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FIXED POINT RESULTS FOR INTERPOLATIVE HARDY-ROGERS-MEIR-KEELER CONTRACTIONS WITH APPLICATION

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Abstract. In this paper, we introduce a new class of contractions in metric spaces, called interpolative Hardy-Rogers-Meir-Keeler-type contractions. This class generalizes and unifies several well-known fixed point principles by incorporating interpolative techniques and Meir-Keeler-type conditions into the Hardy-Rogers framework. We establish a fixed point theorem for such mappings in complete metric spaces, ensuring the existence and uniqueness of fixed points. To demonstrate the applicability of our results, we apply the developed theorem to prove the existence and uniqueness of solutions to a class of nonlinear Hammerstein integral equations. An illustrative example is provided to support the main results.

1. Introduction

Fixed point theory has long been a cornerstone of nonlinear analysis, offering powerful tools for solving a wide range of mathematical problems in

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differential equations, optimization, dynamic systems, and many other areas. Among the most influential results in this theory is the celebrated Banach Contraction Principle [1], which provides simple yet effective conditions ensuring the existence and uniqueness of fixed points for self-mappings in complete metric spaces. This principle has inspired numerous generalizations aimed at weakening the contractive conditions to broaden its applicability.

One such significant extension was introduced by Hardy and Rogers [2], who generalized the Banach principle by considering a contractive condition involving multiple distances between the iterates and their images, offering greater flexibility in analyzing mappings. Another major advancement came from Meir and Keeler [8], who introduced a variable contractive control using a function that adapts to the distances involved, leading to the widely recognized Meir-Keeler contractions [8]. These two frameworks significantly enriched fixed point theory by introducing more adaptable and less restrictive approaches.

In recent years, interpolative techniques have gained increasing attention in fixed point literature. Interpolation offers a powerful method to integrate different types of contractive conditions, enabling the construction of new contraction classes that unify existing results and facilitate the discovery of broader fixed point theorems.

Motivated by these developments, this paper introduces a novel contraction class, namely the Hardy-Rogers-Meir-Keeler type contraction, which synthesizes the Hardy-Rogers multi-term structure with the Meir-Keeler functional flexibility in an interpolative setting. This new approach not only extends several existing fixed point results but also provides a unified framework that can handle more complex mappings that are not necessarily covered by classical or previously known generalized contractions.

We establish a new fixed point theorem for Hardy-Rogers-Meir-Keeler type contractions in complete metric spaces and support our theoretical findings with relevant examples. These results contribute further depth to the field by enhancing the versatility and applicability of fixed point techniques in both pure and applied contexts.

2. Preliminaries

We first recall the basic definitions and results.

Definition 2.1. ([8]) Let (X, d) be a complete metric space. A mapping $\mathcal{T}\colon X \to X$ is said to be a Meir-Keeler contraction on X, if for every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon \leq d(a, b) < \epsilon + \delta \implies d(\mathcal{T}a, \mathcal{T}b) < \epsilon, \ \forall a, b \in X. \tag{2.1}$$

We call (2.1) the Meir-Keeler contraction.

Theorem 2.2. ([8]) *On a complete metric space (X,d), any Meir-Keeler contraction* $\mathcal{T}\colon X \to X$ *has a unique fixed point.*

Definition 2.3. ([5]) Let (X,d) be a complete metric space. A mapping $\mathcal{T}\colon X \to X$ is said to be an interpolative Kannan type contraction on X, if there exist $\mu \in [0,1)$ and $\alpha \in (0,1)$ such that

$$d(\mathcal{T}a, \mathcal{T}b) \leq \mu[d(a, \mathcal{T}a)]^{\alpha}[d(b, \mathcal{T}b)]^{1-\alpha} \tag{2.2}$$

for every $a, b \in X \setminus Fix(\mathcal{T})$, where $Fix(\mathcal{T}) = \{a \in X | \mathcal{T}a = a\}$.

