Nonlinear Functional Analysis and Applications
Vol. 17, No. 1 (2012), pp. 109-118

http://nfaa.kyungnam.ac.kr/jour-nfaa.htm

SENSITIVITY ANALYSIS FOR GENERAL NONLINEAR ORDERED PARAMETRIC VARIATIONAL INEQUALITY WITH RESTRICTED-ACCRETIVE MAPPING IN ORDERED BANACH SPACE

Honggang Li

Institute of Nonlinear Analysis Research
Chongqing University of Posts and TeleCommunications
Chongqing 400065, China
e-mail: lihg12@126.com

Abstract. The purpose of this paper is to discuss an existence theorem for a new class of general nonlinear ordered parametric variational inequalities and prove sensitivity analysis of the solution for this class of parametric variational inequalities in ordered Banach space. The obtained results seem to be general in nature.

1. Introduction

As everyone knows, sensitivity analysis for a class of general nonlinear variational inequalities(inclusions) have wide applications to many fields including, for example, mathematics, physics, optimization and control, nonlinear programming, and economics et al.. In 1999, M. A. Noor and K. I. Noor have studied sensitivity analysis for strongly nonlinear quasi-variational inclusions[24]. From 2000, Agarwal, Cho and Huang have discussed sensitivity analysis for strongly nonlinear quasi-variations in Hilbert spaces by using the resolvent operator technique[2], furthermore, Bi, Han and Fang[3], Lan, Cho and Verma[11]-[12], Dong, Lee and Hang[9], Jin[10], Verma[27], and Li[17] have show the existence of solutions and sensitivity analysis for a class of nonlinear variational inclusions involving generalized nonlinear mappings in Banach spaces, respectively.

[0]Received June 16, 2011. Revised December 21, 2011.
[0]2000 Mathematics Subject Classification: 49J40, 47H06.
[0]Keywords: General nonlinear ordered parametric variational inequality, ordered Banach space, B-restricted-accretive mapping, convergence, sensitivity analysis.

On the other hand, in 1972, the number of solutions of nonlinear equations has been introduced and studied by Amann[1], and in recent years, the nonlinear mapping fixed point theory and application have been intensively studied in ordered Banach space(see [6], [7], [8]). Very recently, Li has studied the approximation solution for general nonlinear ordered variational inequalities and ordered equations in ordered Banach space[18]-[20]. For details, we refer the reader to [1-27] and the references therein.

Inspired and motivated by recent research work in this field, in this paper, an existence theorem for a new class of general nonlinear ordered parametric mixed variational inequalities is studied, and sensitivity analysis of the solution for this class of parametric variational inequalities is proved in ordered Banach space. In this field, the results in the instrument are obtained.

1.1. General Nonlinear Parametric Ordered Variational Inequality.

Definition 1.1. ([18]) Let X be a real Banach space with a norm $\|\cdot\|$, θ be a zero element in the X. A nonempty closed convex subsets $\mathbf{P}$ of X is said to be a cone if,

(i) for any $x \in \mathbf{P}$ and any $\lambda > 0$, holds $\lambda x \in \mathbf{P}$;
(ii) if $x \in \mathbf{P}$ and $-x \in \mathbf{P}$, then $x = \theta$.

Definition 1.2. ([18]) Let the $\mathbf{P}$ be a cone of X. $\mathbf{P}$ is said to be a normal cone if and only if there exists a constant $N > 0$ such that for $\theta \leq x \leq y$, holds $\|x\| \leq N\|y\|$, where the N is called normal constant of $\mathbf{P}$.

Lemma 1.3. ([18]) *Let the $\boldsymbol{P}$ is a cone in X, for arbitrary $x, y \in X$, $x \leq y$ if and only if $x - y \in \boldsymbol{P}$, then the relation $\leq$ in X is a partial ordered relation in X, where, the Banach space X with a ordered relation $\leq$ defined by a normal cone $\boldsymbol{P}$ is called a ordered Banach space.*

Let X be a real ordered Banach space with a norm $\|\cdot\|$, and θ be a zero in the X. Let $\mathbf{P}$ be a normal cone of X, and $\leq$ be a partial ordered relation defined by the cone $\mathbf{P}$. Let Ω be a nonempty open subset of X in which the parametric λ takes values. Let $A, g, f : X \times \Omega \to X$ be single-valued nonlinear ordered compression mappings and $rangeg(\cdot, \lambda) \bigcap domA(\cdot, \lambda) \neq \emptyset$ for any $\lambda \in \Omega$. we consider the following problem:

Find $x = x(\lambda) : \Omega \to X$ such that

$$A(g(x, \lambda), \lambda) + f(x, \lambda) \geq \theta. \tag{1.1}$$

The problem (1.1) is called a new class of general nonlinear ordered parametric variational inequalities(a new class of general nonlinear ordered parametric equations, as changed $\geq$ to $=$) in ordered Banach space.

