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APPROXIMATION OF COMMON FIXED POINTS OF MULTIVALUED NONEXPANSIVE MAPPINGS VIA FASTER ITERATIVE PROCESS

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Abstract. In this paper we suggest a new iterative method for approximating a common fixed point (CFP) in nonexpansive multivalued mappings. The strong convergence theorem of the new approach has been established. To illustrate the convergence rate of the main conclusions, a numerical example is given. This study's findings generalize numerous previously published findings.

1. Introduction

Fixed point theory necessitates an extensive body of literature due to its capacity to offer practical solutions to numerous issues that are applicable in numerous disciplines. Nevertheless, the determination of the value of a fixed point (fp) in certain mappings is a challenging endeavor once it is established. Consequently, iterative processes are employed to determine the value of the fp. Several researchers have studied iterative approaches to approximating fps of nonexpansive single-valued mappings utilizing the Mann or Ishikawa iteration schemes. There is currently a large body of work on iterative fps

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for many types of maps [12, 13, 14, 15, 19]. Markin [10] and Nadler [11] initiated the investigation of fps for nonexpansive mappings and multi-valued contractions. There is a substantial body of literature on multi-valued fp theory, which has been applied in a variety of disciplines [4, 7]. Additionally, Lim [9] verified the existence of fps for multi-valued nonexpansive mappings in uniformly convex Banach spaces (for short, UBS). A variety of iterative procedures have been employed to approximate the fps of multi-valued nonexpansive mappings [5, 6, 8, 17, 21]. The estimation of CFPs is very significant as it is directly linked to the issue of minimizing. Several authors have conducted research on the estimation of CFPs among nonexpansive mappings. For instance, Cholamjiak and Suantai in 2011 [2], presented and analyzed two new iterative methods to estimate CFPs in Banach spaces for quasi-nonexpansive multi-valued maps while Abbas et al. [1] proposed a novel one-step iterative method for approximately the CFPs of two multivalued nonexpansive mappings in UBS.

In 2021, Garodia et al. [3] proposed an iteration scheme to approximate fps of a multivalued nonexpansive mapping. Consider $\mathcal{S}$ to be a subset of UBS. Assume $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3 : \mathcal{S} \to \mathcal{S}$ are three nonexpansive mappings, then the sequence $\{v_n\}$ with an initial point $v_1 \in \mathcal{S}$ defined by

$$\begin{aligned} \mathfrak{J}_n &= (1-\gamma_n)\, v_n + \gamma_n \mathbb{T}_1 v_n, \\ u_n &= (1-\beta_n)\ \mathfrak{J}_n + \beta_n \mathbb{T}_2 \mathfrak{J}_n, \\ v_{n+1} &= (1-\rho_n)\, \mathbb{T}_2 \mathfrak{J}_n + \rho_n \mathbb{T}_3 u_n, \end{aligned} \tag{1.1}$$

where γ_n, β_n and ρ_n are real sequences in (0,1).

Inspired by the work of Cholamjiak and Suantai [2] and Abbas et al. [1], we first provide a multivalued version of Garodia et al. [3]'s iteration technique (1.1) and then analyze its convergence in the context of Banach spaces. The following is the definition of our iteration approach.

$$\begin{aligned} \mathfrak{J}_n &= (1-\gamma_n)\, v_n + \gamma_n \nabla_n, \\ u_n &= (1-\beta_n)\ \mathfrak{J}_n + \beta_n \int_n, \\ v_{n+1} &= (1-\rho_n)\, \int_n + \rho_n \Updownarrow_n, \end{aligned} \tag{1.2}$$

where $\gamma_n, \beta_n, \rho_n \in\ (0,1)$ and $\nabla_n \in \mathbb{T}_1 v_n, \int_n \in \mathbb{T}_2 \mathfrak{J}_n$ and $\Updownarrow_n \in \mathbb{T}_3 u_n$.

