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IMPLICIT MANN TYPE ITERATION METHOD INVOLVING TWO STRICTLY HEMICONTRACTIVE MAPPINGS IN BANACH SPACES

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Abstract. The purpose of this paper is to prove that the modified implicit Mann iteration process can be applied to approximate the common fixed point of two strictly hemicontractive mappings in smooth Banach spaces.

1. Introduction

Let K be a nonempty subset of an arbitrary Banach space X and X^* be its dual space. The symbols $D(T)$, $R(T)$ and $F(T)$ stand for the domain, the range and the set of fixed points of T (for a single-valued map $T : X \to X$, $x \in X$ is called a fixed point of T if $T(x) = x$). We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\},$$

where $\langle ., . \rangle$ denotes the duality pairing. In a smooth Banach space J is single-valued (and denoted by j).

Remark 1.1. 1. X is called uniformly smooth if X^* is uniformly convex.

2. J is uniformly continuous on bounded subsets of X in a uniformly convex Banach spaces.

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Let T be a self-mapping of K.

Definition 1.2. The mapping T is called *Lipshitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \leqslant L \|x - y\|,$$

for all $x, y \in K$. If $L = 1$, then T is called *nonexpansive* and if $0 \leqslant L < 1, T$ is called *contraction.*

Definition 1.3. ([3, 5])

(1) The mapping T is said to be *pseudocontractive* if the inequality

$$\|x - y\| \leqslant \| x - y + t((I - T)x - (I - T)y \|, \tag{1.1}$$

holds for each $x, y \in K$ and for all $t > 0$.

(2) T is said to be *strongly pseudocontractive* if there exists a $t > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rt(Tx - Ty)\| \tag{1.2}$$

for all $x, y \in D(T)$ and $r > 0$.

(3) T is said to be *local strongly pseudocontractive* if for each $x \in D(T)$ there exists a $t_x > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rt_x(Tx - Ty)\| \tag{1.3}$$

for all $y \in D(T)$ and $r > 0$.

(4) T is said to be *strictly hemicontractive* if $F(T) \neq \varphi$ and if there exists a $t > 1$ such that

$$\|x - q\| \leq \|(1 + r)(x - q) - rt(Tx - q)\| \tag{1.4}$$

for all $x \in D(T), q \in F(T)$ and $r > 0$.

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive.

Chidume [3] established that the Mann iteration sequence converges strongly to the unique fixed point of T in case T is a Lipschitz strongly pseudocontractive mapping from a bounded closed convex subset of L_p (*or* l_p) into itself. Schu [14] generalized the result in [3] to both uniformly continuous strongly pseudo-contractive mappings and real smooth Banach spaces. Park [11] extended the result in [3] to both strongly pseudocontractive mappings and certain smooth Banach spaces. Rhoades [12] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear mappings. Afterwards, several generalizations have been made in various directions (see for example [4, 7-8, 10-11, 15]).

In 2001, Xu and Ori [16] introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_i : i \in I\}$ (here $I = \{1, 2, \ldots, N\}$), with $\{\alpha_n\}$ a real sequence in $(0, 1)$, and an initial point $x_0 \in K$:

$$\begin{aligned} x_1 &= (1-\alpha_1)x_0 + \alpha_1 T_1 x_1, \\ x_2 &= (1-\alpha_2)x_1 + \alpha_2 T_2 x_2, \\ &\vdots \\ x_N &= (1-\alpha_N)x_{N-1} + \alpha_N T_N x_N, \\ x_{N+1} &= (1-\alpha_{N+1})x_N + \alpha_{N+1} T_{N+1} x_{N+1}, \\ &\vdots \end{aligned}$$

which can be written in the following compact form:

$$x_n = (1-\alpha_n)x_{n-1} + \alpha_n T_n x_n, \text{ for all } n \geq 1, \tag{XO}$$

where $T_n = T_{n\,(mod\,N)}$ (here the $mod\,N$ function takes values in I). Xu and Ori [16] proved the weak convergence of this process to a common fixed point of the finite family defined in a Hilbert space. They further remarked that it is yet unclear what assumptions on the mappings and/or the parameters $\{\alpha_n\}$ are sufficient to guarantee the strong convergence of the sequence $\{x_n\}$.

