sum_{i=0}^{l} \left(\prod_{j=0}^{i} (sp(a_j, a_m)) \right) sp(a_i, a_{i+1}) k^i.$$

From (2.4), we get

$$\mathcal{C}_b(a_n, a_m) \leq \mathcal{C}_b(a_0, a_1)[k^n sp(a_n, a_{n+1}) + (\mathcal{S}_{m-1} - \mathcal{S}_n)]. \qquad (2.5)$$

Since $p(a, b) \geq 1$, $s \geq 1$, and by applying ratio test, $\lim_{n\to\infty} \mathcal{S}_n$ exists and therefore the real sequence $\{\mathcal{S}_n\}$ is Cauchy.

In the end, if we consider the limit in (2.5) when $n, m \to \infty$, we infer that

$$\lim_{n,m\to\infty} \mathcal{C}_b(a_n, a_m) = 0. \qquad (2.6)$$

Thus, $\{a_n\}$ is a Cauchy sequence in the complete controlled b-metric space $(X, \mathcal{C}_b)$. So there exists $\rho \in X$ such that

$$\lim_{n\to\infty} \mathcal{C}_b(a_n, \rho) = 0, \quad \text{that is,} \;\; a_n \to \rho \;\; \text{as} \;\; n \to \infty. \qquad (2.7)$$

Now, we have to verify ρ is a fixed point of $\mathscr{H}$.
From condition (iii) in Definition 1.4,

$$\mathcal{C}_b(\rho, a_{n+1}) \leq s[p(\rho, a_n)\mathcal{C}_b(\rho, a_n) + p(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})].$$

By using (2.2), (2.3), (2.6) and (2.7), we can deduce that

$$\lim_{n\to\infty} \mathcal{C}_b(\rho, a_{n+1}) = 0. \qquad (2.8)$$

Again, by using the condition (iii) in Definition 1.4 and (2.1)

$$
\begin{aligned}
\mathcal{C}_b(\rho, \mathscr{H}\rho) &\leq s[p(\rho, a_{n+1})\mathcal{C}_b(\rho, a_{n+1}) + p(a_{n+1}, \mathscr{H}\rho)\mathcal{C}_b(a_{n+1}, \mathscr{H}\rho)] \\
&\leq s[p(\rho, a_{n+1})\mathcal{C}_b(\rho, a_{n+1}) + p(a_{n+1}, \mathscr{H}\rho)k\mathcal{C}_b(a_n, \rho)].
\end{aligned}
$$

Allowing $n \to \infty$ and applying (2.3), (2.7) and (2.8), we can easily deduce that $\mathcal{C}_b(\rho, \mathscr{H}\rho) = 0$. This yields that $\rho = \mathscr{H}\rho$. We conclude that the proof is completed. □

We illustrate the above theorem by the subsequent example.

Example 2.2. Assume $X = \{0, 1, 2\}$. Determine $p : X \times X \to [1, \infty)$ and $\mathcal{C}_b : X \times X \to [1, \infty)$ when $p(a, b) = 1 + ab$ and

$$\mathcal{C}_b(2,2) = \mathcal{C}_b(0,0) = \mathcal{C}_b(1,1) = 0;$$

$$\mathcal{C}_b(2,0) = \mathcal{C}_b(0,2) = 5; \mathcal{C}_b(0,1) = \mathcal{C}_b(1,0) = 10;$$

$$\mathcal{C}_b(2,1) = \mathcal{C}_b(1,2) = 30 \quad \text{and} \quad s = 2.$$

Now, define $\mathscr{H} : X \to X$ by

$$\mathscr{H}a = \begin{cases} 0, & \text{if } \ a \in \{0, 2\}, \\ 2, & \text{if } \ a = 1 \end{cases}$$

and choose $k = 0.9$.

Case I: If $a = 0, b = 1$, we have

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) = \mathcal{C}_b(\mathscr{H}0, \mathscr{H}1) = \mathcal{C}_b(0, 2) = 5.$$

Thus,

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) \leq k\mathcal{C}_b(a, b).$$

Case II: If $a = 0, b = 2$, we have

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) = \mathcal{C}_b(\mathscr{H}0, \mathscr{H}2) = \mathcal{C}_b(0, 0) = 0.$$

So

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) \leq k\mathcal{C}_b(a, b).$$

Case III: If $a = 1, b = 2$, we get

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) = \mathcal{C}_b(\mathscr{H}1, \mathscr{H}2) = \mathcal{C}_b(2, 0) = 5$$

that is,

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) \leq k\mathcal{C}_b(a, b).$$

Case IV: If $a = b = 0$ or $a = b = 1$ or $a = b = 2$, we get

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) = 0.$$

Consequently,

$$\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) \leq k\mathcal{C}_b(a, b)$$

for all, $a, b \in X$. Therefore, all the requirements of above theorem are fulfilled and $\mathscr{H}$ has a unique fixed point, which is, $a = 0$.

We may discuss many facts as particular cases of Banach contraction by arranging the below different consecutive values in Definition 1.4.

Special Cases:

(1) Suppose we assume $s = 1$ in Theorem 2.1, then we can get Theorem 1 of Mlaiki et al. [10].
(2) If we take $p(a, c) = p(c, b)$ and $s = 1$ in Theorem 2.1, then we get Theorem 2 of Kamran et al. [8] (since we omit the strong hypothesis concerning the continuity of the extended metric imposed in [8] and it is replaced by a weak hypothesis, as condition (3)).
(3) If we take $p(a, c) = p(c, b) = 1$ Theorem 2.1, then we can get Theorem 1 of Czerwik [5].
(4) If we take $p(a, c) = p(c, b) = s = 1$ Theorem 2.1, then it reducing to standard complete metric space.