2. Preliminaries

Let B be a real Banach space. If for all $\mu \in B$, there exists $p \in \mathcal{P}$ where $\mathcal{P} \subseteq B$ such that $d(\mu, p) = \inf\{\|\mu - \epsilon\| : \epsilon \in \mathcal{P}\} = d(\mu, \mathcal{P})$ then $\mathcal{P}$ is said to be proximal. It is widely recognized that a weakly compact convex subset of a Banach space and closed convex subsets of a UBS are Proximal. The family of all nonempty bounded proximal subsets of $\mathcal{P}$ will be denoted by $\mho(\mathcal{P})$, and the class of all nonempty bounded and closed subsets of $\mathcal{P}$ will be indicated as $\mathbb{CB}(\mathcal{P})$. Let $\mathcal{H}$ be the Hausdorff metric produced through the metric d of B, that is,

$$\mathcal{H}(\mathcal{U},\mathcal{V}) = \max\{\sup_{\mu\in\mathcal{U}} d(\mu,\mathcal{V}),\ \sup_{\epsilon\in\mathcal{V}} d(\epsilon,\mathcal{U})\}$$

for every $\mathcal{U},\mathcal{V} \in \mathbb{CB}(\mathcal{P})$, where $d(\mu,\mathcal{V}) = \inf\{\|\mu-\epsilon\| : \epsilon \in \mathcal{V}\}$.

Lemma 2.1. ([20]) *Assume that $\{a_n\},\{b_n\},\{c_n\}$ are three sequences that meet the criteria:*

$$a_{n+1} \le (1+b_n)\,a_n + c_n,\ n \ge 0,\ \sum_{n=0}^{\infty} b_n < \infty,\ \sum_{n=0}^{\infty} c_n < \infty.$$

Then

(1) $\lim_{n\to\infty} a_n$ *exists,*
(2) *if* $\liminf_{n\to\infty} a_n = 0$ *then* $\lim_{n\to\infty} a_n = 0$.

Lemma 2.2. ([16]) *Let B be a UBS and $\{a_n\}$ be a real sequence with $0 < b_n \le a_n \le c_n < 1$ for each $n \ge 1$. Assume that $\{b_n\},\{c_n\}$ are two sequences of B with $\lim_{s\to\infty} \sup\|b_n\| \le \vartheta, \lim_{s\to\infty}\sup\|c_n\| \le \vartheta$ and $\lim_{s\to\infty}\sup\|a_n b_n + (1-a_n)c_n\| = \vartheta$ hold for some $\vartheta \ge 0$. Then $\lim_{s\to\infty}\sup\|b_n - c_n\| = 0$.*

Lemma 2.3. ([18]) *Assume $\mathbb{T} : \mathcal{P} \to \mho(\mathcal{P})$ is a multivalued mapping and $\mho_{\mathbb{T}(b)} = \{c \in \mathbb{T}(b) : \|b-c\| = d(b,\mathbb{T}b)\}$. Then the following statements are equivalent.*

(1) $b \in F^*(\mathbb{T})$*; where $F^*(\mathbb{T})$ represents the set of all fps,*
(2) $\mho_{\mathbb{T}}(b) = \{b\}$,
(3) $b \in F^*(\mho_{\mathbb{T}})$.

3. Main results

Some strong convergence theorems are proved in this section by use of the iteration method (1.2). To demonstrate the primary findings, we must first provide the following lemmas.

Consider B is a real Banach space and $\mathcal{P} \subseteq B$ (where $\mathcal{P}$ is a nonempty closed and convex). Put $F^* = F^*(\mathbb{T}_1) \cap F^*(\mathbb{T}_2) \cap F^*(\mathbb{T}_3)$ denotes the set of all CFPs of $\mathbb{T}_1, \mathbb{T}_2$ and $\mathbb{T}_3$.

Lemma 3.1. *Suppose $\mathcal{P} \subseteq B$ and $\mathbb{T}_1,\mathbb{T}_2,\mathbb{T}_3 : \mathcal{P} \to \mho(\mathcal{P})$ are three multivalued mappings such that $F^* \neq \emptyset$ and $\mho_{\mathbb{T}_1}, \mho_{\mathbb{T}_2}$ and $\mho_{\mathbb{T}_3}$ are nonexpansive mappings. Consider $\{v_n\}$ is a sequence defined by Eq. (1.2). Then $\lim_{n\to\infty}\|v_n - \omega\|$ exists for all $\omega \in F^*$.*