In [10], Oslilike proved the following results.

Theorem 1.4. *Let E be a real Banach space and K be a nonempty closed convex subset of E. Let $\{T_i : i \in I\}$ be N strictly pseudocontractive self-mappings of K with $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. Let $\{\alpha_n\}_{n=1}^{\infty}$ be a real sequence satisfying the conditions:*

(i) $0 < \alpha_n < 1$,

(ii) $\sum_{n=1}^{\infty}(1-\alpha_n) = \infty$,

(iii) $\sum_{n=1}^{\infty}(1-\alpha_n)^2 < \infty$.

From arbitrary $x_0 \in K$, define the sequence $\{x_n\}$ by the implicit iteration process (XO)*. Then $\{x_n\}$ converges strongly to a common fixed point of the mappings $\{T_i : i \in I\}$ if and only if $\lim_{n\to\infty} \inf d(x_n, F) = 0$.*

Remark 1.5. One can easily see that for $\alpha_n = 1 - \frac{1}{n^{\frac{1}{2}}}$, $\sum(1-\alpha_n)^2 = \infty$. Hence the results of Osilike [10] are needed to be improve.

Let K be a nonempty closed bounded convex subset of an arbitrary smooth Banach space X and $T, S : K \to K$ be two continuous strictly hemicontractive mappings. We proved that the implicit Mann type iteration method converges strongly to the common fixed point of T and S. The results presented in this paper extend and improve the corresponding results particularly in [3-4, 8-11, 14-15].

2. Preliminaries

We need the following results.

Lemma 2.1. ([11]) *Let X be a smooth Banach space. Suppose one of the following holds:*

1. *J is uniformly continuous on any bounded subsets of X,*
2. *$\langle x-y, j(x)-j(y)\rangle \leq \|x-y\|^2$, for all x, y in X,*
3. *for any bounded subset D of X, there is a $c : [0,\infty) \to [0,\infty)$ such that*
$$\operatorname{Re}\langle x-y,\ j(x)-j(y)\rangle \leq c(\|x-y\|),$$
for all $x, y \in D$, where c satisfies $\lim_{t\to 0^+} \frac{c(t)}{t} = 0$.

Then for any $\epsilon > 0$ and any bounded subset K, there exists $\delta > 0$ such that

$$\|sx+(1-s)y\|^2 \leq (1-2s)\|y\|^2 + 2s \operatorname{Re}\langle x,\ j(y)\rangle + 2s\epsilon \tag{2.1}$$

for all x, $y \in K$ and $s \in [0,\delta]$.

Remark 2.2. 1. If X is uniformly smooth, then (1) in Lemma 2.1 holds.
2. If X is a Hilbert space, then (2) in Lemma 2.1 holds.

Lemma 2.3. ([4]) *Let $T : D(T) \subseteq X \to X$ be an operator with $F(T) \neq \varphi$. Then T is strictly hemicontractive if and only if there exists $t > 1$ such that for all $x \in D(T)$ and $q \in F(T)$, there exists $j(x-q) \in J(x-q)$ satisfying*

$$\operatorname{Re}\langle x-Tx,\ j(x-q)\rangle \geq \left(1-\frac{1}{t}\right)\|x-q\|^2. \tag{2.2}$$

Lemma 2.4. ([8]) *Let X be an arbitrary normed linear space and $T : D(T) \subseteq X \to X$ be an operator.*

1. *If T is a local strongly pseudocontractive operator and $F(T) \neq \varphi$, then $F(T)$ is a singleton and T is strictly hemicontractive.*
2. *If T is strictly hemicontractive, then $F(T)$ is a singleton.*

Lemma 2.5. ([8]) *Let* $\{\theta_n\}_{n=0}^{\infty}$, $\{\xi_n\}_{n=0}^{\infty}$ *and* $\{\gamma_n\}_{n=0}^{\infty}$ *be nonnegative real sequences and let* $\epsilon' > 0$ *be a constant satisfying*

$$\xi_{n+1} \leq (1-\theta_n)\xi_n + \epsilon'\theta_n + \gamma_n,\ n \geq 0,$$

where $\sum\limits_{n=0}^{\infty} \theta_n = \infty$, $\theta_n \leq 1$ *for all* $n \geq 0$ *and* $\sum\limits_{n=0}^{\infty} \gamma_n < \infty$. *Then,*

$$\lim_{n\to\infty} \sup \xi_n \leq \epsilon'.$$

3. Main Results

We now prove our main results.