3. On $(\varpi - F_\alpha)$-contractions

We now present the being next definition for proving the main results:

Definition 3.1. Assume that $(X, \mathcal{C}_b)$ is a $\mathcal{C}_b$-metric space. A mapping $\mathscr{H} : X \to X$ is called a $(\varpi - F_\mathcal{C})$-contraction on X provided that there exists $\alpha : X \times X \to \mathbb{R}^+$ and $\varpi : (0, \infty) \to (0, \infty)$ such that

($\mathcal{H}_1$) $F_\mathcal{C}$ satisfies $(F_1), (F_2)$ and (F_3);
($\mathcal{H}_2$) $\liminf_{s \to t^+} \varpi(s) > 0$ for all $t \geq 0$;
($\mathcal{H}_3$) $\varpi(\mathcal{P}(a, b)) + \alpha(a, b) F_\mathcal{C}(\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b)) \leq F_\mathcal{C}(\mathcal{P}(a, b))$, where

$$\mathcal{P}(a, b) = \max\left\{\mathcal{C}_b(a, b), \mathcal{C}_b(a, \mathscr{H}a), \mathcal{C}_b(b, \mathscr{H}b), \frac{\mathcal{C}_b(a, \mathscr{H}a).\mathcal{C}_b(b, \mathscr{H}b)}{1 + \mathcal{C}_b(a, b)}\right\},$$

for all $a, b \in X$ with $\mathcal{C}_b(\mathscr{H}a, \mathscr{H}b) > 0$.

Theorem 3.2. Assume that $(X, \mathcal{C}_b)$ is a complete $\mathcal{C}_b$-metric space and $\mathscr{H} : X \to X$ is a $(\varpi - F_\mathcal{C})$-contraction fulfilling the subsequent properties:

(I) $\mathscr{H}$ is α-admissible;
(II) $\exists\, a_0 \in X \ni \alpha(a_0, \mathscr{H}a_0) \geq 1$;
(III) $\mathscr{H}$ is continuous;
(IV) $\sup\limits_{p \geq 0} \lim\limits_{i \to \infty} \left\{\frac{\mathcal{C}_b(a_{i+1}, a_{i+2})\mathcal{C}_b(a_{i+1}, a_{n+p})}{\mathcal{C}_b(a_i, a_{i+1})}\right\} < 1$.

Then $\mathscr{H}$ has a fixed point.

Proof. Assume that for $a_0 \in X$, such that $\alpha(a_0, \mathscr{H}a_0) \geq 1$. We determine $\{a_n\}$ in X by $a_{n+1} = \mathscr{H}a_n$ for all $n \in \mathbb{N}$. Clearly, if there exists $n_0 \in \mathbb{N}$ for which $a_{n_0+1} = a_{n_0}$, then $\mathscr{H} a_{n_0} = a_{n_0}$ and the proof is finished. Hence, we assume $a_{n+1} \neq a_n$ for $n \in \mathbb{N}$. By applying (I) and (II), clearly

$$\alpha(\mathscr{H} a_n, \mathscr{H} a_{n+1}) \geq 1 \ \forall \ n \in \mathbb{N}.$$

It gives that

$$\begin{aligned} F_{\mathcal{C}}(\mathcal{C}_b(a_{n+1}, a_n)) &= F_{\mathcal{C}}(\mathcal{C}_b(\mathscr{H} a_n, \mathscr{H} a_{n-1})) \\ &\leq \alpha(a_n, a_{n-1}) F_{\mathcal{C}}(\mathcal{C}_b(\mathscr{H} a_n, \mathscr{H} a_{n-1})). \end{aligned}$$

Since $\mathscr{H}$ is a $(\varpi - F_\alpha)$-contraction, for every $n \in \mathbb{N}$, we can write

$$\begin{aligned} &\varpi(\mathcal{C}_b(a_n, a_{n-1})) + F_{\mathcal{C}}(\mathcal{C}_b(a_{n+1}, a_n)) \\ &\quad \leq \varpi(\mathcal{C}_b(a_n, a_{n-1})) + \alpha(a_n, a_{n-1}) F_{\mathcal{C}}(\mathcal{C}_b(\mathscr{H} a_n, \mathscr{H} a_{n-1})) \\ &\quad \leq F_{\mathcal{C}}(\mathcal{P}(a_n, a_{n-1})) \\ &\quad \leq F_{\mathcal{C}}\Bigg(\max \Bigg\{ \mathcal{C}_b(a_n, a_{n-1}), \mathcal{C}_b(a_n, \mathscr{H} a_n), \mathcal{C}_b(a_{n-1}, \mathscr{H} a_{n-1}), \\ &\qquad\qquad \frac{\mathcal{C}_b(a_n, \mathscr{H} a_n).\mathcal{C}_b(a_{n-1}, \mathscr{H} a_{n-1})}{1 + \mathcal{C}_b(a_n, a_{n-1})} \Bigg\} \Bigg) \qquad (3.1) \\ &\quad = F_{\mathcal{C}}\Bigg(\max \Bigg\{ \mathcal{C}_b(a_n, a_{n-1}), \mathcal{C}_b(a_n, a_{n+1}), \mathcal{C}_b(a_{n-1}, a_n), \\ &\qquad\qquad \frac{\mathcal{C}_b(a_n, a_{n+1}).\mathcal{C}_b(a_{n-1}, a_n)}{1 + \mathcal{C}_b(a_n, a_{n-1})} \Bigg\} \Bigg) \\ &\quad = F_{\mathcal{C}}(\max\{\mathcal{C}_b(a_n, a_{n-1}), \mathcal{C}_b(a_n, a_{n+1})\}). \end{aligned}$$

If there exists $n \in \mathbb{N}$ such that $\max\{\mathcal{C}_b(a_n, a_{n-1}), \mathcal{C}_b(a_n, a_{n+1})\} = \mathcal{C}_b(a_n, a_{n+1})$, then (3.1) becomes

$$\varpi(\mathcal{C}_b(a_n, a_{n-1})) + F_{\mathcal{C}}(\mathcal{C}_b(a_{n+1}, a_n)) \leq F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})), \qquad (3.2)$$

which is a contradiction to $(\mathcal{H}_2)$. Therefore $\max\{\mathcal{C}_b(a_n, a_{n-1}), \mathcal{C}_b(a_n, a_{n+1})\} = \mathcal{C}_b(a_n, a_{n-1})$ for all $n \in \mathbb{N}$. Thus from (3.1), we get

$$\varpi(\mathcal{C}_b(a_n, a_{n-1})) + F_{\mathcal{C}}(\mathcal{C}_b(a_{n+1}, a_n)) \leq F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n-1})) \quad \text{for all} \quad n \in \mathbb{N}.$$

Thus,

$$F_{\mathcal{C}}(\mathcal{C}_b(a_{n+1}, a_n)) \leq F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n-1})) - \varpi(\mathcal{C}_b(a_n, a_{n-1})), \ \ \forall \, n \in \mathbb{N}.$$

By referring (3.2) and (F_1), we obtain $\mathcal{C}_b(a_n, a_{n+1})$ is decreasing and therefore $\mathcal{C}_b(a_n, a_{n+1}) \searrow t$, $t \geq 0$. In view of $(\mathcal{H}_2)$, there exist $e > 0$ and $n_0 \in \mathbb{N}$ such that $\varpi(\mathcal{C}_b(a_n, a_{n+1})) > e$ for each $n \geq n_0$.