Remark 1.4. When mapping $f(x, \lambda) = \theta$, then the problem (1.1) reduces to the problem (2.1) in [18].

1.2. **Preliminaries.**

Let us recall some concepts and results.

Definition 1.5. ([26]) Let X be an ordered Banach space, and the $\mathbf{P}$ be a cone of X. The $\leq$ is a partial ordered relation defined by the cone $\mathbf{P}$, for $x, y \in X$, if holds $x \leq y$(or $y \leq x$), then the x and the y is said to be comparison between each other(denoted by $x \propto y$ for $x \leq y$ and $y \leq x$).

Lemma 1.6. ([26]) *Let X be an ordered Banach space, and the $\boldsymbol{P}$ be a cone of X. The $\leq$ is a partial ordered relation defined by the cone $\boldsymbol{P}$, for arbitrary $x, y \in X$, $lub\{x, y\}$ and $glb\{x, y\}$ express the least upper bound of the set $\{x, y\}$ and the greatest lower bound of the set $\{x, y\}$ on the partial ordered relation $\leq$, respectively, Suppose $lub\{x, y\}$ and $glb\{x, y\}$ exist, some binary operators can be defined as following:*

$$
\begin{aligned}
&(1) \quad x \vee y = lub\{x, y\};\\
&(2) \quad x \wedge y = glb\{x, y\};\\
&(3) \quad x \oplus y = (x - y) \vee (y - x).
\end{aligned}
$$

$\vee, \wedge$, and $\oplus$ is called OR, AND, and XOR operation, respectively. For arbitrary $x, y, w \in X$, then holds the following relations:

(1) *if $x \leq y$, then $x \vee y = y, x \wedge y = x$;*
(2) *if x and the y can be compared, then $\theta \leq x \oplus y$;*
(3) *$(x + w) \vee (y + w)$ exists and $(x + w) \vee (y + w) = (x \vee y) + w$;*
(4) *$(x \wedge y) = (x + y) - (x \vee y)$;*
(5) *if $\lambda \geq 0$, then $\lambda(x \vee y) = \lambda x \vee \lambda y$;*
(6) *if $\lambda \leq 0$, then $\lambda(x \wedge y) = \lambda x \vee \lambda y$;*
(7) *if $x \neq y$, then the converse holds for (5) and (6);*
(8) *if for any $x, y \in X$, either $x \vee y$, and $x \wedge y$ exists, then X is a lattice;*
(9) *$(x + w) \wedge (y + w)$ exists and $(x + w) \wedge (y + w) = (x \wedge y) + w$;*
(10) *$(x \wedge y) = -(-x \vee -y)$;*
(11) *$(-x) \wedge (x) \leq \theta \leq (-x) \vee x$.*

Lemma 1.7. ([6]) *If $x \propto y$, then $lub\{x, y\}$, and $glb\{x, y\}$ exist, $x - y \propto y - x$, and $\theta \leq (x - y) \vee (y - x)$.*

Lemma 1.8. ([6]) *If for any natural number n, $x \propto y_n$, and $y_n \to y^*(n \to \infty)$, then $x \propto y^*$.*

Lemma 1.9. ([18]) *Let X be an ordered Banach space, and the $\boldsymbol{P}$ be a cone of X. The $\leq$ is a partial ordered relation defined by the cone $\boldsymbol{P}$, if for $x, y, z, w \in X$, they can be compared each other, then holds the following relations:*

(1) $x \oplus y = y \oplus x$;
(2) $x \oplus x = \theta$;
(3) $\theta \leq x \oplus \theta$;
(4) *let λ be a real, then* $(\lambda x) \oplus (\lambda y) = |\lambda|(x \oplus y)$;
(5) *if x, y and w can be comparative each other, then* $(x \oplus y) \leq x \oplus w + w \oplus y$;
(6) *let* $(x+y) \vee (u+v)$ *exist, and if* $x \propto u, v$ *and* $y \propto u, v$, *then*

$$(x+y) \oplus (u+v) \leq (x \oplus u + y \oplus v) \wedge (x \oplus v + y \oplus u);$$

(7) *if x, y, z, w can be compared with each other, then*

$$(x \wedge y) \oplus (z \wedge w) \leq ((x \oplus z) \vee (y \oplus w)) \wedge ((x \oplus w) \vee (y \oplus z));$$

(8) $\alpha x \oplus \beta x = |\alpha - \beta| x + (\alpha \oplus \beta) x$, *if* $x \propto \theta$.