Lemma 3.1. *Let* X *be a smooth Banach space. Suppose one of the following holds:*

(1) J *is uniformly continuous on any bounded subsets of* X,
(2) $\langle x-y, j(x)-j(y)\rangle \leq \|x-y\|^2$, *for all* x, y *in* X,
(3) *for any bounded subset* D *of* X, *there is a* $c : [0,\infty) \to [0,\infty)$ *such that*

$$\mathrm{Re}\,\langle x-y, j(x)-j(y)\rangle \leq c(\|x-y\|),$$

for all $x, y \in D$, *where* c *satisfies* $\lim\limits_{t\to 0^+} \frac{c(t)}{t} = 0$.

Then for any $\epsilon > 0$ *and any bounded subset* K, *there exists* $\delta > 0$ *such that*

$$\left\|\sum_{i=1}^{j} \alpha_j x_j\right\|^2 \leq (1-2\alpha_1)\|x_1\|^2 + 2\frac{\alpha_1}{1-\alpha_1}\sum_{l=2}^{j} \alpha_i \mathrm{Re}\,\langle x_l,\ j(x_1)\rangle + 2\epsilon\alpha_1, \quad (3.1)$$

for all $x_i \in K$ *and* $\alpha_i \in [0,\delta]$, $i = 1,2,...,j$ *such that* $\sum\limits_{i=1}^{j} \alpha_j = 1$.

Proof. For $\alpha_i \in [0,\delta]$, $i = 1,2,...,j$ such that $\sum\limits_{i=1}^{j} \alpha_j = 1$ and by using (2.1), then we have

$$\begin{aligned}\left\|\sum_{i=1}^{j} \alpha_j x_j\right\|^2 &= \left\|\alpha_1 x + (1-\alpha_1)\sum_{l=2}^{j} \frac{\alpha_l}{1-\alpha_1} x_l\right\|^2 \\ &\leq (1-2\alpha_1)\|x_1\|^2 + 2\epsilon\alpha_1 + 2\alpha_1 \mathrm{Re}\left\langle \sum_{l=2}^{j} \frac{\alpha_l}{1-\alpha_1} x_l, j(x_1)\right\rangle \\ &= (1-2\alpha_1)\|x_1\|^2 + 2\epsilon\alpha_1 + 2\frac{\alpha_1}{1-\alpha_1}\sum_{l=2}^{j} \alpha_i \mathrm{Re}\,\langle x_l,\ j(x_1)\rangle.\end{aligned}$$

This completes the proof. □

Remark 3.2. 1. If X is uniformly smooth, then (1) in Lemma 3.1 holds.
2. If X is a Hilbert space, then (2) in Lemma 3.1 holds.

Theorem 3.3. *Let X be a smooth Banach space satisfying any of the Axioms (1)-(3) of Lemma 3.1. Let K be a nonempty closed bounded convex subset of X and $T, S : K \to K$ be two continuous strictly hemicontractive mappings. Let $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ be real sequences in $[0,1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ and satisfying conditions*

$$\text{(i)} \ \sum_{n=0}^{\infty} \alpha_n = \infty, \quad \textit{and} \quad \text{(ii)} \ 0 = \lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n.$$

Suppose that $\{x_n\}_{n=0}^{\infty}$ is the sequence generated from an arbitrary $x_0 \in K$ by

$$x_n = \alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n, \ n \geq 1. \tag{3.2}$$

Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the common fixed point q of T and S.

Proof. By [5, Corollary 1], T and S have the unique fixed point q in K. It follows from Lemma 2.4 that $F(T) \cap F(S)$ is a singleton. That is, $F(T) \cap F(S) = \{q\}$ for some $q \in K$. Now for $k = \frac{1}{t}$, where t satisfies (2.2).