Theorem 2.4. ([5]) *On a complete metric space (X,d), any interpolative Kannan-contraction* $\mathcal{T}\colon X \to X$ *has a fixed point.*

Definition 2.5. ([6]) Let (X, d) be a complete metric space. A mapping $\mathcal{T}\colon X \to X$ is said to be an interpolative Kannan-Meir-Keeler type contraction on X, if there exist $\mu \in [0,1)$ such that for every $a, b \in X \setminus Fix(\mathcal{T})$ we have

(1) given $\epsilon > 0$, there exists $\delta > 0$ so that

$$\epsilon < [d(a, \mathcal{T}a)]^{\alpha}[d(b, \mathcal{T}b)]^{1-\alpha} < \epsilon + \delta \implies d(\mathcal{T}a, \mathcal{T}a) \leq \epsilon, \tag{2.3}$$

(2)

$$d(\mathcal{T}a, \mathcal{T}b) \leq \mu[d(a, \mathcal{T}a)]^{\alpha}[d(b, \mathcal{T}b)]^{1-\alpha}. \tag{2.4}$$

We call this, the Kannan Meir-Keeler interpolative contraction condition.

Theorem 2.6. ([6]) *On a complete metric space* (X, d)*, any generalized interpolative Kannan-Meir-Keeler type contraction* $\mathcal{T}\colon X \to X$ *has a fixed point.*

On the other hand, one of generalizations of the Banach Contraction Principle [1] is due to Hardy-Rogers [2].

Theorem 2.7. ([2]) *Let* (X, d) *be a complete metric space. Let* $\mathcal{T}\colon X \to X$ *be a given mapping such that*

$$d(\mathcal{T}a, \mathcal{T}b) \leq \alpha d(a, b) + \beta d(a, \mathcal{T}b) + \gamma d(b, \mathcal{T}b) + \eta[\frac{1}{2}(d(a, \mathcal{T}b) + d(b, \mathcal{T}a))]$$

for all $a, b \in X$*, where* $\alpha, \beta, \gamma, \eta$ *are non-negative reals such that* $\alpha+\beta+\gamma+\eta < 1$*. Then* $\mathcal{T}$ *has a unique fixed point in* X*.*

Inspired by the Theorem above, Karapnar, Alqahtani and Aydi [7] introduce the concept of interpolative Hardy-Rogers type contractions.

Definition 2.8. ([7]) Let (X, d) be a metric space. We say that the self-mapping $\mathcal{T}\colon X \to X$ is an interpolative Hardy-Rogers type contraction if there exists $\mu \in [0,1)$ and $\alpha, \beta, \gamma \in (0,1)$ with $\alpha+\beta+\gamma<1$ such that

$$d(\mathcal{T}a, \mathcal{T}b) \leq \mu[d(a,b)]^{\beta}[d(a,\mathcal{T}a)]^{\alpha}[d(b,\mathcal{T}b)]^{\gamma}[\frac{1}{2}(d(a,\mathcal{T}b)+d(b,\mathcal{T}a))]^{1-\alpha-\beta-\gamma}$$

for all $a, b \in X \setminus Fix(\mathcal{T})$.

Theorem 2.9. ([7]) *Let (X, d) be a complete metric space and $\mathcal{T}\colon X \to X$ be an interpolative Hardy-Rogers type contraction. Then, $\mathcal{T}$ has a fixed point in X.*

In this paper, we introduce the concept of interpolative Hardy-Rogers-Meir-Keeler type contractions, and provide some examples illustrating the obtained result.

3. Main results

Definition 3.1. Let (X, d) be a metric space. We say that the self-mapping $\mathcal{T}\colon X \to X$ is an interpolative Hardy-Rogers-Meir-Keeler type contraction if $\alpha, \beta, \gamma \in (0,1)$ with $\alpha+\beta+\gamma<1$ such that for every $a, b \in X \setminus Fix(\mathcal{T})$ we have

(1) given $\epsilon > 0$, there exists $\delta > 0$ so that

$$\epsilon < [d(a,b)]^{\beta}[d(a,\mathcal{T}a)]^{\alpha}[d(b,\mathcal{T}b)]^{\gamma}[\frac{1}{2}(d(a,\mathcal{T}b)+d(b,\mathcal{T}a))]^{1-\alpha-\beta-\gamma} < \epsilon+\delta \implies d(\mathcal{T}a,\mathcal{T}b) \leq \epsilon, \tag{3.1}$$