Definition 1.10. ([18]) Let X be a real ordered Banach space, Ω be a nonempty open subset of X in which the parametric λ takes values, $A, B : X \times \Omega \to X$ be two parametric mappings.

(1) A is said to be comparison with respect to the argument λ, if for any $\lambda \in \Omega$ and each $x(\lambda), y(\lambda) \in X$, $x(\lambda) \propto y(\lambda)$ then $A(y(\lambda), \lambda) \propto A(y(\lambda), \lambda)$, $x(\lambda) \propto A(x(\lambda), \lambda)$, and $y(\lambda) \propto A(y(\lambda), \lambda)$.

(2)A and B is said to be comparison with respect to the argument λ each other, if for each $x(\lambda) \in X$, $A(x(\lambda), \lambda) \propto B(x(\lambda), \lambda)$(denoted by $A \propto B$).

Obviously, if A is comparison, then $A \propto I$(where, I is an identity mapping on the X).

Definition 1.11. ([18]) Let X be a real ordered Banach space, $\mathbf{P}$ be a normal cone with normal constant N in the X, Ω be a nonempty open subset of X in which the parametric λ takes values, $A : X \times \Omega \to X$ be a parametric mapping. A is said to be β-order compression with respect to the second argument λ, if A is comparative with respect to the argument λ, and there exists a constant $0 < \beta < 1$ such that for any $\lambda \in \Omega$,

$$(A(x(\lambda), \lambda) \oplus A(y(\lambda), \lambda)) \leq \beta(x(\lambda) \oplus y(\lambda)),$$

holds.

Definition 1.12. ([18]) Let X be a real ordered Banach space, $\mathbf{P}$ be a normal cone with normal constant N in the X, Ω be a nonempty open subset of X in which the parametric λ takes values, $A, B : X \times \Omega \to X$ be two parametric mappings, I be an identity mapping on the $X \times X$.

(i) A mapping A is said to be restricted-accretive mapping with respect to the argument λ, if A is comparative, and there exist two constants $0 < \alpha_1, \alpha_2 \leq 1$ such that for any $\lambda \in \Omega$ and arbitrary $x(\lambda), y(\lambda) \in X$, holds

$$\begin{aligned}&(A(x(\lambda),\lambda) + I(x(\lambda),\lambda)) \oplus (A(y(\lambda),\lambda) + I(y(\lambda),\lambda))\\ &\leq \alpha_1(A(x(\lambda),\lambda) \oplus A(y(\lambda),\lambda)) + \alpha_2(x(\lambda) \oplus y(\lambda));\end{aligned}$$

(ii) A mapping $A : X \times \Omega \to X$ is said to be B-restricted-accretive mapping with respect to the argument λ, if A, B and $A \wedge B : X \times \Omega \to A(x(\lambda),\lambda) \wedge B(x(\lambda),\lambda) \in X$ all are comparative and they are comparison with respect to the argument λ each other, and there exist two constants $0 < \alpha_1, \alpha_2 \leq 1$ such that for any $\lambda \in \Omega$ and arbitrary $x(\lambda), y(\lambda) \in X$, holds

$$\begin{aligned}&(A(x(\lambda),\lambda)\wedge B(x(\lambda),\lambda)+I(x(\lambda),\lambda)) \oplus (A(y(\lambda),\lambda)\wedge B(y(\lambda),\lambda)+I(y(\lambda),\lambda))\\ &\leq \alpha_1((A(x(\lambda),\lambda)\wedge B(x(\lambda),\lambda))\oplus(A(y(\lambda),\lambda)\wedge B(y(\lambda),\lambda)))+\alpha_2(x(\lambda)\oplus y(\lambda)),\end{aligned}$$

where the $I(x(\lambda),\lambda) = x(\lambda) : X \times X \to X$ is an identity mapping.

Lemma 1.13. *Let X be an ordered Banach space, $\boldsymbol{P}$ be a normal cone with normal constant N in the X, $A : X \to X$ be a comparative, then for any $x, y \in X$*

(1) $\|\theta \oplus \theta\| = \|\theta\| = 0$,
(2) $\|x \vee y\| \leq \|x\| \vee \|y\| \leq \|x\| + \|y\|$,
(3) $\|x \oplus y\| \leq \|x - y\| \leq N\|x \oplus y\|$,
(4) *if* $x \propto y$, *then* $\|x \oplus y\| = \|x - y\|$,
(5) $\lim_{x\to x_0} \|A(x) - A(x_0)\| = 0$, *if and only if* $\lim_{x\to x_0} A(x) \oplus A(x_0) = \theta$.