Consider,

$$\begin{aligned}
&F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})) \\
&\leq F_{\mathcal{C}}(\mathcal{C}_b(a_{n-1}, a_n)) - \varpi(\mathcal{C}_b(a_{n-1}, a_n)) \\
&\leq F_{\mathcal{C}}(\mathcal{C}_b(a_{n-2}, a_{n-1})) - \varpi(\mathcal{C}_b(a_{n-2}, a_{n-1})) - \varpi(\mathcal{C}_b(a_{n-1}, a_n)) \\
&\leq F_{\mathcal{C}}(\mathcal{C}_b(a_{n-3}, a_{n-2})) - \varpi(\mathcal{C}_b(a_{n-3}, a_{n-2})) - \varpi(\mathcal{C}_b(a_{n-2}, a_{n-1})) \\
&\quad - \varpi(\mathcal{C}_b(a_{n-1}, a_n)) \\
&\vdots \\
&\leq F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) - \varpi(\mathcal{C}_b(a_0, a_1)) - \varpi(\mathcal{C}_b(a_1, a_2)) - \cdots \\
&\quad - \varpi(\mathcal{C}_b(a_{n-1}, a_n)) \\
&= F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) - \sum_{i=1}^{n} \varpi(\mathcal{C}_b(a_{i-1}, a_i)).
\end{aligned} \tag{3.3}$$

From (3.3), we get

$$F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})) \leq F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) - ne. \tag{3.4}$$

Letting $n \to \infty$ in (3.4), we get

$$F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})) \to -\infty.$$

By (F_2),

$$\mathcal{C}_b(a_n, a_{n+1}) \to 0 \quad \text{as} \quad n \to \infty. \tag{3.5}$$

To show that $\{a_n\}$ is a Cauchy sequence, from (F_3), there exists $k \in (0,1)$ such that,

$$\lim_{n\to\infty} (\mathcal{C}_b(a_n, a_{n+1}))^k F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})) = 0. \tag{3.6}$$

By (3.4), the being next holds for all $n \in \mathbb{N}$,

$$\begin{aligned}
&(\mathcal{C}_b(a_n, a_{n+1}))^k F_{\mathcal{C}}(\mathcal{C}_b(a_n, a_{n+1})) - (\mathcal{C}_b(a_n, a_{n+1}))^k F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) \\
&\leq (\mathcal{C}_b(a_n, a_{n+1}))^k (F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) - ne) - (\mathcal{C}_b(a_n, a_{n+1}))^k F_{\mathcal{C}}(\mathcal{C}_b(a_0, a_1)) \\
&= -(\mathcal{C}_b(a_n, a_{n+1}))^k ne \\
&\leq 0.
\end{aligned} \tag{3.7}$$

Letting $n \to \infty$ in (3.7) and using (3.5) and (3.6), we obtain

$$\lim_{n\to\infty} n(\mathcal{C}_b(a_n, a_{n+1}))^k = 0. \tag{3.8}$$

By referring (3.8), there exists $n_1 \in \mathbb{N}$ such that $n(\mathcal{C}_b(a_n, a_{n+1}))^k \leq 1$ for all $n \geq n_1$. Consequently, we have

$$\mathcal{C}_b(a_n, a_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}}, \quad \text{for all} \quad n \geq n_1.$$

We will verify $\{a_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence. Consider the triangle inequality for $q \geq 1$,

$$
\begin{aligned}
&\mathcal{C}_b(a_n, a_{n+q})\\
&\leq s[p(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + p(a_{n+1}, a_{n+q})\mathcal{C}_b(a_{n+1}, a_{n+q})]\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + sp(a_{n+1}, a_{n+q})\mathcal{C}_b(a_{n+1}, a_{n+q})\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + sp(a_{n+1}, a_{n+q})[s\{p(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2})\\
&\quad + p(a_{n+2}, a_{n+q})\mathcal{C}_b(a_{n+2}, a_{n+q})\}]\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + s^2p(a_{n+1}, a_{n+q})p(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2})\\
&\quad + s^2p(a_{n+1}, a_{n+q})p(a_{n+2}, a_{n+q})\mathcal{C}_b(a_{n+2}, a_{n+p})\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + s^2p(a_{n+1}, a_{n+q})p(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2})\\
&\quad + s^3p(a_{n+1}, a_{n+q})p(a_{n+2}, a_{n+q})p(a_{n+3}, a_{n+q})\mathcal{C}_b(a_{n+3}, a_{n+p}) + ...\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + s^2p(a_{n+1}, a_m)p(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2})\\
&\quad + s^3p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+2}, a_{n+3})\mathcal{C}_b(a_{n+2}, a_{n+3})\\
&\quad + s^4p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+3}, a_m)p(a_{n+3}, a_{n+4})\mathcal{C}_b(a_{n+3}, a_{n+4})\\
&\quad + s^4p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+3}, a_m)p(a_{n+4}, a_m)\mathcal{C}_b(a_{n+4}, a_m)\\
&\ \vdots\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + s^2p(a_{n+1}, a_m)p(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2})\\
&\quad + s^3p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+2}, a_{n+3})\mathcal{C}_b(a_{n+2}, a_{n+3})\\
&\quad + s^4p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+3}, a_m)p(a_{n+3}, a_{n+4})\mathcal{C}_b(a_{n+3}, a_{n+4})\\
&\quad + s^4p(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+3}, a_m)\\
&\ \vdots\\
&\quad + s^ip(a_{n+1}, a_m)p(a_{n+2}, a_m)p(a_{n+3}, a_m)....p(a_{n+i}, a_m)\mathcal{C}_b(a_{n+i}, a_m)\\
&\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&\quad + \left(\prod_{j=n+1}^{n+1} sp(a_j, a_{n+p})\right)(sp(a_{n+1}, a_{n+2})\mathcal{C}_b(a_{n+1}, a_{n+2}))\\
&\quad + \left(\prod_{j=n+1}^{n+2} sp(a_j, a_{n+q})\right)(sp(a_{n+2}, a_{n+3})\mathcal{C}_b(a_{n+2}, a_{n+3}))\\
&\quad + \left(\prod_{j=n+1}^{n+3} sp(a_j, a_{n+q})\right)(sp(a_{n+3}, a_{n+4})\mathcal{C}_b(a_{n+3}, a_{n+4})) + ...
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\prod_{j=n+1}^{n+q-2} sp(a_j, a_{n+q})\right)(sp(a_{n+q-2}, a_{n+q-1})\mathcal{C}_b(a_{n+q-2}, a_{n+q-1}))\\
&+\left(\prod_{i=n+1}^{n+q-1} sp(a_i, a_{n+q})\right)\mathcal{C}_b(a_{n+q-1}, a_{n+q}))\\
&\le sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} sp(a_j, a_{n+q})\right)(sp(a_i, a_{i+1})\mathcal{C}_b(a_i, a_{i+1}))\\
&+\prod_{i=n+1}^{n+q-1} sp(a_i, a_{n+q})\mathcal{C}_b(a_{n+q-1}, a_{n+q})).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&\mathcal{C}_b(a_n, a_{n+q})\\
&\le sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} sp(a_j, a_{n+q})\right)(sp(a_i, a_{i+1})\mathcal{C}_b(a_i, a_{i+1}))\\
&+\left(\prod_{i=n+1}^{n+q-1} sp(a_i, a_{n+q})\right)\mathcal{C}_b(a_{n+q-1}, a_{n+q}))\\
&\le sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} sp(a_j, a_{n+q})\right)(sp(a_i, a_{i+1})\mathcal{C}_b(a_i, a_{i+1}))\\
&+\left(\prod_{i=n+1}^{n+q-1} sp(a_i, a_{n+q})\right)(sp(a_{n+q-1}, a_{n+q})\mathcal{C}_b(a_{n+q-1}, a_{n+q}))\\
&\le sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&+\sum_{i=n+1}^{n+q-1}\left(\prod_{j=n+1}^{i} sp(a_j, a_{n+q})\right)(sp(a_i, a_{i+1})\mathcal{C}_b(a_i, a_{i+1}))\\
&\le sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1})\\
&+\sum_{i=n+1}^{n+q-1}\left(\prod_{j=0}^{i} sp(a_j, a_{n+q})\right)sp(a_i, a_{i+1})\mathcal{C}_b(a_i, a_{i+1})
\end{aligned}
\tag{3.9}
$$

