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MAXIMAL ELEMENTS AND PAIR OF GENERALIZED GAMES FOR U-MAJORIZED AND CONDENSING CORRESPONDENCES

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Abstract. In this paper, we prove the existence of equilibria for pair of maps in locally convex spaces, in which preference correspondences are U-majorized with any(countable or uncountable) set of players. We also present the generalization of some existence theorems for compact(resp. non-compact) qualitative games and generalized games, in which the constraint or preference correspondences are U-majorized(resp. Ψ-condensing) are obtained in locally convex topological vector spaces.

1. Introduction

In Border [3] established that the results appearing in economics about the existence of equilibria is indeed equivalent to some classical fixed point theorem coming from pure mathematics. It can also be explained that such results in pure mathematics have applications in other disciplines(eg. game theory, optimization theory and economics). As a matter of fact starting from contexts of other disciplines(eg. economics) we can restate and reobtain classical results in mathematics.

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In mathematical economics, it is possible to generalize the results containing several variables to generalized economic spaces. This attempt will help to understand the dimension of abstract economy related to other sector of social system. In this discussion the map P is related to abstract economy whereas the map Q is related to social/organizational /government system etc. Thus, the new concept will be very helpful in dealing with the socio-economic, politico-economic, government or any corporate sector related problems, in which the organizational constraints will begin to play their role and affecting the abstract economy. The maximal elements of $(P \cap Q)(x) = \phi$ be very useful in economy and human relation problems.

Debreu [6], proved the existence of equilibrium in an abstract economy with compact strategy sets in $\mathbb{R}^n$. Since then many generalization of Debreu's theorem appeared in mathematical economics such as Borglin and Keiding [1], Florenzano [10], Gale and Mas-colell [13], Shafer and Sonnenschein [28], Tarafdar [29], Tulcea [30], Tian [31], Tan and Yuan [33], Toussaint [34], Yannelis and Prabhakar [37] and the references wherein.

In this paper, we establish some existence theorem of equilibria for pair of generalized games in which the intersection of constraint and preference correspondences are U-majorized and with any(countable or uncountable) set of players in locally convex topological vector spaces. Further we prove the existence theorem of equilibria for pair of generalized games in which the constraint mappings are Ψ-condensing.

2. Preliminaries

Let E be a vector space and $A \subset E$. We shall denote by coA the convex hull of A. If A is subset of a topological space X, we denote by $cl_X A$ the closure of A in X. If A is a non-empty set, we denote by 2^A the family of all subsets of A. If A is a non empty subset of a topological vector space E and $F, G : A \to 2^E$ are two coresspondences, then coF, $F \cap G : A \to 2^E$ are correspondences defined by $(coF)(x) := coF(x)$ and $(F \cap G)(x) := F(x) \cap G(x)$ for each $x \in A$, respectively.

Suppose X and Y are topological spaces and $F : X \to 2^Y$ is a correspondence, then

(i) F is said to be upper semicontinuous on X if for any open subset U of Y, the set $\{x \in X : F(x) \subset U\}$ is an open in X;
(ii) F is upper semicontinuous (on X) if F is upper semicontinuous at x for each $x \in X$;
(iii) the graph of F, denoted by Graph F, is the set $\{(x, y) \in X \times Y : y \in F(x)\}$;

(iv) the correspondence $\overline{F} : X \to 2^Y$ is defined by $\overline{F}(x) = \{y \in Y : (x, y) \in cl_{X\times Y} graph(F)\}$;
(v) the correspondence $clF : X \to 2^Y$ is defined by $clF(x) = cl_Y(F(x))$ for each $x \in X$; and
(vi) F has a maximal element if there exists a point $x \in X$ such that $F(x) = \phi$.

The existence of maximal element is essential in proving existence of equilibria in generalized games and there have been numerous existence theorems on maximal elements in general settings by several authors(e.g. [1], [8], [13], [29], [30], etc).

