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REVIEW OF RECENT STUDIES ON THE KKM THEORY, II

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Abstract. In our previous survey [33], we gave a short history of the KKM theory and reviewed its current study by recalling our previous comments or surveys in a sequence of papers. The present survey is a continuation of [33] and to review some recent works on the theory mainly due to other authors. On this occasion, we give some corrections on [27].

1. Introduction

Since we introduced the KKM theory as an independent branch of Nonlinear Analysis in 1992 [23,24], there have appeared more than twelve hundred publications related to the theory. Many of them are concerned with KKM type theorems on particular spaces, their equivalent formulations, and their applications to various problems. Recently the author initiated the KKM theory on abstract convex spaces properly including generalized convex spaces (G-convex spaces for short) due to the author. For details, see [27] and the references therein.

In our previous survey [33], we gave a short history of the KKM theory and reviewed its current study by recalling our previous comments or surveys in a sequence of papers. The present paper is a continuation of [33] and to

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review some recent works on the theory mainly due to other authors. On this occasion, we give some corrections on [27].

Section 2 deals with basic concepts on abstract convex spaces. In Section 3, we recall the contents of [27] with some corrections. Section 4 deals with some recent papers of ourselves concerning comments on other authors' works. In Sections 5, we review recent works of other authors which were concerned with particular researches on the KKM theory.

2. Abstract convex spaces

We follow our recent works [27, 33] and the references therein:

Definition 2.1. An *abstract convex space* $(E, D; \Gamma)$ consists of a topological space E, a nonempty set D, and a multimap $\Gamma : \langle D\rangle \multimap E$ with nonempty values $\Gamma_A := \Gamma(A)$ for $A \in \langle D\rangle$, where $\langle D\rangle$ is the set of all nonempty finite subsets of D.

For any $D' \subset D$, the Γ-*convex hull* of D' is denoted and defined by

$$\mathrm{co}_\Gamma D' := \bigcup\{\Gamma_A \mid A \in \langle D'\rangle\} \subset E.$$

A subset X of E is called a Γ-*convex subset* of $(E, D; \Gamma)$ relative to D' if for any $N \in \langle D'\rangle$, we have $\Gamma_N \subset X$, that is, $\mathrm{co}_\Gamma D' \subset X$.

In case $E = D$, let $(E; \Gamma) := (E, E; \Gamma)$.

Definition 2.2. Let $(E, D; \Gamma)$ be an abstract convex space and Z a topological space. For a multimap $F : E \multimap Z$ with nonempty values, if a multimap $G : D \multimap Z$ satisfies

$$F(\Gamma_A) \subset G(A) := \bigcup_{y\in A} G(y) \qquad \text{for all} \quad A \in \langle D\rangle,$$

then G is called a *KKM map* with respect to F. A *KKM map* $G : D \multimap E$ is a KKM map with respect to the identity map 1_E.

Definition 2.3. The *partial KKM principle* for an abstract convex space $(E, D; \Gamma)$ is the statement that, for any closed-valued KKM map $G : D \multimap E$, the family $\{G(y)\}_{y\in D}$ has the finite intersection property. The *KKM principle* is the statement that the same property also holds for any open-valued KKM map.

An abstract convex space is called a (*partial*) *KKM space* if it satisfies the (partial) KKM principle, resp.

Recently, we were concerned with a particular subclass of abstract convex spaces as follows:

Definition 2.4. A *space having a family* $\{\phi_A\}_{A\in\langle D\rangle}$ or simply a ϕ_A-*space*

$$(X, D; \{\phi_A\}_{A\in\langle D\rangle}) \text{ or simply } (X, D; \phi_A)$$

consists of a topological space X, a nonempty set D, and a family of continuous functions $\phi_A : \Delta_n \to X$ (that is, singular n-simplexes) for $A \in \langle D\rangle$ with the cardinality $|A| = n + 1$.