Proof. The results (1), (2) and (4) are obvious. (3) is follows that from Definition 1.2, the definition(3) in Lemma 1.6, $x - y \leq (x \oplus y)$ and (2). (5) is follows that from (4). □

2. Existence For General Nonlinear Ordered Parametric Variational Inequality (1.1)

In this section, we will show the existence of solution and the convergence of the approximation sequences for the problem (1.1).

Theorem 2.1. *Let X be a real ordered Banach space, $\boldsymbol{P}$ be a normal cone with normal constant N in the X, $\leq$ be an ordered relation defined by the cone $\boldsymbol{P}$, Ω be a nonempty open subset of X in which the parametric λ takes values, A, g, f, $A+f$, B and $(A+f)\wedge B : X \times \Omega \to X$ be some comparison parametric mappings each other, and A, B be β_i-ordered compression$(i = 1, 2)$, f be δ-ordered compression, and g be γ-ordered compression with respect to* *the argument λ, respectively. If $A+f$ is a B-restricted-accretive mapping with respect to the argument λ for two constants α_1, α_2, and for any $\rho > 0$,*

$$\rho[(\beta_1\gamma + \delta) \vee \beta_2] < \frac{1 - \alpha_2 N}{\alpha_1 N} \tag{2.1}$$

holds, then the general nonlinear ordered variational inequality (1.1) there exists a solution x^*.

Proof. Let X be a real ordered Banach space, $\mathbf{P}$ be a normal cone with normal constant N in the X, $\leq$ be an ordered relation defined by the cone $\mathbf{P}$, Ω be a nonempty open subset of X in which the parametric λ takes values. For any given $\lambda \in \Omega$ and $x_1 = x_1(\lambda), x_2 = x_2(\lambda) \in X$, for $\rho > 0$, let $x_1(\lambda) \propto x_2(\lambda)$, then

$$\begin{aligned} F(x_i(\lambda), \lambda) = \rho[A(g(x_i(\lambda), \lambda), \lambda) + f(x_i(\lambda), \lambda)] \wedge B(x_i(\lambda), \lambda) \\ +I(x_i(\lambda), \lambda), \end{aligned} \tag{2.2}$$

where $i = 1, 2$. It follows from the conditions, A, g, f, $A + f$, B and $(A + f) \wedge B : X \times \Omega \to X$ are some comparison parametric mappings with respect to the second argument λ each other, and $x_1(\lambda) \propto x_2(\lambda)$ that $F(x_1(\lambda), \lambda) \propto F(x_2(\lambda), \lambda)$. By the restricted-accretive and the β-ordered compression of A, the γ-ordered compression of g with respect to the argument λ, respectively, and Lemma 1.9(6), (7), we have

$$\begin{aligned} &\theta \leq F(x_1, \lambda) \oplus F(x_2, \lambda) \\ &\leq [\rho(A(g(x_1(\lambda), \lambda), \lambda) + f(x_1(\lambda), \lambda)) \wedge B(x_1(\lambda), \lambda) + I(x_1(\lambda), \lambda)] \\ &\oplus[\rho(A(g(x_2(\lambda), \lambda), \lambda) + f(x_2(\lambda), \lambda)) \wedge B(x_2(\lambda), \lambda) + I(x_2(\lambda), \lambda)] \\ &\leq \rho\alpha_1[((A(g(x_1, \lambda), \lambda) + f(x_1(\lambda), \lambda)) \wedge B(x_1(\lambda), \lambda)) \\ &\oplus((A(g(x_2, \lambda), \lambda) + f(x_2(\lambda), \lambda)) \wedge B(x_2(\lambda), \lambda))] + \alpha_2(x_1(\lambda) \oplus x_2(\lambda)) \\ &\leq \rho\alpha_1[((A(g(x_1(\lambda), \lambda), \lambda) + f(x_1(\lambda), \lambda)) \oplus (A(g(x_2(\lambda), \lambda), \lambda) \\ &+f(x_2(\lambda), \lambda))) \vee (B(x_1(\lambda), \lambda) \oplus B(x_2(\lambda), \lambda))] + \alpha_2(x_1(\lambda) \oplus x_2(\lambda)) \\ &\leq \rho\alpha_1[((A(g(x_1(\lambda), \lambda), \lambda) \oplus A(g(x_2(\lambda), \lambda), \lambda)) + (f(x_1(\lambda), \lambda) \\ &\oplus f(x_2(\lambda), \lambda))) \vee (B(x_1(\lambda), \lambda) \oplus B(x_2(\lambda), \lambda))] + \alpha_2(x_1(\lambda) \oplus x_2(\lambda)) \\ &\leq (\rho\alpha_1[(\beta_1\gamma + \delta) \vee \beta_2] + \alpha_2)(x_1(\lambda) \oplus x_2(\lambda)) \end{aligned} \tag{2.3}$$