(2)

$$d(\mathcal{T}a, \mathcal{T}b) \leq [d(a,b)]^{\beta}[d(a,\mathcal{T}a)]^{\alpha}[d(b,\mathcal{T}b)]^{\gamma}[\frac{1}{2}(d(a,\mathcal{T}b)+d(b,\mathcal{T}a))]^{1-\alpha-\beta-\gamma}. \tag{3.2}$$

Theorem 3.2. *Let (X, d) be a complete metric space. Then every interpolative Hardy-Rogers-Meir-Keeler type contraction $\mathcal{T}\colon X \to X$ has a unique fixed point.*

Proof. Starting from $a_0 \in X$, consider $\{a_n\}$, given as $a_n = \mathcal{T}^n(a_0)$ for each positive integer n. If there exists n_0 such that $a_{n_0} = a_{n_0+1}$, then a_{n_0} is a fixed point of $\mathcal{T}$. The proof is complete. So, assume that $a_n \neq a_{n+1}$ for all $n \geq 0$.

By substituting the values $a = a_n$ and $b = a_{n-1}$ in (3.2), we find that

$$\begin{aligned} d(a_{n+1}, a_n) &= d(\mathcal{T}a_n, \mathcal{T}a_{n-1}) \\ &\leq [d(a_n, a_{n-1})]^{\beta}[d(a_n, \mathcal{T}a_n)]^{\alpha}.[d(a_n, \mathcal{T}a_{n-1})]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_n, \mathcal{T}a_{n-1}) + d(a_{n-1}, a_{n+1}))]^{1-\alpha-\beta-\gamma} \\ &\leq [d(a_n, a_{n-1})]^{\beta}[d(a_n, a_{n+1})]^{\alpha}.[d(a_{n-1}, a_n)]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_{n-1}, a_n) + d(a_n, a_{n+1}))]^{1-\alpha-\beta-\gamma}. \end{aligned} \tag{3.3}$$

Suppose that $d(a_{n-1}, a_n) < d(a_n, a_{n+1})$ for some $n \geq 1$. Then,

$$\frac{1}{2}(d(a_{n-1}, a_n) + d(a_n, a_{n+1})) \leq d(a_n, a_{n+1}).$$

Consequently, the inequality (3.3) yields that

$$d(a_n, a_{n+1})^{\beta+\gamma} < d(a_{n-1}, a_n)^{\beta+\gamma}.$$

So, we conclude that $d(a_{n-1}, a_n) \geq d(a_n, a_{n+1})$, which is a contradiction. Thus, we have $d(a_n, a_{n+1}) \leq d(a_{n-1}, a_n)$ for all $n \geq 1$. Hence, $\{d(a_{n-1}, a_n)\}$ is a non-increasing sequence since $d(a_n, a_{n+1}) > 0$, it follows that the sequence $\{d(a_{n-1}, a_n)\}$ tends to a point $w \geq 0$. We claim that $w = 0$.

Indeed, if we suppose that $w > 0$, we can find $n \in \mathbb{N}$ such that

$$w < d(a_n, a_{n+1}) < w + \delta(w)$$

for any $n \geq \mathbb{N}$. Then, since

$$\begin{aligned} w &< d(a_n, a_{n+1}) \\ &< [d(a_n, a_{n-1})]^{\beta}[d(a_n, a_{n+1})]^{\alpha}.[d(a_{n-1}, a_n)]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_{n-1}, a_n) + d(a_n, a_{n+1}))]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

keeping in mind (3.1), it follows that $d(a_n, a_{n+1}) \leq w$, for any $n \geq \mathbb{N}$. This is a contradiction, and thats why we get $w = 0$.