Now we have the following diagram for triples $(E, D; \Gamma)$:

Simplex $\Longrightarrow$ Convex subset of a t.v.s. $\Longrightarrow$ Convex space $\Longrightarrow$ H-space
$\Longrightarrow$ G-convex space $\Longrightarrow$ ϕ_A-space $\Longrightarrow$ KKM space
$\Longrightarrow$ Partial KKM space $\Longrightarrow$ Abstract convex space.

3. Corrections of our paper in NA 73: 2010 [27]

In this section, we recall the abstract of our previous work [27] and some corrections.

Abstract: In this paper [27], we clearly derive a sequence of a dozen statements which characterize the KKM spaces and several equivalent formulations of the partial KKM principle. As their applications, we add more than a dozen statements including generalized formulations of von Neumann minimax theorem, von Neumann intersection lemma, the Nash equilibrium theorem, and the Fan type minimax inequalities for any KKM spaces. Consequently, this paper unifies and enlarges previously known several proper examples of such statements for particular types of KKM spaces.

Therefore, the whole contents of [27] is applicable to any ϕ_A-space $(X, D; \phi_A)$.

Corrections: We replace the incorrect statements in [27] by the following:

Let $(E, D; \Gamma)$ be a KKM space.

(V) **The Fan-Browder fixed point property.** *Let* $S : E \multimap D$, $T : E \multimap E$ *be maps satisfying*

(5.1) *for each* $x \in E$, $\mathrm{co}_\Gamma S(x) \subset T(x)$;
(5.2) $S^-(z)$ *is open* [*resp., closed*] *for each* $z \in D$; *and*
(5.3) $E = \bigcup_{z\in M} S^-(z)$ *for some* $M \in \langle D\rangle$.

Then T *has a fixed point* $x_0 \in E$; *that is,* $x_0 \in T(x_0)$.

(VI) **Existence of maximal elements.** *Let* $S : E \multimap D$, $T : E \multimap E$ *be maps satisfying*

(6.1) $S^-(z)$ *is open* [*resp., closed*] *for each* $z \in D$;
(6.2) *for each* $x \in E$, $\mathrm{co}_\Gamma S(x) \subset T(x)$; *and*
(6.3) *for each* $x \in E$, $x \notin T(x)$.

Then E *is not covered by a finite number of* $S^-(z)$*s.*

Theorem 4. (Minimax inequality) *Let $(E, D; \Gamma)$ be a partial KKM space, $f : D \times E \to \overline{\mathbb{R}}$, $g : E \times E \to \overline{\mathbb{R}}$ extended real-valued functions, and $\gamma \in \overline{\mathbb{R}}$ such that*

(1) *for each $z \in D$, $\{y \in X \mid f(z, y) \leq \gamma\}$ is closed;*
(2) *for each $y \in E$, $\mathrm{co}_\Gamma\{z \in D \mid f(z, y) > \gamma\} \subset \{x \in E \mid g(x, y) > \gamma\}$; and*
(3) *the map $G : D \multimap E$ defined by $G(z) := \{y \in E \mid f(z, y) \leq \gamma\}$ has a coercivity condition.*

Then

(i) *there exists a $y_0 \in E$ such that $f(z, y_0) \leq \gamma$ for all $z \in D$; and*
(ii) *if $\gamma := \sup_{x \in E} g(x, x)$, then we have*

$$\inf_{y \in E} \sup_{z \in D} f(x, y) \leq \sup_{x \in E} g(x, x).$$

Let $(X; \Gamma)$ be a partial KKM space.