by Lemma 1.6 and Definition 1.2, we obtain

$$\|F(x_1(\lambda), \lambda) - F(x_2(\lambda), \lambda)\| \leq hN\|x_1(\lambda) - x_2(\lambda)\|, \tag{2.4}$$

where $h = \rho\alpha_1[(\beta_1\gamma + \delta) \vee \beta_2] + \alpha_2$. It follows from the condition (2.1) that $0 < hN < 1$, we know that the $F(x(\lambda), \lambda)$ has a fixed point $x^* \in X$ and the x^* is a solution of the generalized nonlinear ordered parametric equation

$$[A(g(x(\lambda), \lambda)) + f(x(\lambda), \lambda)] \wedge B(x(\lambda), \lambda) = \theta,$$

for any parametric $\lambda \in \Omega$, and $\theta \in X$. By the Lemma 3.2 in [18], then the generalized nonlinear ordered parametric variational inequality (1.1) there exist a solution x^*. This completes the proof. □

3. Sensitivity Analysis For General Nonlinear Ordered Parametric Variational Inequality (1.1)

In this section, we will show the Sensitivity analysis for general nonlinear ordered parametric variational inequality (1.1).

Theorem 3.1. *Let X be a real ordered Banach space, $\boldsymbol{P}$ be a normal cone with normal constant N in the X, $\leq$ be an ordered relation defined by the cone $\boldsymbol{P}$, Ω be a nonempty open subset of X in which the parametric λ takes values, $A(\cdot,\lambda), B(\cdot,\lambda), g(\cdot,\lambda)$, and $f(\cdot,\lambda) : X \times \Omega \to X$ be the parametric mappings continuous with respect to the argument $\bar{\lambda} \in \Omega$, A, g, f, $A+f$, B and $(A+f)\wedge B$ be comparison each other, and A, B be β_i-ordered compression$(i = 1,2)$, f be δ-ordered compression, and g be γ-ordered compression with respect to the argument λ, respectively. If $A+f$ is a B-restricted-accretive mapping with two constants α_1, α_2, and for any $\rho > 0$,*

$$\rho[(\beta_1\gamma + \delta) \vee \beta_2] < \frac{1-\alpha_2}{\alpha_1} \tag{3.1}$$

holds, then the solution $x(\lambda)$ of parametric problem (1.1) is continuous in Ω.

Proof. For any given $\lambda, \bar{\lambda} \in \Omega$, let $x(\lambda)$ and $x(\bar(\lambda))$ be two solutions of parametric problem (1.1), then for any $\rho > 0$, we have

$$\begin{aligned}
x(\lambda) &= F(x(\lambda),\lambda)\\
&= \rho[A(g(x(\lambda),\lambda),\lambda) + f(x(\lambda),\lambda)] \wedge B(x(\lambda),\lambda) + I(x(\lambda),\lambda),\\
x(\bar{\lambda}) &= F(x(\bar{\lambda},\bar{\lambda})\\
&= \rho[A(g(x(\bar{\lambda}),\bar{\lambda}),\bar{\lambda}) + f(x(\bar{\lambda}),\bar{\lambda})] \wedge B(x(\bar{\lambda}),\bar{\lambda}) + I(x(\bar{\lambda}),\bar{\lambda}).
\end{aligned} \tag{3.2}$$

By the condition that A, g, f, $A+f$, B and $(A+f)\wedge B$ are comparison with respect to the argument λ each other, respectively and Lemma 1.9, we have

$$\begin{aligned}
\theta &\leq x(\lambda) \oplus x(\bar{\lambda}) \leq F(x(\lambda),\lambda) \oplus F(x(\bar{\lambda}),\bar{\lambda})\\
&\leq F(x(\lambda),\lambda) \oplus \theta \oplus F(x(\bar{\lambda}),\bar{\lambda})\\
&\leq [F(x(\lambda),\lambda) \oplus F(x(\bar{\lambda}),\lambda)] \oplus [F(x(\bar{\lambda}),\lambda) \oplus F(x(\bar{\lambda}),\bar{\lambda})].
\end{aligned} \tag{3.3}$$