Proof. Let $\omega \in F^*$. Then $\omega \in \mho_{\mathbb{T}_1}(\omega) = \{\omega\}$, $\omega \in \mho_{\mathbb{T}_2}(\omega) = \{\omega\}$ and $\omega \in \mho_{\mathbb{T}_3}(\omega) = \{\omega\}$ by Lemma 2.3. It follows from (1.2) that:

$$\begin{aligned}
\|\mathfrak{J}_n - \omega\| &\le \|[(1-\gamma_n)\, v_n + \gamma_n \nabla_n]\| - \omega \\
&\le (1-\gamma_n)\, \|v_n - \omega\| + \gamma_n \|\nabla_n - \omega\| \\
&\le (1-\gamma_n)\, \|v_n - \omega\| + \gamma_n \mathcal{H}\left(\mho_{\mathbb{T}_1}(v_n), \mho_{\mathbb{T}_1}(\omega)\right) \\
&\le (1-\gamma_n)\, \|v_n - \omega\| + \gamma_n \|v_n - \omega\| \\
&= \|v_n - \omega\|. \qquad (3.1)
\end{aligned}$$

Again using (1.2) and (3.1), we obtain

$$\begin{aligned}
\|u_n - \omega\| &\le \|[(1-\beta_n)\ \mathfrak{J}_n + \beta_n \int_n] - \omega\| \\
&\le (1-\beta_n)\, \|\mathfrak{J}_n - \omega\| + \beta_n \|\int_n - \omega\| \\
&\le (1-\beta_n)\, \|\mathfrak{J}_n - \omega\| + \beta_n \mathcal{H}\left(\mho_{\mathbb{T}_2}(\mathfrak{J}_n), \mho_{\mathbb{T}_2}(\omega)\right) \\
&\le (1-\beta_n)\, \|\mathfrak{J}_n - \omega\| + \beta_n \|\mathfrak{J}_n - \omega\| \\
&\le (1-\beta_n)\, \|v_n - \omega\| + \beta_n \|v_n - \omega\| \\
&= \|v_n - \omega\|. \qquad (3.2)
\end{aligned}$$

Now using (1.2), (3.1) and (3.2), we obtain

$$\begin{aligned}
\|v_{n+1} - \omega\| &\le \|[(1-\rho_n)\, \int_n + \rho_n \Updownarrow_n] - \omega\| \\
&\le (1-\rho_n)\, \|\int_n - \omega\| + \rho_n \|\Updownarrow_n - \omega\| \\
&\le (1-\rho_n)\, \mathcal{H}\left(\mho_{\mathbb{T}_2}(\mathfrak{J}_n), \mho_{\mathbb{T}_2}(\omega)\right) + \rho_n \mathcal{H}\left(\mho_{\mathbb{T}_3}(u_n), \mho_{\mathbb{T}_3}(\omega)\right) \\
&\le (1-\rho_n)\, \|\mathfrak{J}_n - \omega\| + \rho_n \|u_n - \omega\| \\
&\le (1-\rho_n)\, \|v_n - \omega\| + \rho_n \|v_n - \omega\| \\
&= \|v_n - \omega\|. \qquad (3.3)
\end{aligned}$$

According to Lemma 2.1, $\lim_{n\to\infty} \|v_n - \omega\|$ exists for all $\omega \in F^*$. □

Lemma 3.2. *Assume $\mathcal{P} \subseteq B$ and $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$: $\mathcal{P} \to \mho(\mathcal{P})$ are multivalued mappings such that $F^* \neq \emptyset$ and $\mho_{\mathbb{T}_1}, \mho_{\mathbb{T}_2}$ and $\mho_{\mathbb{T}_3}$ are nonexpansive mappings. Consider $\{v_n\}$ is defined by Eq.* (1.2). *Then* $\lim_{n\to\infty} d(v_n, \mathbb{T}_1 v_n) = 0$, $\lim_{n\to\infty} d(v_n, \mathbb{T}_2 \mathfrak{J}_n) = 0$ *and* $\lim_{n\to\infty} d(v_n, \mathbb{T}_3 u_n) = 0$.