Definition 2.1. ([36]) Let X be a topological space and Y be a non-empty subset of a vector space $E, \theta : X \to E$ be a mapping and $T : X \to 2^Y$ be a correspondence

(1) T is said to be of class U_θ (or U) if
  (a) for each $x \epsilon X$, $\theta(x) \notin T(x)$;
  (b) T is upper semicontinuous with closed and convex values in Y.
(2) A correspondence $T_x : X \to 2^Y$ is said to be a U_θ-majorant of T at x if there exists an open neighborhood $N(x)$ of x such that
  (a) for each $x \in N(x)$, $T(z) \subset T_x(z)$ and $\theta(z) \notin T_x(z)$;
  (b) T_x is upper semicontinuous with closed and convex values.
(3) T is said to be U_θ-majorized if for each $x \in X$ with $T(x) \neq \phi$ there exists an U-majorant T_x of T at x.

Let I be a countable or uncountable set of agents(players). For each $i \in I$, let X_i be a non-empty subset of a topological vector space representing the set of strategy(action) and define $X := \Pi_{i\in I} X_i$. For each $i \in I$, let $P_{2i+1} : X \to 2^{Xi}$ and $Q_{2i+2} : X \to 2^{Xi}$ be a pair of correspondences, the collection $\Gamma_1 = (X_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; Q_{2i+2})_{i\in I}$ are called qualitative games. A point $\hat{x} \in X$ is said to be common equilibrium of the games Γ_1 and Γ_2 if $P_{2i+1}(\hat{x}) = \phi = Q_{2i+2}(\hat{x})$ for all $i \in I$.

An abstract and socio-economy are a family of quadruples $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; Q_{2i+2})_{i\in I}$, respectively, where I is a(finite or infinite) set of players(agents) such that, for each $i \in I, X_i$ is a non-empty subset of a topological vector space and $A_i, B_i : X = \Pi_{j\in I} X_j \to 2^{X_i}$are constraint correspondences and $P_{2i+1}, Q_{2i+2} : X \to 2^{X_i}$are preference correspondences. When $I = \{1, \cdots, N\}$ where N is a positve integer, $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; Q_{2i+2})_{i\in I}$ are, also called a pair of N-person games. A common equilibrium of Γ_1 and Γ_2, is a point $\hat{x} \epsilon X$, such that, for each $i \in I$, $\hat{x}_i = \pi_i(\hat{x}) \in \bar{B}_i(\hat{x})$; $A_i(\hat{x}) \cap P_{2i+1}(\hat{x}) = \phi = A_i(\hat{x}) \cap Q_{2i+2}(\hat{x})$. Here $\bar{B}_i(\hat{x}) = Cl_{X_i} B_i(\hat{x})$.

Let X be a Hausdorff topological vector space. Then a mapping $\Psi : 2^X \to C$ is called measure of non-compactness provided that the following conditions hold for any $A, B \in 2^X$;

(1) $\Psi(A) = 0$ if and only if A is precompact;
(2) $\Psi(\bar{co}A) = \Psi(A)$, where $\bar{co}A$ denotes the closed convex hull of A;
(3) $\Psi(A \cup B) = \max\{\Psi(A), \Psi(B)\}$.

It follows from (3) that if $A \subset B$ then $\Psi(A) \leq \Psi(B)$. The above notion is a generalization of the set-measure of non-compactness of Kuratowski [17] and the ball-measure of non-compactness of Sadovskii [27] defined in terms of either a family of seminorms when X is a locally convex topological vector space or a single norm when X is Banach space. For more detail refer [12].

Let $\Psi : 2^X \to C$ be a measure of non-compactness of X and $D \subset X$. A mapping $T : D \to 2^X$is called $\Psi-$condensing provided that if $Z \subset D$ and $\Psi(T(z)) \geqslant \Psi(z)$, then z is relatively compact. If $T : D \to 2^X$is compact mapping (i.e., T(D)) is precompact. Then T is Ψ- condensing for any measure of non-compactness Ψ. Various Ψ- condensing mappings which are not compact have been considered in [2, 11], [23]-[26] etc. Moreover, when the measure of non-compactness Ψ is either the set-measure of non-compactness or ball-measure of non-compactness, Ψ-condensing mappings are called condensing mappings.

We need the following results to prove the existence theorems in the next section.