Set $M = 1 + diamK$ for all $n \geq 0$. It is easy to verify that

$$M = \sup_{n\geq 1} \|x_n - q\| + \sup_{n\geq 1} \|Tx_n - q\| + \sup_{n\geq 1} \|Sx_n - q\| . \tag{3.3}$$

Consider

$$\begin{aligned} \|x_n - q\|^2 &= \|\alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n - q\|^2 \\ &= \|\alpha_n (x_{n-1} - q) + \beta_n (T x_n - q) + \gamma_n (S x_n - q)\|^2 \\ &\leq \alpha_n \|x_{n-1} - q\|^2 + \beta_n \|T x_n - q\|^2 + \gamma_n \|S x_n - q\|^2 \\ &\leq \|x_{n-1} - q\|^2 + M^2 (\beta_n + \gamma_n), \end{aligned} \tag{3.4}$$

where the first inequality holds by the convexity of $\|.\|^2$.

Using (3.2) and Lemma 3.1, we infer that

$$\begin{aligned} \|x_n - q\|^2 &= \|\alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n - q\|^2 \\ &= \|\alpha_n (x_{n-1} - q) + \beta_n (T x_n - q) + \gamma_n (S x_n - q)\|^2 \\ &\leq (1 - 2\alpha_n) \|x_{n-1} - q\|^2 + 2\frac{\alpha_n \beta_n}{1 - \alpha_n} \text{Re} \langle T x_n - q, j(x_{n-1} - q) \rangle \\ &\quad + 2\frac{\alpha_n \gamma_n}{1 - \alpha_n} \text{Re} \langle S x_n - q, j(x_{n-1} - q) \rangle + 2\epsilon\alpha_n \end{aligned}$$

$$\begin{aligned}
&= (1-2\alpha_n)\|x_{n-1}-q\|^2 + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\mathrm{Re}\,\langle Tx_n-q, j(x_n-q)\rangle \\
&\quad + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\mathrm{Re}\,\langle Tx_n-q, j(x_{n-1}-q)-j(x_n-q)\rangle \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\mathrm{Re}\,\langle Sx_n-q, j(x_n-q)\rangle \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\mathrm{Re}\,\langle Sx_n-q, j(x_{n-1}-q)-j(x_n-q)\rangle + 2\epsilon\alpha_n \\
&\le (1-2\alpha_n)\|x_{n-1}-q\|^2 + 2\frac{\alpha_n\beta_n}{1-\alpha_n}k\|x_n-q\|^2 \\
&\quad + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\|Tx_n-q\|\|j(x_{n-1}-q)-j(x_n-q)\| \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}k\|x_n-q\|^2 \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\|Sx_n-q\|\|j(x_{n-1}-q)-j(x_n-q)\| + 2\epsilon\alpha_n \\
&\le (1-2\alpha_n)\|x_{n-1}-q\|^2 + 2k\alpha_n\|x_n-q\|^2 + 2M\alpha_n\varepsilon_n + 2\epsilon\alpha_n,
\end{aligned} \tag{3.5}$$

where

$$\varepsilon_n = \|j(x_{n-1}-q)-j(x_n-q)\|. \tag{3.6}$$

Since J is uniformly continuous on any bounded subsets of X, we have

$$\begin{aligned}
\|x_{n-1}-x_n\| &= \|x_{n-1}-\alpha_n x_{n-1}-\beta_n Tx_n-\gamma_n Sx_n\| \\
&= \|\beta_n(x_{n-1}-Tx_n)+\gamma_n(x_{n-1}-Sx_n)\| \\
&\le \beta_n\|x_{n-1}-Tx_n\|+\gamma_n\|x_{n-1}-Sx_n\| \\
&\le 2M(\beta_n+\gamma_n) \\
&\to 0,
\end{aligned}$$

as $n\to\infty$, implies

$$\varepsilon_n \to 0 \text{ as } n\to\infty. \tag{3.7}$$

For given any $\epsilon>0$ and the bounded subset K, there exists a $\delta>0$ satisfying (2.1). Note that (3.7) and (ii) ensure that there exists an N such that