Proof. According to Lemma 3.1, $\lim_{n\to\infty} \|v_n - \omega\|$ exists for all $\omega \in F^*$. Let $\lim_{n\to\infty} \|v_n - \omega\| = \wr$ for some $\wr \ge 0$. Then,

$$\begin{aligned}
\limsup_{n\to\infty} \|\Updownarrow_n - \omega\| &\le \limsup_{n\to\infty} \mathcal{H}\left(\mho_{\mathbb{T}_3}(u_n), \mho_{\mathbb{T}_3}(\omega)\right) \\
&\le \limsup_{n\to\infty} \|u_n - \omega\| \\
&\le \limsup_{n\to\infty} \|v_n - \omega\| \\
&= \wr.
\end{aligned}$$

So,

$$\limsup_{n\to\infty}\|\Updownarrow_n-\omega\|\leq\wr. \tag{3.4}$$

Again, since

$$\begin{aligned}\limsup_{n\to\infty}\|\nabla_n-\omega\| &\leq \limsup_{n\to\infty}\mathcal{H}\left(\mho_{\mathbb{T}_1}(v_n),\mho_{\mathbb{T}_1}(\omega)\right)\\ &\leq \limsup_{n\to\infty}\|v_n-\omega\|\\ &=\wr.\end{aligned}$$

So,

$$\limsup_{n\to\infty}\|\nabla_n-\omega\|\leq\wr. \tag{3.5}$$

Similarly,

$$\limsup_{n\to\infty}\|\smallint_n-\omega\|\leq\wr. \tag{3.6}$$

Employing Lemma 2.2, obtain

$$\lim_{n\to\infty}\|\Updownarrow_n-\nabla_n\|=0, \tag{3.7}$$

$$\lim_{n\to\infty}\|\smallint_n-\Updownarrow_n\| \tag{3.8}$$

and

$$\lim_{n\to\infty}\|\nabla_n-\smallint_n\|=0. \tag{3.9}$$

By applying limit sup to both sides of (3.1) and (3.2), one gets

$$\limsup_{n\to\infty}\|\mathfrak{I}_n-\omega\|\leq\wr \tag{3.10}$$

and

$$\limsup_{n\to\infty}\|\nabla_n-\omega\|\leq\wr. \tag{3.11}$$

Also

$$\begin{aligned}\|v_{n+1}-\omega\| &\leq \|[(1-\rho_n)\smallint_n+\rho_n\Updownarrow_n]-\omega\|\\ &\leq (1-\rho_n)\|\smallint_n-\omega\|+\rho_n\|\Updownarrow_n-\omega\|.\end{aligned}$$

It implies that

$$\|\smallint_n-\omega\|\leq\frac{\|\smallint_n-\omega\|-\|v_{n+1}-\omega\|}{\rho_n}+\|\Updownarrow_n-\omega\|. \tag{3.12}$$

By taking the limit of the inequality above on both sides, we get

$$\wr\leq\liminf_{n\to\infty}\|\Updownarrow_n-\omega\|. \tag{3.13}$$

Putting (3.4) and (3.13) together, we get

$$\lim_{n\to\infty}\|\Updownarrow_n-\omega\|=\wr. \tag{3.14}$$

Thus

$$\begin{aligned}\|\Updownarrow_n - \omega\| &\leq \|\Updownarrow_n - \int_n\| + \|\int_n - \omega\| \\ &\leq \Updownarrow_n - \int_n + \mathcal{H}\left(\mho_{\mathbb{T}_2}(\mathfrak{I}_n), \mho_{\mathbb{T}_2}(\omega)\right) \\ &\leq \|\Updownarrow_n - \nabla_n\| + \|\mathfrak{I}_n - \omega\|.\end{aligned}$$

Gives

$$\wr \leq \lim\inf_{n\to\infty}\|\mathfrak{I}_n - \omega\| \tag{3.15}$$

and by combining the virtue of (3.10), we get

$$\lim_{n\to\infty}\|\mathfrak{I}_n - \omega\| = \wr. \tag{3.16}$$

Utilizing Lemma 2.2,

$$\lim_{n\to\infty} \|v_n - \Updownarrow_n\| = 0. \tag{3.17}$$

Also, take notice that

$$\|v_n - \int_n\| \leq \|v_n - \Updownarrow_n\| + \|\Updownarrow_n - \int_n\|.$$

Applying equations (3.8) and (3.17) yields

$$\lim_{n\to\infty} \|v_n - \int_n\| = 0. \tag{3.18}$$

Since

$$\|v_n - \nabla_n\| \leq \|v_n - \Updownarrow_n\| + \|\Updownarrow_n - \nabla_n\|,$$

applying equations (3.7) and (3.17) yields

$$\lim_{n\to\infty} \|v_n - \nabla_n\| = 0. \tag{3.19}$$

Since

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_1 v_n\right) \leq \|v_n - \nabla_n\|,$$

then

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_1 v_n\right) = 0. \tag{3.20}$$

