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SOME NEW RESULTS ON DIFFERENTIAL SUBORDINATIONS AND SUPERORDINATIONS FOR ANALYTIC p-VALENT FUNCTIONS USING NEW HADAMARD PRODUCT OPERATOR

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Abstract. In this work, we obtain sandwich theorems involving a new Hadamard product operator $F^{\alpha}_{\delta,c,p,\gamma,\beta}$ for p-valent (or multivalent) functions in the open unit disk U by employing differential subordinations as well superordinations on p-valent functions using a new Hadamared product operator, we establish new results such as, differential subordination and superordination theorems.

1. Introduction

Letting $\mathfrak{M} = \mathfrak{M}(\mathfrak{A})$ become a collection over analytic functions within $\mathfrak{A} = \{z \in \mathfrak{C} : |z| < 1\}$ open unit disk. Regarding $n \in \aleph$ with $\mathfrak{o} \in \mathfrak{C}$, the subclass $\mathfrak{M}[\mathfrak{o}, n]$ represents a subset of $\mathfrak{M}$. Furthermore

$$\mathfrak{M}[\mathfrak{o}, n] = \left\{\mathfrak{H} \in \mathfrak{M} : \mathfrak{H}(z) = \mathfrak{o} + \mathfrak{o}_n z^n + \mathfrak{o}_{n+1} z^{n+1} + \dots\right\} (\mathfrak{o} \in \mathfrak{C}).$$

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Give A_p a represent the subfamily of $\mathfrak{M}$ included to functions $\mathfrak{H}$ that have a specified format:

$$\mathfrak{H}(z) = z^p + \sum_{n=1}^{\infty} \mathfrak{a}_{n+p} z^{n+p}, \ (p \in \aleph, \ \mathfrak{a}_{n+p} \geq 0), \tag{1.1}$$

that are multivalent analytic within $\mathfrak{A} = \{z \in \mathfrak{C} : |z| < 1\}$. We know the Hadamard product (or convolution):

$$\mathfrak{H}(z) = z^p + \sum_{n=1}^{\infty} \mathfrak{a}_{n+p} z^{n+p} \text{ and } \quad g(z) = z^p + \sum_{n=1}^{\infty} \mathfrak{d}_{n+p} z^{n+p},$$

$$(\mathfrak{H} * g) = z^p + \sum_{n=1}^{\infty} \mathfrak{a}_{n+p} \mathfrak{d}_{n+p} z^{n+p} = (g * f) \qquad (z \in \mathfrak{A}).$$

Suppose $\mathfrak{H}$ with g both analytic functions within $\mathfrak{M}$. $\mathfrak{H}$ is considered subordinate to g, or g is considered superordinate to $\mathfrak{H}$ in $\mathfrak{A}$ composed $\mathfrak{H} \prec g$, when a Schwarz function obtains $\mathfrak{Y}$ in $\mathfrak{A}$, that includes $\mathfrak{Y}(0) = 0$, with $|\mathfrak{Y}(z)| < 1$, $(z \in \mathfrak{A})$, also $\mathfrak{H}(z) = g(\mathfrak{Y}(z))$. Regarding this specific case, we'll represent $\mathfrak{H} \prec g$, also $\mathfrak{H}(z) \prec g(z) \quad (z \in \mathfrak{A})$. When g be univalent within $\mathfrak{A}$, thus $\mathfrak{H} \prec g$ if and only if $\mathfrak{H}(0) = g(0)$, $\mathfrak{H}(\mathfrak{A}) \subset g(\mathfrak{A})$ ([20, 21]).

Definition 1.1. ([20]) Letting $\theta : \mathfrak{C}^3 \times \mathfrak{A} \to \mathfrak{C}$ as well as the function $\mathfrak{T}(z)$ to be univalent in $\mathfrak{A}$. When $p(z)$ be analytic within $\mathfrak{A}$ it fulfils the second-ordar differential subordinetion condition:

$$\theta\left(p(z), zp'(z), z^2 p''(z); z\right) \prec \mathfrak{T}(z), \tag{1.2}$$

therefore, $p(z)$ is referred to be a solutions for differential subordination (1.2). Also, the function $q(z)$, which is univalent, is refarred to as a dominant from the solution as the differential subordination (1.2), alternatively, it can be stated that dominent when $p(z) \prec q(z)$ with all $p(z)$ fulfilling (1.2). A univalent dominent $\widetilde{q}(z)$ which fulfils $\widetilde{q}(z) \prec q(z)$ to each dominating q(z) in formula (1.2) it's claimed to be the best dominant is uniquely determined by a relation of $\mathfrak{A}$.