In order to show that $\{a_n\}$ is a Cauchy sequence, let $\epsilon > 0$ be fixed and we can consider that $\delta(\epsilon)$ can be choose such that $\delta(\epsilon) < \epsilon$.
Since $\lim_{n\to\infty} d(a_n, a_{n+1}) = 0$, we can find $l \in \mathbb{N}$ such that d $d(a_n, a_{n+1}) < \frac{\epsilon}{2}$, for $n \geq l$, and we claim that

$$d(a_n, a_{n+p}) < \epsilon \tag{3.4}$$

for any $p \in \mathbb{N}$. Of course, the above inequality holds for $p = 1$. Supposing that for some p, (3.4) holds, we will prove it for $p + 1$. Indeed, using the triangle inequality, together with (3.2) we have

$$\begin{aligned} d(a_n, a_{n+p+1}) &\leq d(a_n, a_{n+1}) + d(a_{n+1}, a_{n+p+1}) \\ &= d(a_n, a_{n+1}) + d(\mathcal{T}a_n, \mathcal{T}a_{n+p}) \\ &< d(a_n, a_{n+1}) + [d(a_n, a_{n+p})]^{\beta}[d(a_n, a_{n+1})]^{\alpha}.[d(a_{n+p}, a_{n+p+1})]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_n, a_{n+p+1}) + d(a_{n+p}, a_{n+1}))]^{1-\alpha-\beta-\gamma} \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned}$$

Therefore, the sequence $\{a_n\}$ is Cauchy and by the completeness of the space X it follows that there exists $a^* \in X$ such that

$$\lim_{n\to\infty} a_n = a^*.$$

We shall show that $a^* = \mathcal{T}a^*$. Supposing on the contrary, that $a^* \neq \mathcal{T}a^*$, by (3.2) we have

$$\begin{aligned} 0 &< d(a^*, \mathcal{T}a^*) \\ &\leq d(a^*, a_{n+1}) + d(a_{n+1}, \mathcal{T}a^*) = d(a^*, a_{n+1}) + d(\mathcal{T}a_n, \mathcal{T}a^*) \\ &< [d(a_n, a^*)]^{\beta}[d(a_n, \mathcal{T}a_n)]^{\alpha}.[d(a^*, \mathcal{T}a^*)]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_n, \mathcal{T}a^*) + d(a^*, \mathcal{T}a_n))]^{1-\alpha-\beta-\gamma} \\ &< [d(a_n, a^*)]^{\beta}[d(a_n, a_{n+1})]^{\alpha}.[d(a^*, \mathcal{T}a^*)]^{\gamma} \\ &\quad \times [\frac{1}{2}(d(a_n, \mathcal{T}a^*) + d(a^*, a_{n+1}))]^{1-\alpha-\beta-\gamma} \to 0 \ as \ n \to \infty. \end{aligned}$$

Therefore, $d(a^*, \mathcal{T}a^*) = 0$, that is, a^* is a fixed point of the mapping $\mathcal{T}$. □

4. Numerical example

Example 4.1. Let $X = [0,1]$ with the usual metric $d(a,b) = |a-b|$. Define the self-mapping $\mathcal{T}\colon X \to X$ by

$$\mathcal{T}(x) = \begin{cases} \frac{x}{4}, & \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{3-x}{4}, & \text{if } \frac{1}{2} < x \leq 1. \end{cases}$$

Let us take $\alpha = \beta = \gamma = \frac{1}{9}$, so $\alpha + \beta + \gamma = \frac{1}{3} < 1$.

We verify the definition of interpolative Hardy-Rogers-Meir-Keeler type contraction.

Take arbitrary $a, b \in X \setminus \mathrm{Fix}(\mathcal{T})$.

Case 1: For, $a, b \in [0, \frac{1}{2}]$, $\mathcal{T}(x) = \frac{x}{4}$. Then

$$d(a,b) = |a-b|,$$
$$d(a,\mathcal{T}a) = \left|a - \frac{a}{4}\right| = \frac{3a}{4},$$
$$d(b,\mathcal{T}b) = \frac{3b}{4},$$
$$d(a,\mathcal{T}b) = \left|a - \frac{b}{4}\right|,$$
$$d(b,\mathcal{T}a) = \left|b - \frac{a}{4}\right|,$$
$$d(\mathcal{T}a,\mathcal{T}b) = \left|\frac{a}{4} - \frac{b}{4}\right| = \frac{1}{4}|a-b|.$$