(XVI) **Variational inequality.** *Let $f, g : X \times X \to \mathbb{R}$ be functions satisfying*
(16.1) *for any $x, y \in X$, $f(y, y) - f(x, y) \leq g(y, y) - g(x, y)$;*
(16.2) *for each $x \in X$, $\{y \in X \mid f(x, y) < f(y, y)\}$ is open; and*
(16.3) *for each $y \in X$, $\{x \in X \mid g(x, y) < g(y, y)\}$ is Γ-convex.*

Then

(i) *there exists a $y_0 \in X$ such that*

$$f(x, y_0) \geq f(y_0, y_0) \quad for\ all\ x \in X;\ and$$

(ii) *we have*

$$\sup_{y \in X} \inf_{x \in X} f(x, y) \geq \inf_{x \in X} f(x, x).$$

(XVII) **Variational inequality.** *Let $f, g : X \times X \to \mathbb{R}$ be functions satisfying*
(17.1) *$f \leq g$ on the diagonal $\Delta := \{(x, x) \mid x \in X\}$ and $g \leq f$ on $(X \times X) \setminus \Delta$;*
(17.2) *for each $x \in X$, $y \mapsto f(y, y) - f(x, y)$ is l.s.c. on X; and*
(17.3) *for each $y \in X$, $x \mapsto g(x, y)$ is quasiconcave on X.*

Then there exists a $y_0 \in X$ such that

$$f(y_0, y_0) \geq f(x, y_0) \quad for\ all\ x \in X.$$

The proofs of the above statements can be easily obtained by properly modifying the original proofs. On this occasion, the author would like to express his gratitude to some kind readers who indicated the incorrectness of original forms of the above statements in [27].

4. Our recent reviews commenting other authors' works

In this section, we review our previous works which were concerned with comments on particular researches on the KKM theory. Some of them are closely related to modifications of our G-convex space theory.

(1) CANA 19: 2012 [28]

In 2008, Kulpa and Szymanski [19] introduced a series of theorems called Infimum Principles in simplicial spaces. As applications, they derive fixed point theorems due to Schauder, Tychonoff, Kakutani, and Fan-Browder; minimax theorems; the Nash equilibrium theorem; the Gale-Nikaido-Debreu theorem; and the Ky Fan minimax inequality. Their study is based on and utilizes the techniques of simplicial structure and the Fan-Browder map. In [28], we recall that for any KKM spaces, we can deduce such classical theorems without using any Infimum Principles. Moreover, we note that the newly defined L^*-spaces in [19] are particular types of KKM spaces, and add some remarks on them.

(2) NAF 17: 2012 [29]

In our previous works, we showed that every ϕ_A-space $(X,\ D;\{\phi_A\}_{A\in\langle D\rangle})$ can be made into a G-convex space in several ways. In this paper, we show that a ϕ_A-space can be made into a G-convex space $(X, D;\Gamma)$ iff it has a KKM map $G : D \multimap X$, and that it is a KKM space. Moreover, we show that recent examples of GFC-spaces due to Khanh et al. and Ding are not adequate to their claim that GFC-spaces or FC-spaces properly include G-convex spaces.

(3) NAF 17: 2012 [30]

In [30], we introduce equivalent formulations of the minimax theorem for marginally semicontinuous functions and the KKM theorem for marginally closed-valued multimaps due to Greco and Moschen [13]. We note that many results in [13] are equivalent to the Brouwer fixed point theorem. Moreover, we give more equivalent formulations as in our previous works [25,27] and remarks on some related matters.

(4) NAF 17: 2012 [31]

Recently, by means of variational relation problems, Y. J. Pu and Z. Yang [42] obtained new existence theorems of solutions for generalized KKM theorems, variational inclusion problems, the generalized (vector) Ky Fan minimax inequality, generalized Ky Fan section theorems, n-person noncooperative generalized games and n-person noncooperative multi-objective generalized games.

In [31], based on recent works on the KKM theory, we show that the Hausdorffness and the fixed point property imposed on relevant spaces are redundant in all of the results in [42].

(5) CANA 19: 2012 [32]

In a recent paper of Z. Yang and Y. J. Pu [48], a KKM type theorem without convex hull was proved and applied to some existence theorems of solutions for (vector) Ky Fan minimax inequality, Ky Fan section theorem, variational relation problems, n-person noncooperative game, and n-person noncooperative multiobjective game. In the present paper, we show that the KKM type theorem and all results in [48] are consequences of already known results for ϕ_A-spaces and can be improved by eliminating some redundant restrictions.