Definition 1.2. ([20]) Letting $p, k \in A_p$ with $\theta : \mathfrak{C}^3 \times \mathfrak{A} \to \mathfrak{C}$. Assuming p with $\theta\left(p(z), zp'(z), z^2 p''(z); z\right)$ two univalant functions in $\mathfrak{A}$ and if $p(z)$ fulfills the second-type differential superordination:

$$\mathfrak{T}(z) \prec \theta\left(p(z), zp'(z), z^2 p''(z); z\right), \tag{1.3}$$

therefore, $p(z)$ is referred to be a solution for differential superordination (1.3). The function $q(z)$ is refarred to as a subordinant for the solution of this differential superordination (1.3), or, to put it clearly a subordinant when $p \prec q$ with each functions p that fulfill Eq. (1.3). A univalent subordinant $\widetilde{q}$ it fulfills $q \prec \widetilde{q}$ to every the subordinants q of (1.3) is considered the best subordinant.

Many researchers [1, 2, 8, 15, 17, 18, 20, 24, 25, 26, 27, 29, 30] have derived necessary constraints in the functions $p, \mathfrak{T}$, as well θ whose the next conclusions is valid:

$$\mathfrak{T}(z) \prec \theta\left(p(z), zp'(z), z^2p''(z); z\right),$$

thus

$$q(z) \prec p(z). \tag{1.4}$$

Utilizing the outcomes (refer to [4, 6, 11, 12, 13, 16, 21, 28]), it is necessary to establish adequate criteria for analytical functions to fulfill:

$$q_1(z) \prec \frac{z\mathfrak{H}'(z)}{\mathfrak{H}(z)} \prec q_2(z),$$

when q_1 as well q_2 are supplied univalent functions within $\mathfrak{A}$, also $q_1(0) = q_2(0) = 1$. Furthermore, multiple authors (refer to [1, 3, 4, 5, 9, 10, 11, 12, 19, 23, 29]) having obtained some conclusions on differential subordination and superordination using sandwich theorems. To $\mathfrak{H} \in A_p$, let the Komatu operator [22] be denoted by

$$\begin{aligned} K_{c,p}^{\delta}\mathfrak{H}(z) &= \frac{(c+p)^{\delta}}{\Gamma(\delta) z^c}\int_0^z t^{c-1}\left(\log\frac{z}{t}\right)^{\delta-1}\mathfrak{H}(t)\,dt \\ &= z^p + \sum_{n=1}^{\infty}\left(\frac{c+p}{c+p+n}\right)^{\delta} a_{n+p}z^{n+p}\,(c>-p,\ \delta>0). \end{aligned} \tag{1.5}$$

Aouf et al. [7] defined the operator $\mathfrak{R}_{\beta,p}^{\alpha,\gamma}\mathfrak{H}(z)$ as follows:

$$\mathfrak{R}_{\beta,p}^{\alpha,\gamma}\mathfrak{H}(z) = z^p + \frac{\Gamma(p+\alpha-\gamma+1)}{\Gamma(p+\beta)}\sum_{n=1}^{\infty}\left[\frac{\Gamma(\beta+p+n)}{\Gamma(p+\alpha+\beta+n-\gamma+1)}\right]a_{n+p}z^{n+p},$$

$$(\beta>-p; \alpha+1>\gamma; \gamma\in\mathfrak{N}; p\in\aleph; z\in\mathfrak{A}). \tag{1.6}$$

We define a new Hadamard product operator $F_{\delta,c,p,\gamma,\beta}^{\alpha}f(z)$ of function $\mathfrak{H}\in A_p$ as follows:

$$F_{\delta,c,p,\gamma,\beta}^{\alpha}\mathfrak{H}(z) = K_{c,p}^{\delta}\mathfrak{H}(z) * \mathfrak{R}_{\beta,p}^{\alpha,\gamma}\mathfrak{H}(z),$$

where

$$\begin{aligned} F_{\delta,c,p,\gamma,\beta}^{\alpha}\mathfrak{H}(z) &= z^p + \frac{\Gamma(p+\alpha-\gamma+1)}{\Gamma(p+\beta)}\sum_{n=1}^{\infty}\left[\frac{\Gamma(\beta+p+n)}{\Gamma(p+\alpha+\beta+n-\gamma+1)}\right] \\ &\qquad\times\left(\frac{c+p}{c+p+n}\right)^{\delta}a_{n+p}z^{n+p}. \end{aligned} \tag{1.7}$$