Lemma 2.2. ([36]) *Let D be a non-empty closed convex subset of a locally convex topological vector space E and $T : D \to 2^D$ is Ψ-condensing, where $\Psi : 2^E \to C$ is a measure of non-compactness. Then there exists a non-empty compact and convex subset K of X such that $T : K \to 2^K$.*

Lemma 2.3. ([22]) *Let X and Y be two topological spaces and let A be a closed (resp. open) subset of X. Suppose $F_1 : X \to 2^Y, F_2 : A \to 2^Y$ are lower (resp. upper) semicontinuous such that $F_2(x) \subset F_1(x)$ for all $x \in A$. Then the mapping $F : X \to 2^Y$ defined by*

$$F(x) = \begin{cases} F_1(x), & if \quad x \notin A, \\ F_2(x), & if \quad x \in A. \end{cases} \tag{2.1}$$

is, also, lower (resp. upper) semicontinuous.

Lemma 2.4. ([16]) *Let X be a topological space and Y be a normal space. If $F, G : X \to 2^Y$ have closed values and are upper semicontinuous at $x \in X$, then $F \cap G$ is, also, upper semicontinuous at x.*

Theorem 2.5. ([36]) *Let X be a paracompact space and let Y be a non-empty normal subset of a topological vector space E. Let $\theta : X \to E$ and $P : X \to 2^Y$ be U-majorized. Then there exists a correspondence $\Psi : X \to 2^Y$, of class U such that $P(x) \subset \Psi(x)$ for each $x \in X$.*

3. Existence of equilibria pair for generalized games

Theorem 3.1. *Let $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; Q_{2i+2})_{i\in I}$ be a pair of generalized games(abstract economy), where I is any (countable or uncountable) set of agents(players) such that for each $i \in I$;*

(i) *X_i is a non-empty compact and convex subset of a locally Hausdorff topological vector space E_i;*
(ii) *for each $x \in X(= \Pi_{i\in I} X_i), A_i(x)$ is non-empty and $A_i(x) \subset \bar{B}_i(x)$, where $\bar{B}_i(x)$ is convex;*
(iii) *the set $E^i = \{x \in X : A_i(x) \cap P_{2i+1}(x) \neq \phi$ and $A_i(x) \cap Q_{2i+2}(x) \neq \phi\}$ is open and paracompact in X;*
(iv) *the mappings $A_i \cap P_{2i+1} : X \to 2^{X_i}$ and $A_i \cap Q_{2i+2} : X \to 2^{X_i}$ are U-majorized on E^i.*

Then Γ_1 and Γ_2 have a common equilibria point, i.e., there exists a point $x \in X$ such that $\pi_i(x) \in \bar{B}_i(x); A_i(x) \cap P_{2i+1}(x) = \phi$ and $A_i(x) \cap Q_{2i+2}(x) = \phi$ for all $i \in I$.

Proof. If we take $E^i = \phi$ for all $i \in I$, then the conclusion follows by Fan-Glicksberg fixed point theorem(e.g. see [9] and [14]).

Let $I_o = \{i \in I : E^i \neq \phi\}$. Without loss of generality, we may assume that $I_o \neq \phi$.

Case I. For each $i \in I_o$ by (iv) and Theorem 2.5, there exists a mapping $T_i : E^i \to 2^{X_i}$ which is upper semicontinuous with closed and convex values and $A_i(x) \cap P_{2i+1}(x) \subset T_i(x)$ for each $x \in E^i$. Since $\bar{B}_i : X \to 2^{X_i}$ is, also, upper semicontinuous with closed and convex values, the mapping $T_i \cap \bar{B}_i : X \to 2^{X_i}$ is, also, upper semicontinuous with non-empty closed and convex values by Lemma 2.4 on E^i. Define a correspondence $\gamma_i : X \to 2^{X_i}$ by

$$\gamma_i(x) = \begin{cases} \bar{B}_i(x), & \text{if } x \notin E^i, \\ (T_i \cap \bar{B}_i)(x), & \text{if } x \in E^i. \end{cases} \tag{3.1}$$

Then Lemma 2.3 implies that γ_i is upper semicontinuous with non-empty closed and convex values.

Case II. For $i \in I \setminus I_o$, we define a correspondence $\gamma : X \to 2^{X_i}$ by $\gamma_i := \bar{B}_i(x)$ for each $x \in X$. Then γ is upper semicontinuous with non-empty compact and convex values.