$$\leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + \sum_{i=n+1}^{n+q-1} \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}) \frac{1}{i^{\frac{1}{k}}}.$$

Now, consider

$$\begin{aligned} &\sum_{i=n+1}^{n+q-1} \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}) \frac{1}{i^{\frac{1}{k}}} \\ &\qquad = \sum_{i=n+1}^{n+q-1} \frac{1}{i^{\frac{1}{k}}} \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}) \\ &\qquad = \sum_{i=n+1}^{\infty} \frac{1}{i^{\frac{1}{k}}} \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}) \\ &\qquad = \sum_{i=n+1}^{\infty} U_i V_i, \end{aligned}$$

where

$$U_i = \frac{1}{i^{\frac{1}{k}}}$$

and

$$V_i = \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}).$$

Because $\frac{1}{k} > 1$, $\sum\limits_{i=n+1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ converges and additionally

$$V_i = \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1})$$

is increasing and is bounded above. Therefore, $\lim_{i\to\infty}\{V_i\} = \sup(V_i)$, which is non zero and exists. Therefore, the product $\prod_{j=0}^{i} sp(a_j, a_{n+q}) sp(a_i, a_{i+1})$ converges. Therefore, $\sum\limits_{i=n+1}^{\infty} U_i V_i$ converges.

Assume that the partial sum

$$\mathcal{S}_q = \sum_{i=0}^{q} \left(\prod_{j=0}^{i} sp(a_j, a_{n+q}) \right) sp(a_i, a_{i+1}) \frac{1}{i^{\frac{1}{k}}}.$$

Now, from (3.9),

$$\mathcal{C}_b(a_n, a_{n+q}) \leq sp(a_n, a_{n+1})\mathcal{C}_b(a_n, a_{n+1}) + (\mathcal{S}_{n+q-1} - \mathcal{S}_n). \tag{3.10}$$

Letting $n \to \infty$ in (3.10) and using (3.5), we get

$$\lim_{n\to\infty} \mathcal{C}_b(a_n, a_{n+q}) = 0.$$

Thus, $\{a_n\}$ is a Cauchy sequence. Since X is a complete $\mathcal{C}_b$-metric space, there exists $\lambda \in X$ such that,

$$\lim_{n\to\infty} \mathcal{C}_b(a_n, \lambda) = 0.$$

In view of the assumptions that the mapping $\mathscr{H}$ and the controlled b-metric are continuous, since $a_n \to \lambda$, we have that $\mathscr{H}a_n \to \mathscr{H}\lambda$ and hence we have

$$\lim_{n\to\infty} \mathcal{C}_b(\mathscr{H}a_n, \mathscr{H}\lambda) = 0 = \lim_{n\to\infty} \mathcal{C}_b(a_{n+1}, \mathscr{H}\lambda) = \mathcal{C}_b(\lambda, \mathscr{H}\lambda),$$

and hence $\mathscr{H}\lambda = \lambda$. Thus λ is a fixed point of $\mathscr{H}$. □

We may discuss many facts as particular cases on $(\varpi - F_{\mathcal{C}})$-contractions by arranging the following various consecutive values in Definition 1.4.

Special Cases:

(1) If we take $s = 1$ in Theorem 3.2, then Theorem 3.2 reduces to a controlled metric type space, as in [10].
(2) If we take $p(a, c) = p(c, b)$ and $s = 1$ in Theorem 3.2, then Theorem 3.2 reduces to an extended b-metric space, as in [8].
(3) If we take $p(a, c) = p(c, b) = 1$ in Theorem 3.2, then Theorem 3.2 reduces to a b-metric space.