It could easily be simply noted from Eq. (1.7) that

$$z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)\right)' = \left(\alpha+\beta+p-\gamma+1\right)F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right) - \left(\alpha+\beta-\gamma+1\right)F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right). \tag{1.8}$$

The primary aim of this study is to establish adequate situations for a specific normalized analytic function to fulfill:

$$q_1\left(z\right) \prec \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma} \prec q_2\left(z\right)$$

and

$$q_1\left(z\right) \prec \left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^p}\right]^{\eta} \prec q_2\left(z\right),$$

when q_1 , q_2 provided multivalent functions within $\mathfrak{A}$, also $q_1\left(0\right)=q_2\left(0\right)=1$.

This paper presents a derivation of several sandwich theorems that include the operator $F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)$.

2. Preliminaries

The definitions as well as lemmas classified below are necessary to establish our conclusions.

Definition 2.1. ([20]) Setting Q a collection of every functions q, which are both analytic as well injective over $\bar{\mathfrak{A}}\setminus E(q)$, when $\bar{\mathfrak{A}}=\mathfrak{A}\cup\{z\in\partial\mathfrak{A}\}$, with

$$E\left(q\right)=\left\{\varepsilon\in\partial\mathfrak{A}:\lim_{z\to\varepsilon}q\left(z\right)=\infty\right\},$$

in a manner witch $q'(z)\neq 0$ when $\varepsilon\in\partial\mathfrak{A}\setminus E\left(q\right)$. Additionally, assume us represent the subfamils of Q in which $q(0)$ as $Q(a)$, with $Q(0)=Q_0$, $Q\left(1\right)=Q_1=\{q\in Q:q\left(0\right)=1\}$.

Lemma 2.2. ([14]) *Letting $q(z)$ be convex as well univalent functions within $\mathfrak{A}$, assume that $\alpha\in\mathfrak{C}$, $\beta\in\mathfrak{C}\setminus\{0\}$ through*

$$\mathfrak{R}\left\{1+\frac{zq''\left(z\right)}{q'\left(z\right)}\right\}>\max\left\{0,-\mathfrak{R}\left(\frac{\alpha}{\beta}\right)\right\}. \tag{2.1}$$

If p is analytic within $\mathfrak{A}$, with

$$\alpha p\left(z\right)+\beta zp'\left(z\right)\prec\alpha q\left(z\right)+\beta zq'\left(z\right), \tag{2.2}$$

then, $p\left(z\right)\prec q\left(z\right)$ with q is the best dominant for (2.2).

Lemma 2.3. ([6]) *Consider $q(z)$ as a univalent function within $\mathfrak{A}$, assume that θ with ϕ is analytic within a dominant $\mathfrak{O}$ that includes $q(\mathfrak{A})$ also $\theta(w) \neq 0$, as well $w \in q(\mathfrak{A})$. Setting $Q(z) = zq'(z)\theta(q(z))$ as well $\mathfrak{Y}(z) = \phi(q(z)) + Q(z)$. Assume as*

(1) *$Q(z)$ is star like univalent within $\mathfrak{A}$,*
(2) *$\Re\left\{\frac{z\mathfrak{Y}'(z)}{Q(z)}\right\} > 0$, regarding $z \in \mathfrak{A}$.*

If p is analytic function within $\mathfrak{A}$, also $p(0) = q(0), p(\mathfrak{A}) \subseteq \mathfrak{O}$ as well

$$\phi(p(z)) + zp'(z)\theta(p(z)) \prec \phi(q(z)) + zq'(z)\theta(q(z)), \tag{2.3}$$

then $p \prec q$ as well q is the best dominant to (2.3).