(6) NAF 18: 2012 [34]

Recently, Lu and Zhang [22] introduced the concepts of FWC-spaces (short form of finite weakly convex spaces) as a unified form of many known modifications of G-convex spaces, and the better admissible class of multimaps on them. In [34], we showed that their FWC-spaces and their better admissible classes are inadequately defined and that their results can not be true.

(7) NFAA 18: 2013 [35]

In 2012, Colao, Lopez, Marino, and Martin-Marquez [9] developed an equilibrium theory in Hadamard manifolds. In [35], we show that three of their key results (the KKM lemma, the Ky Fan type minimax inequality, and Nash equilibrium theorem) on Hadamard manifolds can be extended to hyperbolic spaces and are particular ones for abstract convex spaces in the sense of Park. Similarly, most of main theorems in the KKM theory on abstract convex spaces can be applied to hyperbolic spaces and Hadamard manifolds.

(8) NAF 18: 2013 [36]

Earlier we found that our ϕ_A-spaces can be made into G-convex spaces in several ways and that GFC-spaces due to Khanh et al. are all ϕ_A-spaces. Recently, they [17] gave an example of a GFC-space which is a 'trivial' G-convex space. In this paper, we show that a GFC-space can be made into a nontrivial G-convex space $(X, D; \Gamma)$ iff it has a nontrivial KKM map $G : D \multimap X$. Consequently, their example has only a trivial KKM map and is not adequate to show that GFC-spaces properly extend G-convex spaces.

(9) NFAA 18: 2013 [37]

Recently, Allmohammady et al. [1, 2, 3], Darzi et al. [10], and Delavar et al. [11] dealt with some results in the KKM theory on generalized convex minimal spaces. By establishing a kind of the KKM principle in these spaces, they obtained some results on coincidence or fixed point theorems and others. The aim in [37] is to show that their results are consequences of corresponding ones for abstract convex minimal spaces in our previous paper [16] and hence, can be extended to more general setting.

(10) AFPT : 2013 [40]

In the KKM theory, G-convex spaces are extended to KKM spaces or abstract convex spaces in 2006. Various types of ϕ_A-spaces $(X, D; \{\phi_A\}_{A\in\langle D\rangle})$ appeared until 2007 can be made into G-convex spaces in several ways. Moreover, various types of generalized KKM maps on ϕ_A-spaces are simply KKM maps on G-convex spaces. Therefore, our G-convex space theory can be applied to various types of ϕ_A-spaces. However, Khanh et al. in 2009 introduced a disguised form of ϕ_A-spaces called GFC-spaces. In the present paper, we review their works on GFC-spaces and clarify that their basic results are consequences of known ones. Finally, further comments on each of seven papers on GFC-spaces are given.

(11) RIMS : 2013 [38]

In [38], firstly, we recall Ky Fan's contributions to the KKM theory based on his celebrated 1961 KKM lemma (or the Fan-KKM theorem). Secondly, we introduce relatively recent applications of the Fan lemma due to other authors in the 21st century. Finally, some historical remarks on related works are added.

(12) PNAO-Asia : 2013 [39]

In [39], firstly, we recall our versions of general KKM type theorems for abstract convex spaces. Secondly, we introduce relatively recent applications of various generalized KKM type theorems due to other authors in the 21st century. Finally, some general comments to improve such applications are added.