4. Applications to our results in the pursuit of 2019-nCoV modelling

There is regularly a compromise between oversimplified or key models in the mathematical displaying of ailment transmission, as in most different regions of numerical demonstrating, which exclude a large portion of the points of interest and are planned principally to show general subjective conduct, and mind-boggling or strategic models, normally intended for specific conditions, including transient quantitative expectations. All in all, definite models are troublesome or difficult to logically unravel and their handiness is consequently restricted for hypothetical purposes, in spite of the fact that their key worth might be high. For example, extremely oversimplified pestilence models anticipate that after some time, a plague will cease to exist, leaving a segment of the populace unaffected by contamination, and this is likewise valid for models that include insurance measures. This factual hypothesis isn't, without anyone else, extremely fruitful in surveying what well-being measures in a given situation will be the best, yet it implies that an exhaustive model might be valuable for general well-being experts to report the reason as correctly as could reasonably be expected. Specialist based models, which partition the network into people or gatherings of individuals with comparable activities, are the benchmark in thorough models.

One of the significant reasons for death overall keeps on being irresistible illnesses. With the unexpected variants of coronaviruses, bringing up issues about regular mental fighting as well as developing worries about natural animosity, sickness demonstration has gotten applicable to assume a part in network well being methodology making. Mathematics and additionally logical models of irresistible maladies may help us to comprehend the idea of ailments and the pace of transmission.

To build up and break down various mediation techniques to evade or fortify defilement and to all the more likely appropriate available assets (for instance, choosing the objective populace, time for intercession, and area), models regularly empower us to re-authorize the spread of ailments in different viewpoints and angles.

Note that the utilization of mathematical modeling in the evaluation of epidemiological illnesses is developing drastically. In this interest, we present the existence results for the new Coronavirus fractional order models.

4.1. **Existence of the solution under the setting of complete metric space:** In this section, the mathematical model capable of depicting the spread of the Coronavirus-19 was recently suggested by Baleanu et al. [4], which model has considered several variables in the setting of Caputo-Fabrizio fractional derivative as stated below:

$$
\begin{aligned}
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathscr{S}_p = \bigwedge_p - a_p\mathscr{S}_p - \frac{b_p\mathscr{S}_p(\mathscr{I}_p + \Theta\mathscr{A}_p)}{\mathcal{N}_p} - b_w\mathscr{S}_p\mathscr{M};\\
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathscr{A}_p = \frac{b_p\mathscr{S}_p(\mathscr{I}_p + \Theta\mathscr{A}_p)}{\mathcal{N}_p} + b_w\mathscr{S}_p\mathscr{M} - (1-\Upsilon_p)\omega_p\mathscr{A}_p - \Upsilon_p\xi_p\mathscr{A}_p - a_p\mathscr{A}_p;\\
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathscr{I}_p = (1-\Upsilon_p)\omega_p\mathscr{A}_p - (\tau_p + a_p)\mathscr{I}_p;\\
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathscr{A}_p = \Upsilon_p\xi_p\mathscr{A}_p - (\tau_{\vartheta p} + a_p)\mathscr{A}_p;\\
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathcal{R}_p = \tau_p\mathscr{I}_p + \tau_{\vartheta p}\mathscr{A}_p - a_p\mathcal{R}_p;\\
&{}^{\mathscr{CF}}\mathscr{D}^{\rho}\mathscr{M} = c_p\mathscr{I}_p + e_p\mathscr{A}_p - \pi\mathscr{M}.
\end{aligned}
\tag{4.1}
$$

By applying fractional integral operator, we convert model (4.1) into below integral form:

$$
\mathscr{S}_p(t) - \mathscr{S}_p(0) = {}^{\mathscr{CF}}\mathscr{I}^{\rho}\left[\bigwedge_p - a_p\mathscr{S}_p - \frac{b_p\mathscr{S}_p(\mathscr{I}_p + \Theta\mathscr{A}_p)}{\mathcal{N}_p} - b_w\mathscr{S}_p\mathscr{M}\right];
$$

$$\begin{aligned}
\mathscr{A}_p(t) - \mathscr{A}_p(0) &= {}^{\mathscr{C}\mathscr{F}}\ \mathscr{I}^{\rho}\bigg[\frac{b_p\mathscr{S}_p(\mathscr{I}_p + \Theta\mathscr{A}_p)}{\mathscr{N}_p} + b_w\mathscr{S}_p\mathscr{M} \\
&\qquad - (1-\Upsilon_p)\omega_p\mathscr{A}_p - \Upsilon_p\xi_p\mathscr{A}_p - a_p\mathscr{A}_p\bigg]; \\
\mathscr{I}_p(t) - \mathscr{I}_p(0) &= {}^{\mathscr{C}\mathscr{F}}\ \mathscr{I}^{\rho}\bigg[(1-\Upsilon_p)\omega_p\mathscr{A}_p - (\tau_p + a_p)\mathscr{I}_p\bigg]; \\
\mathscr{A}_p(t) - \mathscr{A}_p(0) &= {}^{\mathscr{C}\mathscr{F}}\ \mathscr{I}^{\rho}\bigg[\Upsilon_p\xi_p\mathscr{A}_p - (\tau_{\vartheta p} + a_p)\mathscr{A}_p\bigg]; \\
\mathcal{R}_p(t) - \mathcal{R}_p(0) &= {}^{\mathscr{C}\mathscr{F}}\ \mathscr{I}^{\rho}\bigg[\tau_p\mathscr{I}_p + \tau_{\vartheta p}\mathscr{A}_p - a_p\mathcal{R}_p\bigg]; \\
\mathscr{M}_p(t) - \mathscr{M}_p(0) &= {}^{\mathscr{C}\mathscr{F}}\ \mathscr{I}^{\rho}\bigg[c_p\mathscr{I}_p + e_p\mathscr{A}_p - \pi\mathscr{M}\bigg].
\end{aligned} \tag{4.2}$$

