Nonlinear Functional Analysis and Applications
Vol. 31, No. 2 (2026), pp. 391-402
ISSN: 1229-1595(print), 2466-0973(online)
https://doi.org/10.22771/nfaa.2026.31.02.05
http://nfaa.kyungnam.ac.kr/journal-nfaa

SOME COMMON FIXED POINT THEOREMS OF COMPATIBLE MAPPINGS OF TYPE (K) IN FUZZY METRIC SPACES

Nandu Prasad Koiri[1], Rajeev Kumar[2] and Ajay Kumar Chaudhary[3]

[1]Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Ghantaghar, Kathmandu, Nepal,
PhD Scholar of Arunodaya University, Arunachal, India
e-mail: kushwahanandu072@gmail.com

[2]Department of Mathematics, Arunodaya University, Itanagar, Arunachal, India
e-mail: rjv2878@gmail.com

[3]Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Ghantaghar, Kathmandu, Nepal
e-mail: akcsaurya81@gmail.com

Abstract. In this paper, we prove common fixed point theorems for four self-mappings in the framework of fuzzy metric spaces, using the notion of compatible mappings of type (K). Our results unify, generalize, and extend earlier findings of Rohen et al. [25], Chaudhary et al. [8], and Manandhar et al. [21], thereby contributing to the ongoing development of fixed point theory in fuzzy settings.

1. Introduction

The concept of fuzzy sets was first introduced by Zadeh [31] in 1965, who laid the foundation for fuzzy mathematics and provided a new framework for dealing with uncertainty, vagueness, and imprecision. Since then, the theory has been extensively developed and successfully applied in diverse areas such

[0]Received October 3, 2025. Revised December 10, 2025. Accepted December 13, 2025.
[0]2020 Mathematics Subject Classification: 54H25, 54E50.
[0]Keywords: Common fixed point, compatible mappings, compatible mappings of type (K), fuzzy metric space.
[0]Corresponding author: Ajay Kumar Chaudhary(akcsaurya81@gmail.com).

as control theory, decision making, information sciences, and engineering. Extending these ideas into analysis, Kramosil and Michalek [19] introduced the notion of fuzzy metric spaces in 1975, thereby generalizing classical metric spaces into fuzzy environments. Later, George and Veeramani [11] refined this definition in 1994, which led to substantial advances in the field. The discovery that any metric space induces a fuzzy metric space, creating a natural bridge between the classical and fuzzy frameworks, was one of their main findings.

Fixed point theory was developing rapidly through generalizations of commuting mappings. Jungck [15] initiated this line of inquiry in 1986 by introducing the concept of compatible mappings, which extended the classical notion of commuting self-maps and proved instrumental in deriving common fixed point results. Jungck, Murthy and Cho [16] formulated compatible mappings of type (A) in 1993, Pathak and Khan [24] introduced compatible mappings of type (B) in 1995. Several variations of compatible mappings were proposed thereafter. Pathak and Cho [23] introduced compatible mappings of type (P) in 1994. Singh and Chauhan [28] in 2000 extended the concept to fuzzy metric spaces and obtained common fixed point theorems. To unify these developments, Rohen et al. [26] presented the concept of compatible mappings of type (R) in complete metric spaces in 2008, and Bhadauriya and Gangil [1] in 2016 subsequently established fixed point theorems for such mappings in fuzzy metric spaces.

Another significant extension was introduced by Singh and Singh [30] who defined compatible mappings of type (E) in both metric and fuzzy metric spaces in 2007. In 2014, Manandhar and Pathak [21, 20] extended this concept to prove a common fixed-point theorem for compatible mappings of type (K) in metric space and fuzzy metric space as well. Chaudhary et al. [8] generalized it in Menger space. For more works in this field, see [2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 22, 25, 29].