Lemma 2.4. ([21]) *Letting $q(z)$ is a convex univalent within $\mathfrak{A}$ also $q(0) = 1$. Assume $\beta \in \mathfrak{C}$, which $\Re(\beta) > 0$. If $p(z) \in \mathfrak{M}[1,1] \cap Q$ with $p(z) + \beta zp'(z)$ is univalent within $\mathfrak{A}$, then*

$$q(z) + \beta zq'(z) \prec p(z) + \beta zp'(z), \tag{2.4}$$

it indicates $q(z) \prec p(z)$ with $q(z)$ is the best subordinant of (2.4).

Lemma 2.5. ([14]) *Consider $q(z)$ as univalent functions with convex defined within $\mathfrak{A}$, assume that θ as well ϕ is analytic within a domain $\mathfrak{O}$ that includes $q(\mathfrak{A})$. Say that*

(1) *$\Re\left\{\frac{\theta'(q(z)}{\phi(q(z))}\right\} > 0$, regarding $z \in \mathfrak{A}$,*
(2) *$Q(z) = zq'(z)\phi(q(z))$ is starlike univalent within $\mathfrak{A}$.*

If $p \in \mathfrak{M}[1,1] \cap Q$, as well $p(\mathfrak{A}) \subset \mathfrak{O}, \theta(p(z)) + zp'(z)\phi(p(z))$ denoted univalent within $\mathfrak{A}$ with

$$\theta(q(z)) + zq'(z)\phi(q(z)) \prec (p(z))\theta + zp'(z)\phi(p(z)), \tag{2.5}$$

then $q \prec p$ as well q denoted the best subordinant to (2.5).

3. Differential subordination results

We introduce several differential subordination findings can be obtained by employing the Hadamard product operator $F^{\alpha}_{\delta,c,p,\gamma,\beta}$.

Theorem 3.1. *Consider $q(z)$ as a univalent convex functions that exists within $\mathfrak{A}$, also $q(0) = 1$, $\varepsilon \in \mathfrak{C}^*$, $\sigma > 0$. Letting q which fulfills:*

$$\Re\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\Re\left(\frac{\sigma}{\varepsilon}\right)\right\}. \tag{3.1}$$

If $\mathfrak{H} \in A_p$ fulfills the subordination

$$\varepsilon\left(\alpha+\beta+p-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right) + \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma} \prec\; q\left(z\right)+\frac{\varepsilon}{\sigma}zq^{'}\left(z\right), \tag{3.2}$$

then

$$\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma} \prec q\left(z\right), \tag{3.3}$$

where q is the best dominant to (3.2)*.*

Proof. Given $r(z)$ is defined as:

$$r\left(z\right)=\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma}, \tag{3.4}$$

therefore, the function $r(z)$ exhibits analytic within $\mathfrak{A}$, also $r(0) = 1$. Consequently, by having the derivative of Eq. (3.4) with respect to z with putting this resulting equation into identity (1.8), that we've

$$\frac{zr'\left(z\right)}{r(z)}=\sigma\left[\frac{z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)\right)'}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-p\right], \tag{3.5}$$

thus

$$\frac{zr'\left(z\right)}{r(z)}=\sigma\left[\left(\alpha+\beta+p-\gamma+1\right)\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right)\right],$$

so,

$$\frac{zr'\left(z\right)}{\sigma}=\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma}\left[\left(\alpha+\beta+p-\gamma+1\right)\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right)\right].$$

The hypothesis is transformed into a subordinate term (3.2):

$$r\left(z\right)+\frac{\varepsilon}{\sigma}zr^{'}\left(z\right)\prec q\left(z\right)+\frac{\varepsilon}{\sigma}\varepsilon zq^{'}\left(z\right).$$

By applying the Lemma 2.2 for $\beta=\frac{\varepsilon}{\sigma}$ as well $\alpha\;=1$, we find (3.3). The proof is complete. □

By substituting $q\left(z\right)=\frac{1+z}{1-z}$ into theorem 3.1, it's derive the subsequent conclusion.