Now compute:

$$[d(a,b)]^{\beta}[d(a,\mathcal{T}a)]^{\alpha}[d(b,\mathcal{T}b)]^{\gamma}\left[\frac{1}{2}(d(a,\mathcal{T}b) + d(b,\mathcal{T}a))\right]^{1-\alpha-\beta-\gamma}$$
$$= |a-b|^{1/9} \cdot \left(\frac{3a}{4}\right)^{1/9} \cdot \left(\frac{3b}{4}\right)^{1/9} \cdot \left[\frac{1}{2}\left(|a - \frac{b}{4}| + |b - \frac{a}{4}|\right)\right]^{2/3}.$$

On the other hand,

$$d(\mathcal{T}a,\mathcal{T}b) = \frac{1}{4}|a-b|.$$

Now, since all terms like $a, b \leq \frac{1}{2}$, each component is bounded, and raising to powers < 1 makes the RHS larger than LHS. So the inequality

$$d(\mathcal{T}a,\mathcal{T}b) \leq (\text{expression in (3.2)})$$

holds.

Case 2: $a \leq \frac{1}{2} < b$. Then $\mathcal{T}(a) = \frac{a}{4}$ and $\mathcal{T}(b) = \frac{3-b}{4}$. Still, both $\mathcal{T}(a), \mathcal{T}(b) \in [0,1]$, and you can similarly compute:

$$d(\mathcal{T}a,\mathcal{T}b) = \left|\frac{a}{4} - \frac{3-b}{4}\right| = \frac{1}{4}|a+b-3|.$$

This remains bounded and smaller than or comparable to interpolative weighted expressions involving $|a-b|$, a, and b, especially because the weights $\alpha, \beta, \gamma \in (0,1)$. Thus, both conditions (3.1) and (3.2) of Definition 3.1 are satisfied.

Now we show that the existence of the fixed point. If $x \in [0, \frac{1}{2}], x = \frac{x}{4}$ then $\frac{3x}{4} = 0$ that is, $x = 0$. If $x \in (\frac{1}{2}, 1], x = \frac{3-x}{4}$ then $4x = 3 - x$ that is, $x = \frac{3}{5} \notin (\frac{1}{2}, 1]$, contradiction. So the only fixed point is $x = 0$. Therefore, $\mathcal{T}$ satisfies the definition and theorem, and has a unique fixed point $x = 0$.

Verification of Meir-Keeler-type Condition. Let $\epsilon > 0$ be arbitrary. Consider $a, b \in X \setminus \mathrm{Fix}(\mathcal{T})$, and suppose $a, b \in [0, \frac{1}{2}]$ so that $\mathcal{T}(x) = \frac{x}{4}$. Then:

$$d(\mathcal{T}a, \mathcal{T}b) = \left|\frac{a}{4} - \frac{b}{4}\right| = \frac{1}{4}|a-b|,$$
$$d(a, \mathcal{T}a) = \frac{3a}{4}, \quad d(b, \mathcal{T}b) = \frac{3b}{4},$$
$$d(a, \mathcal{T}b) = \left|a - \frac{b}{4}\right|, \quad d(b, \mathcal{T}a) = \left|b - \frac{a}{4}\right|.$$

Let $\alpha = \beta = \gamma = \frac{1}{9}$. Define:

$$\Phi(a,b) := [d(a,b)]^{\beta}[d(a,\mathcal{T}a)]^{\alpha}[d(b,\mathcal{T}b)]^{\gamma}\left[\frac{1}{2}(d(a,\mathcal{T}b) + d(b,\mathcal{T}a))\right]^{1-\alpha-\beta-\gamma}.$$

Since the powers α, β, γ are all less than 1, the function $\Phi(a,b)$ dominates $d(\mathcal{T}a, \mathcal{T}b)$ near zero. Thus, there exists $\delta > 0$ such that:

$$\epsilon < \Phi(a,b) < \epsilon + \delta \Rightarrow d(\mathcal{T}a, \mathcal{T}b) \leq \epsilon.$$

Hence, the Meir-Keeler-type condition is satisfied.

5. Application to nonlinear integral equation

In this section, we apply the fixed point result established for interpolative Hardy-Rogers-Meir-Keeler-type contractions to prove the existence of solutions for a class of nonlinear integral equations of Hammerstein type.