Again since

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_2 \mathfrak{I}_n\right) \leq \|v_n - \int_n\|,$$

then

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_2 \mathfrak{I}_n\right) = 0. \tag{3.21}$$

Likewise, since

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_3 u_n\right) \leq \|v_n - \Updownarrow_n\|,$$

then

$$\lim_{n\to\infty} d\left(v_n, \mathbb{T}_3 u_n\right) = 0. \tag{3.22}$$

This concludes the evidence. □

Theorem 3.3. *Assume $\mathcal{P} \subseteq B$ ($\mathcal{P}$ is a compact and convex) and $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3 : \mathcal{P} \to \mho(\mathcal{P})$ are multivalued mappings such that $F^* \neq \emptyset$ and $\mho_{\mathbb{T}_1}, \mho_{\mathbb{T}_2}$ and $\mho_{\mathbb{T}_3}$ are nonexpansive mappings. Assume $\{v_n\}$ is a sequence defined by Eq.* (1.2). *Then, $\{v_n\}, \{\mathfrak{J}_n\}$ and $\{u_n\}$ converge strongly to a CFP of $\mathbb{T}_1, \mathbb{T}_2$ and $\mathbb{T}_3$.*

Proof. By Lemma 3.2, we've accomplished $\lim_{n\to\infty} d(v_n, \mathbb{T}_1 v_n) = 0$. According to the hypothesis, $\mathcal{P}$ is a compact convex set, so for any $b^* \in B$ there is a subsequence $\{v_{n_k}\}\, of\, \{v_n\}$ such that $\lim_{k\to\infty} \|v_{n_k} - b^*\| = 0$. Thus

$$\begin{aligned} d(b^*, \mathbb{T}_1 b^*) &\leq \|v_{n_k} - b^*\| + d(v_{n_k}, \mathbb{T}_1 v_{n_k}) + \mathcal{H}(\mho_{\mathbb{T}_1}(v_{n_k}), \mho_{\mathbb{T}_1}(b^*)) \\ &\leq 2\|v_{n_k} - b^* + v_{n_k}\| - \|\nabla_n\| \\ &\to 0 \; as \; k \to \infty. \end{aligned}$$

This shows that b^* is fixed for $\mathbb{T}_1$. It then follows from Lemma 3.1

$$\lim_{n\to\infty} \|v_n - b^*\| = 0.$$

Once again, it follows from Lemma 3.2 that

$$\begin{aligned} \|\mathfrak{J}_n - v_n\| &= \|[(1-\gamma_n) v_n + \gamma_n \nabla_n] - v_n\| \\ &\leq \gamma_n \|\nabla_n - v_n\| \\ &\leq \|\nabla_n - v_n\| \\ &\to 0 \; as \; n \to \infty \end{aligned}$$

and

$$\begin{aligned} \|u_n - v_n\| &= \|[(1-\beta_n)\ \mathfrak{J}_n + \beta_n \int_n] - v_n\| \\ &\leq (1-\beta_n) \|\mathfrak{J}_n - v_n\| + \beta_n \|\int_n - v_n\| \\ &\to 0 \; as \; n \to \infty. \end{aligned}$$

Consequently $\lim_{n\to\infty} \|\mathfrak{J}_n - b^*\| = 0$ and $\lim_{n\to\infty} \|u_n - b^*\| = 0$. Thus, the intended result follows. □