Corollary 3.2. *Letting* $\varepsilon \in \mathfrak{C}^*$, $\sigma > 0$ *with*

$$\mathfrak{R}\left\{1+\frac{2z}{1-z}\right\} > max\left\{0, -\mathfrak{R}\left(\frac{\sigma}{\varepsilon}\right)\right\}.$$

If it fulfills the subordination

$$\varepsilon\left(\alpha+\beta-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma}\left(\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right) + \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma} \prec \left(\frac{1-z^2+2\frac{\varepsilon}{\sigma}z}{\left(1-z\right)^2}\right),$$

then

$$\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^p}\right]^{\sigma} \prec \left(\frac{1+z}{1-z}\right),$$

where $q\left(z\right)=\left(\frac{1+z}{1-z}\right)$ *is the best dominant.*

Theorem 3.3. *Letting the function* $q(z)$*, which is both convex and univalent within* $\mathfrak{A}$*, also* $q\left(0\right)=1, q'\left(z\right)\neq 0,\ \left(z\in\mathfrak{A}\right)$*. Suppose it* $q(z)$ *fulfills the given condition:*

$$\mathfrak{R}\left\{1+\frac{\psi}{\tau}q\left(z\right)+\frac{2\mu}{\tau}q^2\left(z\right)+\frac{zq''\left(z\right)}{q\left(z\right)}-\frac{zq'\left(z\right)}{q\left(z\right)}\right\}>0. \tag{3.6}$$

Assume that $q(z)$ *is starlike as well univalent within* $\mathfrak{A}$*. Additionally, we consider that* $t_1, t_2, \psi, \mu, \tau \in \mathfrak{C}^* = \mathfrak{C}\setminus\{0\}$*, with* $t_1+t_2\neq 0$*,*

$$\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^p}\neq 0,\ z\in\mathfrak{A}.$$

If $\mathfrak{H}\in A_p$ *fulfills*

$$G\left(z\right)\prec 1+\psi q\left(z\right)+\mu q^2\left(z\right)+\tau\frac{zq'\left(z\right)}{q\left(z\right)}, \tag{3.7}$$

which

$$\begin{aligned} G\left(z\right)=1&+\left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^p}\right]^{\eta}\\ &+\left(\psi+\mu\left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^p}\right]^{\eta}\right)\\ &+\tau\eta\left[\frac{t_1z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)\right)'+t_2z\left(F^{\alpha}_{\delta,c,\gamma,\beta}\mathfrak{H}\left(z\right)\right)'}{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-p\right], \end{aligned} \tag{3.8}$$

then

$$\left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{(t_1+t_2)\, z^p}\right]^{\eta} \prec (z)\,, \tag{3.9}$$

where $q(z)$ *be the best dominant of* (3.7)*.*

Proof. Assuming $r(z)$ is written as follows:

$$r(z) = \left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{(t_1+t_2)\, z^p}\right]^{\eta}. \tag{3.10}$$

Then, the function $r(z)$ will be analytic within $\mathfrak{A}$ as well $r(0) = 1$, differentiating (3.10) with respect to z, applying our identities (1.8), we acquire

$$\frac{zr'(z)}{r(z)} = \eta\left[\frac{t_1 z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)\right)' + t_2 z\left(F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)\right)'}{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)} - p\right].$$

By establishing $\theta(w) = 1 + \psi w + \mu w^2$ with $\phi(w) = \frac{\tau}{w}$, $w \neq 0$. It's clear that $\theta(w)$, also $\phi(w)$ are analytic within $\mathfrak{C}, \mathfrak{C}\setminus\{0\}$, respectively. As well $\phi(w) \neq 0, w \in \mathfrak{C}\setminus\{0\}$. Furthermore, it's acquire

$$Q(z) = \mathrm{z}\mathrm{q}'(z)\,\phi(q(z)) = \tau z\frac{q'(z)}{q(z)},$$

with

$$\mathfrak{Y}(z) = \theta(q(z)) + Q(z) = 1 + \psi q(z) + \mu q^2(z) + \tau\frac{zq'(z)}{q(z)}.$$

Evidently, $Q(z)$ is starlike univalent within $\mathfrak{A}$,

$$\mathfrak{R}\left\{\frac{z\mathfrak{Y}'(z)}{Q(z)}\right\} = \mathfrak{R}\left\{1 + \frac{\psi}{\tau}q(z) + \frac{2\mu}{\tau}q^2(z) + \frac{zq''(z)}{q(z)} - \frac{zq'(z)}{q(z)}\right\} > 0.$$

Through a simple calculation, we derive

$$G(z) = \psi r(z) + \mu r^2(z) + \tau\frac{zr'(z)}{r(z)} + 1. \tag{3.11}$$

By utilising Eq. (3.8), that we get

$$1 + \psi r(z) + \mu r^2(z) + \tau\frac{zr'(z)}{r(z)} \prec 1 + \psi q(z) + \mu q^2(z) + \tau\frac{zq'(z)}{q(z)}. \tag{3.12}$$