Let $\mathbb{S} = \{\mathbb{A} \in \mathcal{C}(I, \mathbb{R}) : \mathbb{A}(\theta) > 0 \text{ for all } \theta \in I = [0, \mathscr{H}], \mathscr{H} > 0\}$. Define a mapping $\mathscr{D} : \mathbb{S} \times \mathbb{S} \to [0, \infty)$ as $\mathscr{D}(u, v) = |u - v|$ for all $u, v \in \mathbb{S}$. Then $(\mathbb{S}, \mathscr{D})$ is a complete metric space. Define a mapping $\mathscr{H} : \mathbb{S} \to \mathbb{S}$ by

$$\begin{aligned}
\mathscr{H}\mathscr{S}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\mathbb{A}(\theta, \mathscr{S}_p(\theta)) \\
&\quad + \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\int_0^{\theta}\mathbb{A}(v, \mathscr{S}_p(v))dv; \\
\mathscr{H}\mathscr{A}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\mathbb{B}(\theta, \mathscr{A}_p(\theta)) \\
&\quad + \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\int_0^{\theta}\mathbb{B}(v, \mathscr{A}_p(v))dv; \\
\mathscr{H}\mathscr{I}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\mathbb{C}(\theta, \mathscr{I}_p(\theta)) \\
&\quad + \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\int_0^{\theta}\mathbb{C}(v, \mathscr{I}_p(v))dv; \\
\mathscr{H}\mathscr{A}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\mathbb{D}(\theta, \mathscr{A}_p(\theta)) \\
&\quad + \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\int_0^{\theta}\mathbb{D}(v, \mathscr{A}_p(v))dv; \\
\mathscr{H}\mathcal{R}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\mathbb{E}(\theta, \mathcal{R}_p(\theta)) \\
&\quad + \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}\int_0^{\theta}\mathbb{E}(v, \mathcal{R}_p(v))dv;
\end{aligned} \tag{4.3}$$

$$\begin{aligned}\mathscr{H}\mathscr{M}_p(\theta) &= \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\mathbb{F}(\theta, \mathscr{M}_p(\theta)) \\ &\quad + \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\int_0^\theta \mathbb{F}(v, \mathscr{M}_p(v))dv.\end{aligned}$$

Then, we see that $\mathscr{H}$ is continuous. For each $u, v \in \mathbb{S}$ and $\theta \in I$, we have,

$$\mathscr{H}\mathscr{S}_p(\theta) = \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\mathbb{A}(\theta, \mathscr{S}_p(\theta)) + \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\int_0^\theta \mathbb{A}(v, \mathscr{S}_p(v))dv.$$

We now discuss the existence and uniqueness of solutions of (4.3) provided the following conditions satisfied:

(i) $|\mathbb{A}(\theta, \mathscr{S}_{p_1}(\theta)) - \mathbb{A}(\theta, \mathscr{S}_{p_2}(\theta))| \leq |\mathscr{S}_{p_1}(\theta) - \mathscr{S}_{p_2}(\theta)|$,

(ii) $\frac{2}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)} < \zeta, \quad \text{where} \quad 0 < \zeta < 1.$

Now consider,

$$\begin{aligned}&|\mathscr{H}\mathscr{S}_{p_1}(\theta) - \mathscr{H}\mathscr{S}_{p_2}(\theta)| \\ &= \left| \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\mathbb{A}(\theta, \mathscr{S}_{p_1}(\theta)) + \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\int_0^\theta \mathbb{A}(v, \mathscr{S}_{p_1}(v))dv \right. \\ &\quad \left. - \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\mathbb{A}(\theta, \mathscr{S}_{p_2}(\theta)) - \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\int_0^\theta \mathbb{A}(v, \mathscr{S}_{p_2}(v))dv \right| \\ &= \left| \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\Big(\mathbb{A}(\theta, \mathscr{S}_{p_1}(\theta)) - \mathbb{A}(\theta, \mathscr{S}_{p_2}(\theta))\Big) \right. \\ &\quad \left. + \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\int_0^\theta \Big(\mathbb{A}(v, \mathscr{S}_{p_1}(v))dv - \mathbb{A}(v, \mathscr{S}_{p_2}(v))\Big)dv \right| \\ &\leq \left| \frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)} \right| \left| \Big(\mathbb{A}(\theta, \mathscr{S}_{p_1}(\theta)) - \mathbb{A}(\theta, \mathscr{S}_{p_2}(\theta))\Big) \right| \\ &\quad + \left| \frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)} \right| \left| \int_0^\theta \Big(\mathbb{A}(v, \mathscr{S}_{p_1}(v))dv - \mathbb{A}(v, \mathscr{S}_{p_2}(v))\Big)dv \right|\end{aligned}$$

$$
\begin{aligned}
&\le \left|\frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\right|\left|\Big(\mathbb{A}(\theta,\mathscr{S}_{p_1}(\theta))-\mathbb{A}(\theta,\mathscr{S}_{p_2}(\theta))\Big)\right| \\
&\quad+\left|\frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\right|\left|\Big(\mathbb{A}(\theta,\mathscr{S}_{p_1}(\theta))-\mathbb{A}(\theta,\mathscr{S}_{p_2}(\theta))\Big)\right| \\
&\le \left|\frac{2-2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}+\frac{2\sigma}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\right|\left|\Big(\mathbb{A}(\theta,\mathscr{S}_{p_1}(\theta))-\mathbb{A}(\theta,\mathscr{S}_{p_2}(\theta))\Big)\right| \\
&\le \left|\frac{2}{2\mathscr{B}(\sigma)-\sigma\mathscr{B}(\sigma)}\right|\left|\mathscr{S}_{p_1}(\theta)-\mathscr{S}_{p_2}(\theta)\right| \\
&\le \zeta|\mathscr{S}_{p_1}(\theta)-\mathscr{S}_{p_2}(\theta)| \quad \text{where} \ \ 0<\zeta<1.
\end{aligned}
$$

Therefore,

$$
|\mathscr{H}\mathscr{S}_{p_1}(\theta)-\mathscr{H}\mathscr{S}_{p_2}(\theta)| \le \zeta|\mathscr{S}_{p_1}(\theta)-\mathscr{S}_{p_2}(\theta)|. \tag{4.4}
$$

Using the above method we discussed above, one can get

$$
\begin{aligned}
&|\mathscr{H}\mathscr{A}_{p_1}(\theta)-\mathscr{H}\mathscr{A}_{p_2}(\theta)| \le \zeta|\mathscr{A}_{p_1}(\theta)-\mathscr{A}_{p_2}(\theta)|;\\
&|\mathscr{H}\mathscr{I}_{p_1}(\theta)-\mathscr{H}\mathscr{I}_{p_2}(\theta)| \le \zeta|\mathscr{I}_{p_1}(\theta)-\mathscr{I}_{p_2}(\theta)|;\\
&|\mathscr{H}\mathscr{A}_{p_1}(\theta)-\mathscr{H}\mathscr{A}_{p_2}(\theta)| \le \zeta|\mathscr{A}_{p_1}(\theta)-\mathscr{A}_{p_2}(\theta)|;\\
&|\mathscr{H}\mathcal{R}_{p_1}(\theta)-\mathscr{H}\mathcal{R}_{p_2}(\theta)| \le \zeta|\mathcal{R}_{p_1}(\theta)-\mathcal{R}_{p_2}(\theta)|;\\
&|\mathscr{H}\mathscr{M}_{p_1}(\theta)-\mathscr{H}\mathscr{M}_{p_2}(\theta)| \le \zeta|\mathscr{M}_{p_1}(\theta)-\mathscr{M}_{p_2}(\theta)|.
\end{aligned} \tag{4.5}
$$