Example 3.4. Let $\mathcal{P} = [0,1]$ with the Euclidean norm $\|\cdot\| = |\cdot|$ and let $\mathbb{T} : \mathcal{P} \to \mathcal{CB}(\mathcal{P})\, (\mathcal{CB}(\mathcal{P})$ represents the family of closed and bounded subsets of $\mathcal{P})$ be defined by $\mathbb{T}_1(v) = [0, \frac{v}{6}]$, $\mathbb{T}_2(v) = [0, \frac{v}{5}]$ and $\mathbb{T}_3(v) = [0, \frac{v}{4}]$. Then for any $v, \ell \in \mathcal{P}$,

$$\begin{aligned} \mathcal{H}(\mathbb{T}_1(v), \mathbb{T}_1(\ell)) &= \max\left\{\left\lceil \frac{v}{6} - \frac{\ell}{6} \right\rceil, 0\right\} = \left\lceil \frac{v}{6} - \frac{\ell}{6} \right\rceil = \left\lceil \frac{v-\ell}{6} \right\rceil \leq \lceil v - \ell \rceil, \\ \mathcal{H}(\mathbb{T}_2(v), \mathbb{T}_2(\ell)) &= \max\left\{\left\lceil \frac{v}{5} - \frac{\ell}{5} \right\rceil, 0\right\} = \left\lceil \frac{v}{5} - \frac{\ell}{5} \right\rceil = \left\lceil \frac{v-\ell}{5} \right\rceil \leq \lceil v - \ell \rceil, \\ \mathcal{H}(\mathbb{T}_3(v), \mathbb{T}_3(\ell)) &= \max\left\{\left\lceil \frac{v}{4} - \frac{\ell}{4} \right\rceil, 0\right\} = \left\lceil \frac{v}{4} - \frac{\ell}{4} \right\rceil = \left\lceil \frac{v-\ell}{4} \right\rceil \leq \lceil v - \ell \rceil. \end{aligned}$$

Hence $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$ are multivalued nonexpansive mappings and

$$F^*(\mathbb{T}_1) \cap F^*(\mathbb{T}_2) \cap F^*(\mathbb{T}_3) = 0.$$

Thus $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$ have a unique CFP in $\mathcal{P}$.

In the following example, we demonstrate via numerical and graphical analysis that the iterative technique provided in Eq. (1.2) converges to a CFP across different cases.

Example 3.5. Let $\mathcal{P} = [0, \infty)$ with the Euclidean norm $\| \cdot \| = |\cdot|$ and let $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3 : \mathcal{P} \to \mathcal{CB}(\mathcal{P})$ where $\mathbb{T}_1(v) = \frac{v}{3}$, $\mathbb{T}_2(v) = \frac{v}{4}$ and $\mathbb{T}_3(v) = \frac{v}{5}$. It is clear that $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$ are nonexpansive mappings and 0 is their CFP. Set $\gamma_n = \beta_n = \rho_n = 0.7$. Then, the following tables and figures are produced utilizing different starting values.

TABLE 1. Values of the iteration.

Step	New iteration when v=0.1	New iteration when v=0.5	New iteration when v=1	New iteration when v=3
1	0.1	0.5	1	3
2	0.0075466666	0.03773333	0.07546666	0.22640000
3	$5.69521777 \times 10^{-4}$	0.00284760	0.00569521	0.01708565
4	$4.29799101 \times 10^{-5}$	$2.14899550 \times 10^{-4}$	$4.29799101 \times 10^{-4}$	0.00128939
5	$3.24355055 \times 10^{-6}$	$1.62177527 \times 10^{-5}$	$3.24355055 \times 10^{-5}$	$9.73065166 \times 10^{-5}$
6	$2.44779948 \times 10^{-7}$	$1.22389974 \times 10^{-6}$	$2.44779948 \times 10^{-6}$	$7.34339845 \times 10^{-6}$
7	$1.84727267 \times 10^{-8}$	$9.23636338 \times 10^{-8}$	$1.847272677 \times 10^{-7}$	$5.54181803 \times 10^{-7}$
8	$1.39407511 \times 10^{-9}$	$6.97037557 \times 10^{-9}$	$1.39407511 \times 10^{-8}$	$4.18222534 \times 10^{-8}$

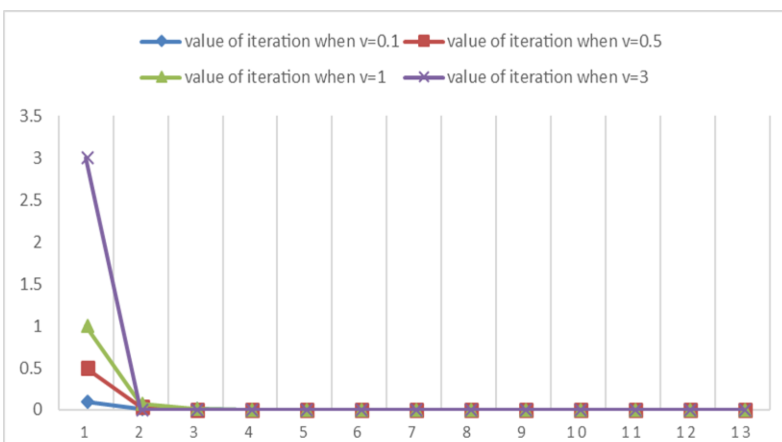

FIGURE 1. Graph corresponding to Table 1.