Hence, according to Lemma 2.3, which we obtain $r(z) \prec q(z)$. Applying Eq. (3.8), that we derive the outcome. Thus, the proof has been complete. □

Setting $q(z) = \left(\frac{1+Az}{1+Bz}\right)$, as well $(-1 \le B < A \le 1)$, within Theorem 3.3, the conclusion next is as follows:

Corollary 3.4. *Letting* $-1 \leq B < A \leq 1$ *with*

$$\mathfrak{R}\left\{1+\frac{\psi}{\tau}\left(\frac{1+Az}{1+Bz}\right)+\frac{2\mu}{\tau}\left(\frac{1+Az}{1+Bz}\right)^{2}+\frac{2Bz}{1+Bz}+\frac{(A-B)\,z}{(1+Bz)\,(1+Az)}\right\}>0,$$

were $\psi, \mu \in \mathfrak{C}, \tau \in \mathfrak{C}^{*} = \mathfrak{C} \setminus \{0\}$, *and* $z \in \mathfrak{A}$, *if* $\mathfrak{H} \in A_p$ *fulfils*

$$G\,(z) \prec 1+\psi\left(\frac{1+Az}{1+Bz}\right)+\mu\left(\frac{1+Az}{1+Bz}\right)^{2}+\tau\frac{(A-B)\,z}{(1+Bz)\,(1+Az)},$$

where $G(z)$ *stated as Eq.* (3.8), *then*

$$\left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\,(z)+t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\,(z)}{(t_1+t_2)\,z^{p}}\right]^{\eta} \prec \left(\frac{1+Az}{1+Bz}\right),$$

and $q\,(z) = \left(\frac{1+Az}{1+Bz}\right)$ *is the best dominant.*

Setting $q\,(z) = \left(\frac{1+z}{1-z}\right)^{\omega}$, as well $(-1 \leq \omega \leq 1)$ within Theorem 3.3, the conclusion next is as follows:

Corollary 3.5. *Letting* $-1 \leq \omega \leq 1$ *with*

$$\mathfrak{R}\left\{1+\frac{\psi}{\tau}\left(\frac{1+z}{1-z}\right)^{\omega}+\frac{2\mu}{\tau}\left(\frac{1+z}{1-z}\right)^{2\omega}+\frac{2\omega z}{1+z^{2}}+\frac{2z^{2}}{1+z^{2}}\right\}>0,$$

where $\psi, \mu \in \mathfrak{C},\ \tau \in \mathfrak{C}^{*} = \mathfrak{C} \setminus \{0\}$, *also* $z \in U$, *if* $\mathfrak{H} \in A_p$ *fulfills*

$$G\,(z) \prec 1+\psi\left(\frac{1+z}{1-z}\right)^{\omega}+\mu\left(\frac{1+z}{1-z}\right)^{2\omega}+\tau\frac{2z^{2}}{1+z^{2}},$$

where $G(z)$ *defined in* (3.8), *then*

$$\left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\,(z)+t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\,(z)}{(t_1+t_2)\,z^{p}}\right]^{\eta} \prec \left(\frac{1+z}{1-z}\right)^{\omega},$$

and $q\,(z) = \left(\frac{1+z}{1-z}\right)^{\omega}$ *is the best dominant.*

4. Differential superordination results

We examine many differential superordination outcomes utilizing the new Hadamard product operator $F^{\alpha+1}_{\delta,c,p,\gamma,\beta} f\,(z)$.

Theorem 4.1. *Consider* $q(z)$ *as a univalent function also convex within* $\mathfrak{A}$, *also* $q\,(0) = 1,\ \sigma > 0$ *with* $\mathfrak{R}\,\{\varepsilon\} > 0$. *Let* $\mathfrak{H} \in A_p$ *fulfills*

$$\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\,(z)}{z^{p}}\right]^{\sigma} \in \mathfrak{M}\,[q\,(0)\,,1] \cap Q,$$

and

$$\varepsilon\left(\alpha+\beta+p-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right)+\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma},$$

denote univalent within $\mathfrak{A}$. *If*

$$\begin{aligned} q\left(z\right)+\frac{\varepsilon}{\sigma}zq^{'}\left(z\right)\prec &\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\\ &+\varepsilon\left(\alpha+\beta+p-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right), \end{aligned} \tag{4.1}$$

then

$$q(z)\prec\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma} \tag{4.2}$$

and $q(z)$ *is the best subordinant of* (4.1).