Further, since $A+f$ is a B-restricted-accretive mapping with constants α_1, α_2, f is a δ-ordered compression , g is a γ-ordered compression, A is a β_1-ordered compression and B is a β_2-ordered compression with respect to the argument

λ, respectively, so that by the Theorem 2.1, we have

$$\begin{aligned}
&F(x(\lambda),\lambda)\oplus F(x(\bar\lambda),\lambda)\\
&\le(\rho(A(g(x(\lambda),\lambda),\lambda)+f(x(\lambda),\lambda))\wedge B(x(\lambda),\lambda)+I(x(\lambda),\lambda))\\
&\oplus(\rho(A(g(x(\bar\lambda),\lambda),\lambda)+f(x(\bar\lambda),\lambda))\wedge B(x(\bar\lambda),\lambda)+I(x(\bar\lambda),\lambda))\\
&\le[\alpha_1(\rho((A(g(x(\lambda),\lambda),\lambda)+f(x(\lambda),\lambda))\wedge B(x(\lambda),\lambda))\\
&\oplus\rho(A(g(x(\bar\lambda),\lambda),\lambda)+f(x(\bar\lambda),\lambda))\wedge B(x(\bar\lambda),\lambda))+\alpha_2(x(\lambda)\oplus x(\bar\lambda))]\\
&\le h(x(\lambda)\oplus x(\bar\lambda)),
\end{aligned}\tag{3.4}$$

where $h=\rho\alpha_1[(\beta_1\gamma+\delta)\vee\beta_2]+\alpha_2<1$ for the condition 3.1, and

$$\begin{aligned}
&F(x(\bar\lambda),\lambda)\oplus F(x(\bar\lambda),\bar\lambda)\\
&\le(\rho(A(g(x(\bar\lambda),\lambda),\lambda)+f(x(\bar\lambda),\lambda))\wedge B(x(\bar\lambda),\lambda)+I(x(\bar\lambda),\lambda))\\
&\oplus(\rho(A(g(x(\bar\lambda),\bar\lambda),\bar\lambda)+f(x(\bar\lambda),\bar\lambda))\wedge B(x(\bar\lambda),\bar\lambda)+I(x(\bar\lambda),\bar\lambda))\\
&\le(\rho(A(g(x(\bar\lambda),\lambda),\lambda)+f(x(\bar\lambda),\lambda))\wedge B(x(\bar\lambda),\lambda))\\
&\oplus(\rho(A(g(x(\bar\lambda),\bar\lambda),\bar\lambda)+f(x(\bar\lambda),\bar\lambda))\wedge B(x(\bar\lambda),\bar\lambda))+\theta\\
&\le(\rho(A(g(x(\bar\lambda),\lambda),\lambda)+f(x(\bar\lambda),\lambda))\wedge B(x(\bar\lambda),\lambda))\\
&\oplus(\rho(A(g(x(\bar\lambda),\bar\lambda),\bar\lambda)+f(x(\bar\lambda),\bar\lambda))\wedge B(x(\bar\lambda),\bar\lambda))\\
&\le(\rho(A(g(x(\bar\lambda),\lambda),\lambda)\oplus A(g(x(\bar\lambda),\bar\lambda),\bar\lambda))\\
&+\rho(f(x(\bar\lambda),\bar\lambda)\oplus f(x(\bar\lambda),\lambda)))\vee(B(x(\bar\lambda),\bar\lambda)\oplus B(x(\bar\lambda),\lambda))\\
&\le\rho[(A(g(x(\bar\lambda),\lambda),\lambda)\oplus A(g(x(\bar\lambda),\bar\lambda),\lambda))\oplus(A(g(x(\bar\lambda),\bar\lambda),\lambda)\\
&\oplus A(g(x(\bar\lambda),\bar\lambda),\bar\lambda))+(f(x(\bar\lambda),\bar\lambda)\oplus f(x(\bar\lambda),\lambda))]\\
&\vee(B(x(\bar\lambda),\bar\lambda)\oplus B(x(\bar\lambda),\lambda)).
\end{aligned}\tag{3.5}$$