$$\beta_n, \gamma_n < \min\left\{\delta, \frac{\epsilon}{8M^2k}\right\}, \tag{3.8}$$

$$\varepsilon_n \le \frac{\epsilon}{4M}, \qquad n\ge N\ .$$

Now substituting (3.4) in (3.5) to obtain

$$\begin{aligned}
\|x_n-q\|^2 &\le (1-2(1-k)\alpha_n)\|x_{n-1}-q\|^2 \\
&\quad + 2M^2k\alpha_n(\beta_n+\gamma_n) + 2M\alpha_n\varepsilon_n + 2\epsilon\alpha_n \\
&\le (1-2(1-k)\alpha_n)\|x_{n-1}-q\|^2 + 3\epsilon\alpha_n,
\end{aligned} \tag{3.9}$$

for all $n \geq N$. Put

$$\begin{aligned} \xi_n &= \|x_{n-1} - q\|, \\ \theta_n &= 2(1-k)\alpha_n, \\ \epsilon' &= \frac{3\epsilon}{2(1-k)}, \\ \gamma_n &= 0, \end{aligned}$$

we have from (3.9)

$$\xi_{n+1} \leq (1-\theta_n)\xi_n + \epsilon'\theta_n + \gamma_n, \qquad n \geq 1.$$

Set $\delta = \frac{1}{2(1-k)}$. Because $\alpha_n \leq \delta$, implies $2(1-k)\alpha_n \leq 1$. Observe that $\sum_{n=0}^{\infty} \theta_n = \infty$, $\theta_n \leq 1$ for all $n \geq 1$. It follows from Lemma 2.5 that

$$\lim_{n\to\infty} \sup \|x_n - q\|^2 \leq \epsilon'.$$

Letting $\epsilon' \to 0^+$, we obtain that $\lim_{n\to\infty} \sup \|x_n - q\|^2 = 0$, which implies that $x_n \to q$ as $n \to \infty$. □

Corollary 3.4. *Let X be a smooth Banach space satisfying any of the Axioms* (1)-(3) *of Lemma* 3.1. *Let K be a nonempty closed bounded convex subset of X and $T, S : K \to K$ be two Lipschitz strictly hemicontractive mappings. Let $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ be real sequences in* $[0,1]$ *such that $\alpha_n + \beta_n + \gamma_n = 1$ and satisfying conditions* (i) $\sum_{n=0}^{\infty} \alpha_n = \infty$, *and* (ii) $0 = \lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n$.

From arbitrary $x_0 \in K$, define the sequence $\{x_n\}$ by the implicit iteration process (3.2). Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the common fixed point q of T and S.

Corollary 3.5. *Let X be a smooth Banach space satisfying any of the Axioms* (1)-(3) *of Lemma* 2.1. *Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Suppose that $\{\alpha_n\}_{n=0}^{\infty}$ be a sequence in* $[0,1]$ *satisfying conditions* (i) $\lim_{n\to\infty} \alpha_n = 0$ *and* (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$.

From arbitrary $x_0 \in K$, define the sequence $\{x_n\}$ by the implicit iteration process (XO). Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to a unique fixed point q of T.

Corollary 3.6. *Let X be a smooth Banach space satisfying any of the Axioms* (1)-(3) *of Lemma* 2.1. *Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a Lipschitz strictly hemicontractive mapping. Suppose* *that* $\{\alpha_n\}_{n=0}^{\infty}$ *be a sequence in* $[0,1]$ *satisfying conditions* (i) $\lim_{n\to\infty}\alpha_n = 0$ *and* (ii) $\sum_{n=0}^{\infty}\alpha_n = \infty$.

From arbitrary $x_0 \in K$, define the sequence $\{x_n\}$ by the implicit iteration process (XO). Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to a unique fixed point q of T.

Remark 3.7. Similar results can be found for the iteration processes involved error terms, we omit the details.

Remark 3.8. Theorem 3.3 and Corollary 3.4 extend and improve the Theorem 1.4 in the following directions:

We do not need the assumption $\lim_{n\to\infty}\inf d(x_n, F) = 0$ as in Theorem 1.4.

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