Proof. Letting $r(z)$ is written as

$$r\left(z\right)=\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}, \tag{4.3}$$

taking the derivative of (4.3) with respect to z, which we acquire

$$\frac{zr'\left(z\right)}{r(z)}=\sigma\left[\frac{z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)\right)'}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-p\right]. \tag{4.4}$$

By performing calculations and utilizing Eq. (1.8) form (4.4), we get

$$\begin{aligned} \varepsilon\left(\alpha+\beta-\gamma+1\right)&\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right)\\ &+\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}=r\left(z\right)+\frac{\varepsilon}{\sigma}zr'\left(z\right), \end{aligned}$$

applying Lemma 2.4, we achieve the required outcome. This complete the proof. □

Setting $q\left(z\right)=\left(\frac{1+z}{1-z}\right)$ within Theorem 4.1, it get the next outcome:

Corollary 4.2. *Letting* $\sigma>0$ *with* $\mathfrak{R}\left\{\varepsilon\right\}>0$. *Assume* $\mathfrak{H}\in A_{p}$ *fulfill*

$$\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\in\mathfrak{M}\left[q\left(0\right),1\right]\cap Q,$$

and

$$\varepsilon\left(\alpha+\beta-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}-1\right)+\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma},$$

denote univalent in $\mathfrak{A}$. *If*

$$\left(\frac{1-z^{2}+2\frac{\varepsilon}{\sigma}z}{\left(1-z\right)^{2}}\right)\prec\varepsilon\left(\alpha+\beta+p-\gamma+1\right)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f\left(z\right)}{z^{p}}\right]^{\sigma}\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}f\left(z\right)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f\left(z\right)}-1\right)+\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f\left(z\right)}{z^{p}}\right]^{\sigma},$$

then

$$\left(\frac{1+z}{1-z}\right)\prec\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{z^{p}}\right]^{\sigma}$$

and $q\left(z\right)=\left(\frac{1+z}{1-z}\right)$ *is the best subordinant.*

Theorem 4.3. *Consider* $q(z)$ *as a convex univalent function within* $\mathfrak{A}$, *also* $q\left(0\right)=1$, $q'\left(z\right)\neq0$ *for each* $z\in\mathfrak{A}$, $t_1,t_2,\psi,\mu,\tau\in\mathfrak{C}^{*}=\mathfrak{C}\setminus\{0\},t_1+t_2\neq0$. *For* $\mathfrak{H}\in A_p$, *assuming that*

$$\mathfrak{R}\left\{\frac{\psi}{\tau}q\left(z\right)q'\left(z\right)+\frac{2\mu}{\tau}q^{2}\left(z\right)q'(z)\right\}>0,\ \textit{where}\ \left(z\in U\right). \tag{4.5}$$

If

$$0\neq\left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^{p}}\right]^{\eta}\in\mathfrak{M}\left[1,1\right]\cap\mathcal{Q},$$

and the function $G(z)$, *established in Eq.* (3.8) *is univalent within* $\mathfrak{A}$, *also*

$$1+\psi q\left(z\right)+\mu q^{2}\left(z\right)+\tau\frac{zq'\left(z\right)}{q\left(z\right)}\prec G\left(z\right), \tag{4.6}$$

then

$$q\left(z\right)\prec\left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^{p}}\right]^{\eta} \tag{4.7}$$

and $q(z)$ *is the best subordinant of* (4.6).

Proof. Suppose $r(z)$ denoted:

$$r\left(z\right)=\left[\frac{t_1F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)+t_2F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}\left(z\right)}{\left(t_1+t_2\right)z^{p}}\right]^{\eta}. \tag{4.8}$$

Calculating a derivative of (4.8) with respect to z, we obtain

$$\frac{zr'(z)}{r(z)} = \eta\left[\frac{t_1 z\left(F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)\right)' + t_2 z\left(F^{\alpha}_{\delta,c,\gamma,\beta}\mathfrak{H}(z)\right)'}{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)} - p\right].$$