From Eq.(4.4) we can write,

$$
\mathscr{D}(\mathscr{H}\mathscr{S}_{p_1}(\theta),\mathscr{H}\mathscr{S}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathscr{S}_{p_1}(\theta),\mathscr{S}_{p_2}(\theta)).
$$

Similarly, we have

$$
\mathscr{D}(\mathscr{H}\mathscr{A}_{p_1}(\theta),\mathscr{H}\mathscr{A}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathscr{A}_{p_1}(\theta),\mathscr{A}_{p_2}(\theta));
$$

$$
\mathscr{D}(\mathscr{H}\mathscr{I}_{p_1}(\theta),\mathscr{H}\mathscr{I}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathscr{I}_{p_1}(\theta),\mathscr{I}_{p_2}(\theta));
$$

$$
\mathscr{D}(\mathscr{H}\mathscr{A}_{p_1}(\theta),\mathscr{H}\mathscr{A}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathscr{A}_{p_1}(\theta),\mathscr{A}_{p_2}(\theta));
$$

$$
\mathscr{D}(\mathscr{H}\mathcal{R}_{p_1}(\theta),\mathscr{H}\mathcal{R}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathcal{R}_{p_1}(\theta),\mathcal{R}_{p_2}(\theta));
$$

$$
\mathscr{D}(\mathscr{H}\mathscr{M}_{p_1}(\theta),\mathscr{H}\mathscr{M}_{p_2}(\theta)) \le \zeta\mathscr{D}(\mathscr{M}_{p_1}(\theta),\mathscr{M}_{p_2}(\theta)).
$$

Hence the system of novel Coronavirus model gratified all the assertions of special case for standard metric space. Hence Eq. (4.3) has unique fixed point. Thus Eq.(4.3) has a unique solution.

4.2. **Existence of solution under the setting of complete extended b-metric space:** Let
$\mathbb{S} = \{\mathbb{A} \in \mathcal{C}(I, \mathbb{R}) : \mathbb{A}(\theta) > 0 \text{ for all } \theta \in I = [0, \mathscr{H}], \mathscr{H} > 0\}$. Define a mapping $\mathscr{D} : \mathbb{S} \times \mathbb{S} \to [0, \infty)$ and $p : \mathbb{S} \times \mathbb{S} \to [1, \infty)$ as $\mathscr{D}(u, v) = |u - v|^2$ and

$$p(u,v) = \begin{cases} |u-v|^2, \text{ if } u \neq v, \\ 1, \quad \text{if } u = v, \end{cases} \tag{4.6}$$

for all $u, v \in \mathbb{S}$. Then $(\mathbb{S}, \mathscr{D})$ is a complete extended b-metric space. Define a mapping $\mathscr{H} : \mathbb{S} \to \mathbb{S}$ by

$$\begin{aligned}
\mathscr{H}\mathscr{S}_p(\theta) &= \mathscr{S}_p(0) + ¥(\sigma)\mathbb{A}(\theta, \mathscr{S}_p) + £(\sigma) \int_0^\theta \mathbb{A}(u, \mathscr{S}_p) du; \\
\mathscr{H}\mathscr{A}_p(\theta) &= \mathscr{A}_p(0) + ¥(\sigma)\mathbb{B}(\theta, \mathscr{A}_p) + £(\sigma) \int_0^\theta \mathbb{B}(u, \mathscr{A}_p) du; \\
\mathscr{H}\mathscr{I}_p(\theta) &= \mathscr{I}_p(0) + ¥(\sigma)\mathbb{C}(\theta, \mathscr{I}_p) + £(\sigma) \int_0^\theta \mathbb{C}(u, \mathscr{I}_p) du; \\
\mathscr{H}\mathscr{A}_p(\theta) &= \mathscr{A}_p(0) + ¥(\sigma)\mathbb{D}(\theta, \mathscr{A}_p) + £(\sigma) \int_0^\theta \mathbb{D}(u, \mathscr{A}_p) du; \\
\mathscr{H}\mathcal{R}_p(\theta) &= \mathcal{R}_p(0) + ¥(\sigma)\mathbb{E}(\theta, \mathcal{R}_p) + £(\sigma) \int_0^\theta \mathbb{E}(u, \mathcal{R}_p) du; \\
\mathscr{H}\mathscr{M}_p(\theta) &= \mathscr{M}_p(0) + ¥(\sigma)\mathbb{F}(\theta, \mathscr{M}_p) + £(\sigma) \int_0^\theta \mathbb{F}(u, \mathscr{M}_p) du,
\end{aligned} \tag{4.7}$$

where $¥(\sigma) = \frac{2-2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}$ and $£(\sigma) = \frac{2\sigma}{2\mathscr{B}(\sigma) - \sigma\mathscr{B}(\sigma)}$. Then, we see that $\mathscr{H}$ is continuous. For each $u, v \in \mathbb{S}$ and $\theta \in I$, we have,

$$\mathscr{H}\mathscr{S}_p(\theta) = \mathscr{S}_p(0) + ¥(\sigma)\mathbb{A}(\theta, \mathscr{S}_p) + £(\sigma) \int_0^\theta \mathbb{A}(u, \mathscr{S}_p) du.$$

We now verify the existence and uniqueness of solutions of (4.7) provided the following conditions satisfied:

(1) $|\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2})|^2 \leq |\mathscr{S}_{p_1}(\theta) - \mathscr{S}_{p_2}(\theta)|^2$,

(2) $|¥(\sigma) + £(\sigma)\theta|^2 < \xi, \quad \text{where} \quad 0 < \xi < 1.$