Example 3.6. Let $\mathcal{P} = [0,1]$ with the Euclidean norm $\|\cdot\| = |\cdot|$ and let $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$: $\mathcal{P} \to \mathcal{CB}(\mathcal{P})$ where $\mathbb{T}_1(v) = \frac{v}{4}, \mathbb{T}_2(v) = \frac{v}{3}$ and $\mathbb{T}_3(v) = \frac{v}{2}$. It is clear that $\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3$ are nonexpansive mappings and 0 is their CFP. Set $\gamma_n = \beta_n = \rho_n = 0.8$. Then, the following tables and figures are produced utilizing different starting values.

TABLE 2. Values of the iteration.

Step	New iteration when v=0.25	New iteration when v=0.5	New iteration when $v = 0.75$	New iteration when v=1
1	0.25	0.5	0.75	1
2	0.0025671111	0.0506666666	0.0760000000	0.1013333333
3	2.6013392592 $\times 10^{-4}$	0.0051342222	0.0077013333	0.0102684444
4	2.6360237827 $\times 10^{-5}$	5.2026785185 $\times 10^{-4}$	7.8040177777 $\times 10^{-4}$	0.0010405357
5	2.6711707664 $\times 10^{-6}$	5.2720475654 $\times 10^{-5}$	7.9080713481 x 10-5	1.0544095130 $\times 10^{-4}$
6	2.7067863767 $\times 10^{-7}$	5.3423415329 $\times 10^{-6}$	8.0135122994 $\times 10^{-6}$	1.0684683065 x 10-5
7	2.7428768617 $\times 10^{-8}$	5.4135727534 $\times 10^{-7}$	8.12035913014 $\times 10^{-7}$	1.0827145506 $\times 10^{-6}$
8	2.7794485532 $\times 10^{-9}$	5.4857537234 $\times 10^{-8}$	8.2286305851 $\times 10^{-8}$	1.0971507446 $\times 10^{-7}$

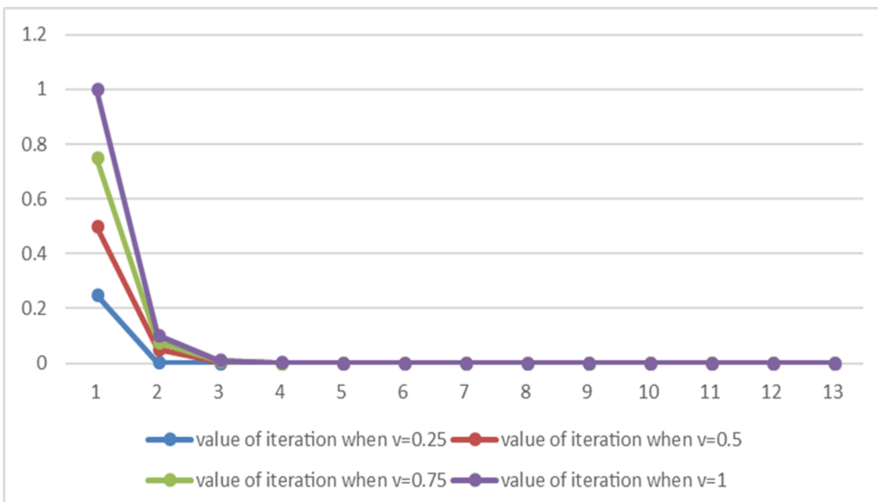

FIGURE 2. Graph corresponding to Table 2.

4. CONCLUSION

In this study, we present an iterative procedure for multivalued nonexpansive mappings. We further prove that a sequence from such a process in a non-empty, compact, convex subset of a Banach space satisfies a strong convergence theorem. Furthermore, some numerical examples are shown to support the primary findings.

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