(13) JNAO : 2013 [41]

In 2005, Ben-El-Mechaiekh, Chebbi, and Florenzano [5] obtained a generalization of Ky Fan's 1984 KKM theorem on the intersection of a family of closed sets on non-compact convex sets in a topological vector space. They also obtained a Fan-Browder type fixed point theorem to a set-valued maps on non-compact convex sets. In 2011, Chebbi, Gourdel, and Hammami [8] introduced a generalized coercivity type condition for set-valued maps defined on topological spaces endowed with a generalized convex structure and extended Fan's KKM theorem. In this paper, we show that better forms of theorems in [5] and [6, 7, 8] can be deduced from a KKM theorem on abstract convex spaces in Park's sense.

5. Comments on recent works of other authors

(1) Jabarootian and Zafarani 2008 – JOTA 136 [16]

Abstract: Using a generalized Fan's KKM theorem, some existence results for generalized vector variational-like inequalities in noncompact settings are established. Some applications to vector optimization problems are given. The results presented in this paper extend and unify corresponding results of other authors.

Comments: Their generalized KKM theorem (due to Fakhar and Zafarani, J. Optim. Theory Appl. 126 (2005), 109–124) is artificial, not practical, and not general enough.

(2) Amini-Harandi et al. 2009 – NFAA 14(1) [4]

Abstract: We present fixed point theory for condensing multimaps on abstract convex uniform spaces. Also we obtain a nonlinear alternative of Leray-Schauder type for Mönch type maps. Our main results unify and improve some well-known results in the literature.

Comments: Some known results are restated for Park's abstract convex spaces.

(3) Yang et al. 2011 – FPTA [47]

Abstract: We first prove that the product of a family of $L\Gamma$-spaces is also an $L\Gamma$-space. Then, by using a Himmelberg type fixed point theorem in $L\Gamma$-spaces, we establish existence theorems of solutions for systems of generalized quasivariational inclusion problems, systems of variational equations, and systems of generalized quasiequilibrium problems in L-spaces. Applications of the existence theorem of solutions for systems of generalized quasiequilibrium problems to optimization problems are given in $L\Gamma$-spaces.

Comments: The authors of [47] stated that: "More recently, Park introduced the concept of $L\Gamma$-spaces which include LC-spaces and LG-spaces as special cases. He also established a Himmelberg type fixed point theorem in $L\Gamma$-spaces."

(4) Ding 2012 – JOGO 53 [12]

Abstract: In this paper, we introduce and study a class of multi-leader-follower generalized multiobjective games in FC-spaces where the number of leaders and followers may be finite or infinite and the objective functions of leaders and followers get their values in infinite-dimensional spaces. By using a Pareto equilibrium existence theorem of generalized constrained multiobjective games in FC-spaces due to author, some equilibrium existence theorems for the multi-leader-follower generalized multiobjective games are established in noncompact FC-spaces. These results improve and generalize some corresponding results in recent literatures.

Comments: Ding [12] still maintains his incorrect claim that his obsolete FC-spaces contain G-convex spaces.

(5) Du 2012 – AAA [13]

Abstract: We firstly prove some new fixed point theorems for set-valued mappings in noncompact abstract convex space. Next, two existence theorems of maximal elements for class of $\mathcal{A}_{C,\theta}$ mapping and $\mathcal{A}_{C,\theta}$-majorized mapping are obtained. As in applications, we establish new equilibria existence theorems for qualitative games and generalized games. Our theorems improve and generalize the most known results in recent literature.

Comments: In this paper, its author is based on a particular form of a Fan-Browder type fixed point theorem for the case $E \supset D$. Its author made an incorrect claim that his or her result improves the corresponding ones of many authors.

(6) Yang and Huang 2012 – BKMS 49(6) [46]

Abstract: In this paper, a coincidence theorem for a compact $\mathfrak{KC}$-map is proved in an abstract convex space. Several more general coincidence theorems for noncompact $\mathfrak{KC}$-maps are derived in abstract convex spaces. Some examples are given to illustrate our coincidence theorems. As applications, an alternative theorem concerning the existence of maximal elements, an alternative theorem concerning equilibrium problems and a minimax inequality for three functions are proved in abstract convex spaces.