In this paper, we focus on compatible mappings of type (K) in fuzzy metric spaces. More specifically, we establish common fixed point theorems for four self-mappings under the assumption that two of them are continuous. Our results generalize and extend the earlier contributions of Rohen et al. [18], Chaudhary et al. [8], and Manandhar et al. [21], and thus contribute to the ongoing development of fixed point theory in fuzzy metric spaces.

2. Preliminaries

Definition 2.1. ([5]) A mapping $T : [0,1]\times[0,1] \to [0,1]$ is called a triangular norm (t-norm) if, for all $x, y, z, w \in [0,1]$:

(i) $T(x,1) = x$ (Boundary Condition),
(ii) $T(x,y) = T(y,x)$ (Commutativity),

(iii) $T(x, T(y, z)) = T(T(x, y), z)$ (Associativity),
(iv) $T(x, y) \leq T(z, w)$, whenever $x \leq z$ and $y \leq w$ (Monotonicity).

Definition 2.2. ([27]) A triangular norm T is a continuous t-norm if, for all $x, y, z, w \in [0, 1]$:
(i) T is associative and commutative,
(ii) T is continuous,
(iii) $T(x, 1) = x$ (Boundary Condition),
(iv) $T(x, y) \leq T(z, w)$, whenever $x \leq z$ and $y \leq w$ (Monotonicity).

Example 2.3. $T(a, b) = ab$ for $a, b \in [0, 1]$ is a continuous t-norm.

Definition 2.4. ([31]) If X is a universal set and $x \in X$, then a fuzzy set A defined on X is a collection of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0, 1]\},$$

where $\mu_A : X \to [0, 1]$ is a membership function.

Example 2.5. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. Then A is a fuzzy set of clever students.

$$A = \{(x_1, 0.3), (x_2, 0), (x_3, 0.4), (x_4, 0.7), (x_5, 0.9)\},$$

where $\mu_A(x)$ is the degree of cleverness.

Definition 2.6. ([19]) The triplet $(X, M, *)$ is a fuzzy metric space if X is a set, $*$ is a continuous t-norm, and $M : X \times X \times [0, \infty) \to [0, 1]$ is a fuzzy set satisfying the following postulates: for all $x, y, z \in X$ and $t, s > 0$,
(KM1) $M(x, y, 0) = 0$;
(KM2) $M(x, y, t) = 1$ if and only if $x = y$;
(KM3) $M(x, y, t) = M(y, x, t)$;
(KM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(KM5) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left-continuous.

Note that $M(x, y, t)$ can be considered as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$.

Definition 2.7. ([11]) The triplet $(X, M, *)$ is a fuzzy metric space if X is a set, $*$ is a continuous t-norm, and $M : X \times X \times (0, \infty) \to [0, 1]$ is a fuzzy set satisfying the following axioms: for all $x, y, z \in X$ and $t, s > 0$,
(GV1) $M(x, y, t) > 0$;
(GV2) $M(x, y, t) = 1$ iff $x = y$;
(GV3) $M(x, y, t) = M(y, x, t)$;
(GV4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(GV5) $M(x, y, \bullet) : (0, \infty) \to [0, 1]$ is continuous.

Example 2.8. Let (X, d) be a metric space. Define $a * b = ab$ for $a, b \in [0, 1]$ and

$$M(x, y, t) = \frac{kt^n}{kt^n + md(x, y)}, \quad k, m, n \in \mathbb{R}.$$

Then $(X, M, *)$ is a fuzzy metric space induced by d. If $k = m = n = 1$, then the above equation reduces to $M(x, y, t) = \frac{t}{t+d(x,y)}$ is called the standard fuzzy metric.

Remark 2.9. $(X, M, *)$ is not a fuzzy metric space if the t-norm defined by $a * b = \min(a, b)$, for $a, b \in [0, 1]$ and the metric d on X and satisfying $M(x, y, t) = \frac{t}{t+d(x,y)}$.