Combining (3.3), (3.4) and (3.5), and by using the Lemma 1.9(8), we can get

$$\begin{aligned}
&(1-h)(x(\lambda)\oplus x(\bar\lambda))\\
&\le\rho[(A(g(x(\bar\lambda),\lambda),\lambda)\oplus A(g(x(\bar\lambda),\bar\lambda),\lambda))\oplus(A(g(x(\bar\lambda),\bar\lambda),\lambda)\\
&\oplus A(g(x(\bar\lambda),\bar\lambda),\bar\lambda))+(f(x(\bar\lambda),\bar\lambda)\oplus f(x(\bar\lambda),\lambda))]\vee(B(x(\bar\lambda),\bar\lambda)\oplus B(x(\bar\lambda),\lambda)).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&(x(\lambda)\oplus x(\bar\lambda))\le\frac{1}{1-h}[\rho((A(g(x(\bar\lambda),\lambda),\lambda)\oplus A(g(x(\bar\lambda),\bar\lambda),\lambda))\\
&\oplus(A(g(x(\bar\lambda),\bar\lambda),\lambda)\oplus A(g(x(\bar\lambda),\bar\lambda),\bar\lambda))+(f(x(\bar\lambda),\bar\lambda)\oplus f(x(\bar\lambda),\lambda)))\\
&\vee(B(x(\bar\lambda),\bar\lambda)\oplus B(x(\bar\lambda),\lambda))].
\end{aligned}\tag{3.6}$$

Since $A(\cdot,\lambda), B(\cdot,\lambda), g(\cdot,\lambda)$, and $f(\cdot,\lambda) : X \times \Omega \to X$ are the parametric mappings continuous with respect to the argument $\bar{\lambda} \in \Omega$, so that

$$\lim_{\lambda\to\bar{\lambda}} \|g(x(\bar{\lambda}),\lambda) - g(x(\bar{\lambda}),\bar{\lambda})\| = 0,$$
$$\lim_{\lambda\to\bar{\lambda}} \|f(x(\bar{\lambda}),\lambda) - f(x(\bar{\lambda}),\lambda)\| = 0,$$
$$\lim_{\lambda\to\bar{\lambda}} \|B(x(\bar{\lambda}),\lambda) - B(x(\bar{\lambda}),\lambda)\| = 0,$$
$$\lim_{\lambda\to\bar{\lambda}} \|A(\cdot,\lambda) - A(\cdot,\bar{\lambda})\| = 0.$$

By the Lemma (1.13)(5), we have

$$\lim_{\lambda\to\bar{\lambda}} x(\lambda) \oplus x(\bar{\lambda}) = \theta,$$

and

$$\lim_{\lambda\to\bar{\lambda}} \|x(\lambda) - x(\bar{\lambda})\| = 0, \tag{3.7}$$

which implies the solution $x(\lambda)$ of problem (1.1) is continuous at $\lambda = \bar{\lambda}$. This completes the proof. □

References

[1] Amann H., *On the number of solutions of nonlinear equations in ordered Banach space*, J. Funct. Anal., 11 (1972), 346–384.

[2] R. P. Agarwal, Y. J. Cho and N. J. Huang, *Sensitivity analysis for strongly nonlinear quasi-variations*, Appl. Math. Lett., 13 (2000), 19–24.

[3] Z. S. Bi, Han and Y. P. Fang, *Sensitivity analysis for nonlinear variational inclusions involing generalized m-accretive mapping*, J. Sichuan Univ., 40(2) (2003), 240–243.

[4] Y. J. Cho and H. Y. Lan, *A New Class of Generalized Nonlinear Multi-valued Quasi-variational-like-inclusions with H-Monotone Mappings*, Math. Inequal. Appl., 10 (2007), 389–401.

[5] Y. J. Cho, X. L. Qin, M. J. Shang and Y. F. Su, *Generalized Nonlinear Variational Inclusions involving (A,η)-Monotone Mappings in Hilbert Spaces*, Fixed Point Theory Appl., Vol. 2007, Article ID 29653, 6 pages.

[6] Du Y H., *Fixed points of incresing operators in ordered Banach spaces and applications*, Appl. Anal., 38 (1990), 1-20.

[7] Ge D. J. and Lakshmikantham V., *Couple fixed points of nonlinear operators with applictions*, Nonlinear Anal. TMA, 11 (1987), 623–632.

[8] Ge D. J., *Fixed points of mixed monotne operators with applications*, Appl. Anal., 31 (1988), 215–224.

[9] H. Dong, B. S. Lee and N. J. Hang, *Sensitivity analysis for generalized quasi-variational-like inclusions*, Nonlinear Anal. Forum, 6(2) (2001), 313–320.