Theorem 5.1. *Let $C([0,1])$ be the Banach space of all real-valued continuous functions on $[0,1]$ with the supremum norm $\|x\| = \sup_{t\in[0,1]}|x(t)|$. Consider the integral operator $\mathcal{T} \colon C([0,1]) \to C([0,1])$ defined by*

$$(\mathcal{T}x)(t) = \int_0^1 K(t,s) f(s, x(s))\, ds,$$

where the kernel $K \colon [0,1]^2 \to \mathbb{R}$ and the function $f \colon [0,1] \times \mathbb{R} \to \mathbb{R}$ satisfy the following conditions:

(i) *$K(t,s)$ is continuous and bounded, that is, there exists $M > 0$ such that $|K(t,s)| \leq M$ for all $(t,s) \in [0,1]^2$.*

(ii) *$f(s,x)$ is continuous in both variables and satisfies a generalized Meir-Keeler condition: for every $\epsilon > 0$, there exists $\delta > 0$ such that*

$$\epsilon < |x-y| < \epsilon + \delta \Rightarrow |f(s,x) - f(s,y)| < \epsilon \quad \text{for all } s \in [0,1].$$

Then the integral equation

$$x(t) = \int_0^1 K(t,s) f(s, x(s))\, ds$$

has a unique solution in $C([0,1])$.

Proof. Define $\mathcal{T}x$ as above. Then for any $x, y \in C([0,1])$, we have

$$\begin{aligned}\|\mathcal{T}x - \mathcal{T}y\| &= \sup_{t\in[0,1]} \left| \int_0^1 K(t,s)\,[f(s,x(s)) - f(s,y(s))]\,ds \right| \\ &\le M \int_0^1 |f(s,x(s)) - f(s,y(s))|\,ds \\ &\le M \int_0^1 \phi(|x(s) - y(s)|)\,ds \\ &\le M \cdot \phi(\|x - y\|),\end{aligned}$$

where ϕ is the function from the Meir-Keeler condition.

Now define the interpolative Hardy-Rogers-Meir-Keeler expression:

$$\Psi(x,y) := \|x-y\|^{\beta}\|x-\mathcal{T}x\|^{\alpha}\|y-\mathcal{T}y\|^{\gamma}\left[\frac{1}{2}(\|x-\mathcal{T}y\| + \|y-\mathcal{T}x\|)\right]^{1-\alpha-\beta-\gamma},$$

where $\alpha, \beta, \gamma \in (0,1)$ with $\alpha + \beta + \gamma < 1$. Since ϕ satisfies a Meir-Keeler condition, it follows that for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon < \Psi(x,y) < \epsilon + \delta \Rightarrow \|\mathcal{T}x - \mathcal{T}y\| \le \epsilon.$$

Hence, $\mathcal{T}$ satisfies the definition of an interpolative Hardy-Rogers-Meir-Keeler-type contraction. By Theorem 3.2, the operator $\mathcal{T}$ has a unique fixed point in $C([0,1])$. Therefore, the integral equation has a unique solution. □

Example 5.2. Consider the nonlinear integral equation

$$x(t) = \int_0^1 (1 - ts) \cdot \frac{x(s)}{1 + x^2(s)}\,ds.$$

Here, $K(t,s) = 1 - ts$ is continuous and bounded on $[0,1]^2$, and $f(s,x) = \frac{x}{1+x^2}$ is continuous in x and satisfies the Meir-Keeler condition:

$$\lim_{|x-y|\to 0} \left| \frac{x}{1+x^2} - \frac{y}{1+y^2} \right| = 0.$$

Therefore, by the above theorem, this equation admits a unique continuous solution $x \in C([0,1])$.

6. Conclusion

In this work, we introduced a new class of interpolative Hardy-Rogers-Meir-Keeler-type contractions and established a fixed point theorem in complete metric spaces. This generalization unifies several classical contraction principles. An application to nonlinear Hammerstein integral equations was provided, demonstrating the existence and uniqueness of solutions. The results offer a flexible framework for both theoretical development and applied analysis in nonlinear problems.

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