Establishing $\theta(w) = 1 + \psi w + \mu w^2$ with $\phi(w) = \frac{\tau}{w}, w \neq 0$, it is evident $\theta(w)$, also $\phi(w)$ denote analytic within $\mathfrak{C}, \mathfrak{C}\setminus\{0\}$, respectively. As well $\phi(w) \neq 0, w \in \mathfrak{C}\setminus\{0\}$. Additionally, it is acquire

$$\mathcal{Q}(z) = zq'(z)\ \ \phi(q(z)) = \tau z\frac{q'(z)}{q(z)}.$$

$\mathcal{Q}(z)$ is evidently a starlike univalent function within $\mathfrak{A}$

$$\mathfrak{R}\left\{\frac{\theta'(q(z))}{\phi(q(z))}\right\} = \mathfrak{R}\left\{\frac{\psi}{\tau}q(z)q'(z) + \frac{2\mu}{\tau}q^2(z)q'(z)\right\} > 0.$$

With a simple calculation, we derive

$$G(z) = \psi r(z) + \mu r^2(z) + \tau z\frac{r'(z)}{r(z)} + 1, \tag{4.9}$$

where $G(z)$ is defined by Eq. (3.8). Utilizing equations (4.6) as well as (4.9), we can conclude that

$$1 + \psi q(z) + \mu q^2(z) + \tau z\frac{q'(z)}{q(z)} \prec 1 + \psi r(z) + \mu r^2(z) + \tau z\frac{r'(z)}{r(z)}.$$

Thus, according to Lemma 2.5, that we acquire $q(z) \prec r(z)$, and q is the best subordinant. □

5. Sandwich results

By comparing Theorem 3.1 as well as Theorem 4.1, that we acquire the subsequent sandwich conclusion:

Theorem 5.1. *Consider q_1 as well q_2 as convex univalent functions within $\mathfrak{A}$ and $q_1(0) = q_2(0) = 1, \sigma > 0$ with $\mathfrak{R}\{\varepsilon\} > 0$, $\varepsilon \in \mathfrak{C}\setminus\{0\}$, where q_2 satisfies Theorem 3.1 and q_1 satisfies Theorem 4.1. Let $\mathfrak{H} \in A_p$ satisfies*

$$\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{z^p}\right]^\sigma \in \mathfrak{M}[1,1] \cap Q,$$

with

$$\varepsilon(\alpha+\beta+p-\gamma+1)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f(z)}{z^p}\right]^\sigma\left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}f(z)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f(z)} - 1\right) + \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}f(z)}{z^p}\right]^\sigma,$$

represent univalent within $\mathfrak{A}$*. If*

$$q_1(z) + \frac{\varepsilon}{\sigma} z q_1'(z) \prec \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{z^p}\right]^\sigma + \varepsilon(\alpha+\beta+p-\gamma+1)\left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{z^p}\right]^\sigma \left(\frac{F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)} - 1\right) \prec q_2(z) + \frac{\varepsilon}{\sigma} z q_2'(z),$$

then

$$q_1(z) \prec \left[\frac{F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{z^p}\right]^\sigma \prec q_2(z)$$

and q_1 *as well* q_2 *represent the best subordinant and dominant, respectively.*

Theorem 5.2. *Consider* q_1 *as well* q_2 *as convex univalent functions inside* $\mathfrak{A}$ *with* $q_1(0) = q_2(0) = 1$*. Assume that* q_1 *fulfill* (4.5) *and also* q_2 *fulfill* (3.6)*. Let* $\mathfrak{H} \in A_p$ *fulfill*

$$0 \neq \left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{(t_1+t_2) z^p}\right]^\eta \in \mathfrak{M}[1,1] \cap Q.$$

Furthermore, $G(z)$ *is a univalent function within* $\mathfrak{A}$*, according to by Eq.* (3.8)*. If*

$$1 + \psi q_1(z) + \mu {q_1}^2(z) + \tau z \frac{{q_1}'(z)}{q_1(z)} \prec G(z) \prec 1 + \psi q_2(z) + \mu {q_2}^2(z) + \tau z \frac{{q_2}'(z)}{q_2(z)},$$

then

$$q_1(z) \prec \left[\frac{t_1 F^{\alpha+1}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z) + t_2 F^{\alpha}_{\delta,c,p,\gamma,\beta}\mathfrak{H}(z)}{(t_1+t_2) z^p}\right]^\eta \prec q_2(z)$$

and q_1 *as well* q_2 *represent the best subordinant and dominant, respectively.*

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