Comments: All results are consequences of Lemma 2.2 (which is a particular form of a KKM theorem due to Park) and hence of our previous results. Moreover, our $\mathfrak{KC}$-maps are denoted $\mathfrak{RC}$ incorrectly.

(7) Wu and Wang 2012 – CANA 19(4) [44]

Abstract: In this paper, some new existence theorems of equilibrium and maximal element for generalized abstract fuzzy economies with uncountable number of agents and qualitative fuzzy games are proved in locally FC-uniform spaces, respectively. As applications, some existence theorems of equilibria for abstract economies are given in locally FC-uniform spaces. The results presented in this paper generalize some known results in the literature.

Comments: This is based on the obsolete FC-spaces.

(8) He and Li 2013 – BKMS 50(1) [15]

Abstract: By using some existence theorems of maximal elements for a family of set-valued mappings involving a better admissible set-valued mapping under noncompact setting of FC-spaces, we present some non-empty intersection theorems for a family $\{G_i\}_{i\in I}$ in product FC-spaces. Then, as applications, some new existence theorems of equilibrium for a system of generalized vector equilibrium problems are proved in product FC-spaces. Our results improve and generalize some recent results.

Comments: Obsolete terms like ccl, cint, and FC-spaces are used to obtain artificial modifications of known results without giving any proper examples or any applications. Park's better admissible multimaps are exploited.

(9) H. Kim 2013 – BKMS 50(1) [18]

Abstract: Topological semilattices with path-connected intervals are special abstract convex spaces. In this paper, we obtain generalized KKM type theorems and their analytic formulations, maximal element theorems and collectively fixed point theorems on abstract convex spaces. We also apply them to topological semilattices with path-connected intervals, and obtain generalized forms of the results of Horvath and Ciscar, Luo, and Al-Homidan et al.

(10) Salahuddin et al. 2013 – TMPA 1(1) [43]

Abstract: In this paper, we establish some existence theorems for a new class of generalized vector quasi-variational inequalities in Banach spaces by using the KKM-Fan Theorem and an equivalent fixed point theorem.

Comments: This paper is based on the 1984 KKM-Fan theorem without assuming Hausdorffness and its equivalent form of the Fan-Browder type fixed point theorem.

(11) Kulpa and Szymanski 2013 – Preprint [20]

Abstract: We discuss S. Park's abstract convex spaces and their relevance to convexities and L^*-operators. We construct an example of a space satisfying the partial KKM principle that is not a KKM space, thus solving a problem by Park. We show that if a compact Hausdorff space admits a 2-continuous L^*-operator, then the space must be locally connected continuum and it has the fixed point property provided the covering dimension is 1. We also show that the unit circle admits no 2-continuous L^*-operators.

Comments: The authors found some partial KKM spaces which are not KKM spaces.

(12) Lu 2013 – JFSA 2013 [21]

Abstract: The main purpose of this paper is to establish a new collectively fixed point theorem in noncompact abstract convex spaces. As applications of this theorem, we obtain some new existence theorems of equilibria for generalized abstract economies in noncompact abstract convex spaces.

Comments: Known results are extended to Park's abstract convex spaces.

(13) Xiang et al. 2013 – FPTA 2013 [45]

Abstract: The purpose of this paper is to give some further results in a type of generalized convexity spaces. First, we prove that an abstract convex space has KKM property if and only if it has a strong Fan-Browder property. Then we introduce an abstract convex structure via an upper semi-continuous multi-valued mapping and establish some generalized versions of KKM lemma. By employing our general KKM lemmas, we derive some generalizations of minimax inequalities, which contain several existing ones as special cases.

Comments: The authors' abstract convexity space is particular to the abstract convex space in the sense of Park. Their weakly convex-valued multimap has a fixed point whenever it has a nonempty value. Hence their new definitions (without giving any proper examples) seem to be incorrect. Moreover, the correct form of their first claim of equivalence was already known for a long time ago.

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