Finally, we define a correspondence $\Psi : X \to 2^X$ by $\Psi(x) := \Pi_{i\in I}\gamma_i(x)$. Then Ψ is also upper semicontinuous with non-empty compact and convex values. Fan-Glicksberg fixed point theorem implies that there exists a point $x \in X$ such that $x \in \Psi(x)$. If there exists $i \in I_o$ such that $x \in E^i$ then $\pi_i(x) \in \gamma_i(x) = \bar{B}_i(x) \cap T_i(x) \subset T_i(x)$ which contradicts that T_i is U-majorized on E^i. Therefore, $x \notin E^i$ for all $i \in I_o$, i.e., there exists an $i^{'} \notin I_o$ such that $x \in E^{i'}$. By the definition of Ψ, we must have $\pi_i(x) \in \bar{B}_i(x)$ and $A_i(x) \cap P_{2i+1}(x) = \phi$ for all $i \in I$. Similarly, it can be established that for each $i \in I$, $\pi_i(x) \in \bar{B}_i(x)$ and $A_i(x) \cap Q_{2i+2}(x) = \phi$ for all $i \in I$, i.e., Γ_1 and Γ_2 have a common equilibria point. □

By Theorem 3.1, we have the following existence theorem of equilibria for a pair of qualitative games:

Theorem 3.2. *Let $\Gamma_1 = (X_i, P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i, Q_{2i+2})_{i\in I}$ be a pair of qualitative games such that for each $i \in I$,*

(a) *X_i is a non-empty compact and convex subset of a Hausdorff locally convex topological vector space E_i;*
(b) *the set $E^i = \{x \in X : P_{2i+1}(x) \neq \phi$ and $Q_{2i+2}(x) \neq \phi\}$ is open and paracompact in X; and*
(c) *P_{2i+1} and Q_{2i+2} are U-majorized on E^i.*

Then there exists a common point $x \in X$ such that $P_{2i+1}(x_i) = \phi = Q_{2i+2}(x_i)$ for all $i \in I$.

Proof. For each $i \in I$, let $A_i, B_i : X \to 2^{X_i}$ be defined by $A_i(x) = B_i(x) = X_i$ for each $x \in X$, then the generalized games $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; Q_{2i+2})_{i\in I}$ satisfy all hypotheses of Theorem 3.1. Therefore, the conclusion of Theorem 3.2 is that, to replace the condition (iii) of Theorem 3.1 by the condition that "the set $E^i = \{x \in X : A_i(x) \cap P_{2i+1}(x) \neq \phi$ and $A_i(x) \cap Q_{2i+2}(x) \neq \phi\}$ is closed" for each $i \in I$. □

4. Maximal elements and equilibria for U-majorized condensing mappings

We give the following existence theorem of equilibria for pair of generalized games in which the constraint mappings are Ψ-condensing.

Theorem 4.1. *Let $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; Q_{2i+2})_{i\in I}$ be a pair of generalized games and $X = \Pi_{i\in I} X_i$ such that for each $i \in I$,*

(i) *for each $i \in I$, X_i is non-empty closed convex subset of a locally convex Hausdorff topological vector space E_i;*

(ii) *for each* $i \in I$, $A_i : X \to 2^{X_i}$ *is such that for each* $x \in X$, $A_i(x)$ *is non-empty and* $coA_i(x) \subset \bar{B}_i(x)$;
(iii) *for each* $i \in I$, *the set* $E^i = \{x \in X : (A_i \cap P_{2i+1})(x) \neq \phi$ *and* $(A_i \cap Q_{2i+2})(x) \neq \phi\}$ *is open and paracompact in X;*
(iv) *for each* $i \in I$, $A_i \cap P_{2i+1}$ *and* $A_i \cap Q_{2i+2}$ *are U-majorized on* E^i;
(v) *for mapping* $B : X \to 2^X$ *defined by* $B(x) = \Pi_{i \in I}\bar{B}_i(x)$ *for each* $x \in X$ *is* Ψ*-condensing, where* $\Psi : 2^{\Pi_{i \in I}E_i} \to C$ *is a measure of non-compactness.*

Then Γ_i *and* Γ_2 *have a common equilibrium point in X, i.e., there exists a point* $\hat{x} = (\hat{x}_i)_{i \in I} \in X$ *such that for each* $i \in I$, $\hat{x}_i \in \bar{B}_i(\hat{x})$; $A_i(\hat{x}) \cap P_{2i+1}(\hat{x}) = \phi$ *and* $A_i(\hat{x}) \cap Q_{2i+2}(\hat{x}) = \phi$.