[10] M. M. Jin, *Sensitivity analysis for quasi-variational inequalities*, Nonlinear Anal. Forum, 8(1)(2003), 93–99.

[11] H. Y. Lan, Y. J. Cho and R. U. Verma, *On nonlinear relaxed cocoercive inclusions involving (A,η)-accretive mappings in Banach spaces*, Math. Appl. Comput., 51 (2006).

[12] H. Y. Lan, Y. J. Cho and R. U. Verma, *On Solution Sensitivity of Generalized Relaxed Cocoercive Implicit Quasivariational Inclusions with A-Monotone Mappings*, J. Comput. Anal. Appl., 8 (2006), 75–87.

[13] H. Y. Lan, *(A,η)-accretive mappings and set-valued quasi-variational inclusions with cocoercive mappings in Banach spaces*, Appl. Math. Lett., 20 (2007), 571–577.

[14] H. Y. Lan, Y. J. Cho and N. J. Huang, *Stability of Iterative Procedures for a Class of Generalized Nonlinear Quasi-Variational-like Inclusions involving Maximal η-Monotone mappings*, Fixed Point Theory and Applications, Edited by Y. J. Cho, J. K. Kim and S. M. Kang, Vol. 6 (2006), 107–116.

[15] H. Y. Lan, J. H. Kim and Y. J. Cho, *On a New System of Nonlinear A-Monotone Multivalued Variational Inclusions*, J. Math. Anal. Appl., 327 (2007), 481–493.

[16] H. Y. Lan, J. I. Kang and Y. J. Cho, *Nonlinear (A,η)-Monotone Operator Inclusion Systems Involving Non-monotone Set-valued Mappings*, Taiwan. J. Math., 11 (2007), 683–701.

[17] H. G. Li, *Sensitivity Analysis For Generalized Set-valued Parametric Mixed Quasi-Variational Inclusion With (A,η)-accretive Mappings*, Nonlinear Analysis Forum, 13(1) (2008), pp.27–37.

[18] H. G. Li, *Approximation Solution For General Nonlinear Ordered Variational Inequalities And Ordered Equations In Ordered Banach Space*, Nonlinear Analysis Forum, 13(2), pp.205–214, 2008.

[19] H. G. Li, *Approximation Solution For A New Class Of General Nonlinear Ordered Variational Inequalities And Ordered Equations In Ordered Banach Space*, Nonlinear Analysis Forum, 14, pp.1–9, 2009.

[20] H. G. Li, *Nonlinear Inclusion Problems For Ordered RME Set-Valued Mappings In Ordered Hilbert Spaces, Nonlinear Functional Analysis and Applications*, Vol.16, No.1 (2011), pp.1–8.

[21] H. G. Li, *Iterative Algorithm for A New Class of Generalized Nonlinear Fuzzy Set-Valude Variational Inclusions Involving (H,η)-monotone Mappings*, Advances in Nonl. Vari. Ineq., 10(1) (2007), 89–100.

[22] H. G. Li, A. J. Xu and M. M. Jin, *A Hybrid Proximal Point Three-Step Algorithm For Nonlinear Set-Valued Quasi-Variational Inclusions System Involving (A,η)-Accretive Mappings*, Fixed Piont Theory And Applications, Hindawi Publishing Corporation, Vol. 2010, Article ID 635382, 24 pages doi:10.1155/2010/635382.

[23] H. G. Li, A. J. Xu and M. M. Jin, *An Ishikawa-Hybrid Proximal Point Algorithm For Nonlinear Set-Valued Inclusions Problem Based on (A,η)-Accretive Framework*, Fixed Piont Theory And Applications, Hindawi Publishing Corporation, Vol. 2010, Article ID 501293, 12 pages doi:10.1155/2010/501293.

[24] M. A. Noor and K. I. Noor, *Sensitivity analysis for strongly nonlinear quasi-variational inclusions*, J. Math. Anal. Appl., 236 (1999), 290–299.

[25] X. B. Pan, H. G. Li and A. J. Xu, *The Over-relaxed A-Proximal Point Algorithm For General Nonlinear Mixed Set-Valued Inclusion Framework*, Fixed Piont Theory And Applications, Hindawi Publishing Corporation, Vol. 2011, Article ID 840978, 12 pages doi:10.1155/2011/840978.

[26] H. H. Schaefer, *Banach lattices and positive operators*, Springer, 1974.

[27] R. U. Verma, *Sensitivity analysis for relaxed cocoercive nonlinear quasivariational inclusions*, J. Appl. Math. Stoch. Anal., 2006, 9 pp. Art. ID 52041.