Now consider,

$$
\begin{aligned}
|\mathscr{H}\mathscr{S}_{p_1}(\theta) - \mathscr{H}\mathscr{S}_{p_2}(\theta)|^2 &= |¥(\sigma)\mathbb{A}(\theta, \mathscr{S}_{p_1}) + £(\sigma)\int_0^\theta \mathbb{A}(u, \mathscr{S}_{p_1})du \\
&\quad - ¥(\sigma)\mathbb{A}(\theta, \mathscr{S}_{p_2}) - £(\sigma)\int_0^\theta \mathbb{A}(u, \mathscr{S}_{p_2})du|^2 \\
&= |¥(\sigma)(\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2})) \\
&\quad + £(\sigma)[\int_0^\theta [\mathbb{A}(u, \mathscr{S}_{p_1}) - \mathbb{A}(u, \mathscr{S}_{p_2})]du]|^2 \\
&= |¥(\sigma)(\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2}))|^2 \\
&\quad + |£(\sigma)[\int_0^\theta [\mathbb{A}(u, \mathscr{S}_{p_1}) - \mathbb{A}(u, \mathscr{S}_{p_2})]du]|^2 \\
&\quad + 2|¥(\sigma)(\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2}))| \\
&\quad \times |£(\sigma)[\int_0^\theta [\mathbb{A}(u, \mathscr{S}_{p_1}) - \mathbb{A}(u, \mathscr{S}_{p_2})]du]| \\
&= |¥(\sigma)|^2|(\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2}))|^2 \\
&\quad + |£(\sigma)|^2|\mathbb{A}(u, \mathscr{S}_{p_1}) - \mathbb{A}(u, \mathscr{S}_{p_2})|^2|\int_0^\theta du|^2 \\
&\quad + 2|¥(\sigma)||\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2})| \\
&\quad \times |£(\sigma)||\mathbb{A}(u, \mathscr{S}_{p_1}) - \mathbb{A}(u, \mathscr{S}_{p_2})||\int_0^\theta du| \\
&= |\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2})|^2 \\
&\quad \times \{|¥(\sigma)|^2 + |£(\sigma)|^2\theta^2 + 2¥(\sigma)£(\sigma)|\theta|\} \\
&= |\mathbb{A}(\theta, \mathscr{S}_{p_1}) - \mathbb{A}(\theta, \mathscr{S}_{p_2})|^2|¥(\sigma) + £(\sigma)\theta|^2 \\
&\leq \xi|\mathscr{S}_{p_1}(\theta) - \mathscr{S}_{p_2}(\theta)|^2.
\end{aligned}
$$

Therefore,

$$
|\mathscr{H}\mathscr{S}_{p_1}(\theta) - \mathscr{H}\mathscr{S}_{p_2}(\theta)| \leq \xi|\mathscr{S}_{p_1}(\theta) - \mathscr{S}_{p_2}(\theta)|. \tag{4.8}
$$

Using the above method we discussed above, one can get

$$
\begin{aligned}
&|\mathscr{H}\mathscr{A}_{p_1}(\theta) - \mathscr{H}\mathscr{A}_{p_2}(\theta)| \leq \xi|\mathscr{A}_{p_1}(\theta) - \mathscr{A}_{p_2}(\theta)|; \\
&|\mathscr{H}\mathscr{I}_{p_1}(\theta) - \mathscr{H}\mathscr{I}_{p_2}(\theta)| \leq \xi|\mathscr{I}_{p_1}(\theta) - \mathscr{I}_{p_2}(\theta)|; \\
&|\mathscr{H}\mathscr{A}_{p_1}(\theta) - \mathscr{H}\mathscr{A}_{p_2}(\theta)| \leq \xi|\mathscr{A}_{p_1}(\theta) - \mathscr{A}_{p_2}(\theta)|; \\
&|\mathscr{H}\mathcal{R}_{p_1}(\theta) - \mathscr{H}\mathcal{R}_{p_2}(\theta)| \leq \xi|\mathcal{R}_{p_1}(\theta) - \mathcal{R}_{p_2}(\theta)|; \\
&|\mathscr{H}\mathscr{M}_{p_1}(\theta) - \mathscr{H}\mathscr{M}_{p_2}(\theta)| \leq \xi|\mathscr{M}_{p_1}(\theta) - \mathscr{M}_{p_2}(\theta)|.
\end{aligned} \tag{4.9}
$$

From Eq.(4.8) we can write,

$$\mathscr{D}(\mathscr{H}\mathscr{S}_{p_1}(\theta), \mathscr{H}\mathscr{S}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathscr{S}_{p_1}(\theta), \mathscr{S}_{p_2}(\theta)).$$

Similarly, we have

$$\mathscr{D}(\mathscr{H}\mathscr{A}_{p_1}(\theta), \mathscr{H}\mathscr{A}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathscr{A}_{p_1}(\theta), \mathscr{A}_{p_2}(\theta));$$
$$\mathscr{D}(\mathscr{H}\mathscr{I}_{p_1}(\theta), \mathscr{H}\mathscr{I}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathscr{I}_{p_1}(\theta), \mathscr{I}_{p_2}(\theta));$$
$$\mathscr{D}(\mathscr{H}\mathscr{A}_{p_1}(\theta), \mathscr{H}\mathscr{A}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathscr{A}_{p_1}(\theta), \mathscr{A}_{p_2}(\theta));$$
$$\mathscr{D}(\mathscr{H}\mathcal{R}_{p_1}(\theta), \mathscr{H}\mathcal{R}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathcal{R}_{p_1}(\theta), \mathcal{R}_{p_2}(\theta));$$
$$\mathscr{D}(\mathscr{H}\mathscr{M}_{p_1}(\theta), \mathscr{H}\mathscr{M}_{p_2}(\theta)) \leq \xi \mathscr{D}(\mathscr{M}_{p_1}(\theta), \mathscr{M}_{p_2}(\theta)).$$

Hence the system of novel Coronavirus model gratified all the assertions of Theorem 2.1 in the setting of extended b-metric space. Hence Eq.(4.7) has unique fixed point. Thus Eq.(4.7) has a unique solution.

5. Conclusion

There is still a dramatic increase for using mathematical modelling in the study of epidemiology diseases. Mathematical models were developed to predict how infectious diseases advance to explain the potential outcome of an outbreak, and better facilitate initiatives in global policy. We too propose a solution for existence and uniqueness solutions of the novel Coronavirus-2019 model via fractional derivatives and nonlinear $(\varpi - F_{\mathcal{C}})$-contractions.

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