Proof. Since the mapping $B : X \to 2^X$ is Ψ-condensing, by Lemma 2.2, there exists a non-empty compact and convex subset K in X such that $B : K \to 2^K$.

Now, we follow the proof of Theorem 3.1. Note that if $E^i = \phi$ for all $i \in I$, then the conclusion follows by Fan-Glicksberg fixed point theorem again(e.g. [9] and [14]).

Let $I_o = \{i \in I : E^i \neq \phi\}$. Without loss of generality, we may assume that $I_o \neq \phi$.

Case I. For each $i \in I_o$ by (iv) and Theorem 2.5, there exists a mapping $T_i : E^i \to 2^{X_i}$ which is upper semicontinuous with closed and convex values and $A_i(x) \cap P_{2i+1}(x) \subset T_i(x)$ for each $x \in E^i$. Since $\bar{B}_i : X \to 2^{X_i}$ is upper semicontinuous with closed and convex values, the mapping $T_i \cap \bar{B}_i : X \to 2^{X_i}$ is, also, upper semicontinuous with non-empty closed and convex values by Lemma 2.4 on E^i. Define a correspondence $\gamma_i : X \to 2^{X_i}$ by

$$\gamma_i(x) = \begin{cases} \bar{B}_i(x), & \text{if } x \notin E^i, \\ (T_i \cap \bar{B}_i)(x), & \text{if } x \in E^i. \end{cases} \tag{4.1}$$

Then Lemma 2.3 implies that γ_i is upper semicontinuous with non-empty closed and convex values.

Case II. For each $i \in I \setminus I_o$, we define a correspondence $\gamma : X \to 2^{X_i}$ by $\gamma_i := \bar{B}_i(x)$ for each $x \in X$. Then γ is upper semicontinuous with non-empty compact and convex values.

Finally, we define a correspondence $\Psi : X \to 2^X$ by $\Psi(x) := \Pi_{i \in I}\gamma_i(x)$ for each $x \in X$. Then Ψ is, also, upper semicontinuous with non-empty compact and covex values. Since $\Psi(x) \subset B(x)$ for each $x \in X$ and B is self-mapping in K, the restriction of Ψ on K is also self-map. Now, Fan-Glicksberg fixed point theorem implies that there exists a point $x \in K$ such that $x \in \Psi(x)$. If there exists $i \in I_o$ such that $x \in E^i$, then $\pi_i(x) \in \gamma_i(x) = \bar{B}_i(x) \cap T_i(x) \subset T_i(x)$ which contradicts that T_i is U-majorized on E^i. Therefore, $x \notin E^i$ for all $i \in I_o$, i.e., there exists an $i^{'} \notin I_o$, such that $x \in E^{i'}$. By the definition of Ψ, we must have $\pi_i(x) \in \bar{B}_i(x)$ and $A_i(x) \cap P_{2i+1}(x) = \phi$ for $i \in I$. Similarly it can be established that for each $i \in I$, $\pi_i(x) \in \bar{B}_i(x)$ and $A_i(x) \cap Q_{2i+2}(x) = \phi$. $\square$

Theorem 4.2. *Let X be a non-empty closed and convex subset of a Hausdorff locally convex topological vector space E. Let $P, Q : X \to 2^X$ be U-majorized and Ψ-condensing, where $\Psi : 2^{\Pi_{i\in I} E_i} \to C$ is a measure non-compactness. Then there exists a point $x \in X$ such that $(P \cap Q)(x) = \phi$.*

Proof. By Lemma 2.2, there exists a non-empty compact and convex subset K of X such that $P, Q : K \to 2^K$. Then it is the same as that of Theorem 3.1, except for the application of Fan-Glicksberg fixed point theorem to K. $\square$

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