Definition 2.10. A function f is upper semi-continuous at x_0 if for every $\epsilon > 0$, there exists a neighborhood U of x_0 such that for all $x \in U$, $f(x) < f(x_0) + \epsilon$. It is lower semi-continuous if $f(x) > f(x_0) - \epsilon$. If f is continuous at x_0, then for every $\epsilon > 0$ there exists U such that $f(x_0) - \epsilon < f(x) < f(x_0) + \epsilon$ for all $x \in U$.

Definition 2.11. ([13]) A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is convergent to $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if for each $\epsilon > 0$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.

George and Veeramani [1] showed that a sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ converges to a point $x \in X$ if and only if

$$\lim_{n\to\infty} M(x_n, x, t) = 1.$$

Definition 2.12. ([11]) A sequence $\{x_n\}$ in $(X, M, *)$ is Cauchy sequence if for each $\epsilon > 0$ and $t > 0$ there exists n_0 such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$. A fuzzy metric space $(X, M, *)$ is complete if every Cauchy sequence converges to a point in X.

Definition 2.13. ([17]) Let U, V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. The mappings U, V are said to be compatible if

$$\lim_{n\to\infty} M(UVx_n, VUx_n, t) = 1 \quad \text{for all} \quad t > 0,$$

where $\{x_n\}$ is a sequence in X with $\lim_{n\to\infty} Ux_n = \lim_{n\to\infty} Vx_n = s$ for some $s \in X$.

Definition 2.14. ([17]) Let U and V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. The mappings U and V are said to be compatible of type (A) if

$$\lim_{n\to\infty} M\big(UVx_n,\ VVx_n,\ t\big) = 1 \quad \text{and} \quad \lim_{n\to\infty} M\big(VUx_n,\ UUx_n,\ t\big) = 1$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ux_n \ = \ \lim_{n\to\infty} Vx_n \ = \ s$$

for some $s \in X$.

Definition 2.15. ([18]) Let U and V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. The mappings U and V are said to be compatible of type (R) if

$$\lim_{n\to\infty} M\big(UVx_n,\ VUx_n,\ t\big) = 1 \quad \text{and} \quad \lim_{n\to\infty} M\big(UUx_n,\ VVx_n,\ t\big) = 1$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ux_n \ = \ \lim_{n\to\infty} Vx_n \ = \ s$$

for some $s \in X$.

Definition 2.16. ([21]) Let U and V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. The mappings U and V are said to be compatible of type (K) if

$$\lim_{n\to\infty} M\big(UUx_n,\ Vs,\ t\big) = 1 \quad \text{and} \quad \lim_{n\to\infty} M\big(VVx_n,\ Us,\ t\big) = 1$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ux_n \ = \ \lim_{n\to\infty} Vx_n \ = \ s$$

for some $s \in X$.

Example 2.17. ([21]) Let $X = [0, 2]$ with the usual metric $d(x, y) = |x - y|$. Define

$$M(x, y, t) = \frac{t}{t + d(x, y)} \quad \text{for all } x, y \in X,\ t > 0$$

and

$$a * b = ab \quad \text{for all } a, b \in [0, 1].$$

Then $(X, M, *)$ is a fuzzy metric space. We define self-mappings U and V as follows:

$$U(x) = 2, \quad V(x) = 0, \quad \text{for } x \in [0, 1) - \left\{\tfrac{1}{2}\right\},$$
$$U\big(\tfrac{1}{2}\big) = 0, \quad V\big(\tfrac{1}{2}\big) = 2,$$
$$U(x) = \frac{2 - x}{2}, \quad V(x) = \frac{x}{2} \quad \text{for } x \in (1, 2].$$

Let us consider a sequence $\{x_n\}$ in X such that

$$x_n = 1 + \frac{1}{n} \quad \text{for all } n \in \mathbb{N}.$$

Hence, U and V are compatible mappings of type (K).

3. Some lemma, propositions, and common fixed point theorems

Lemma 3.1. ([20]) *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing in t for all $x, y \in X$.*

We can easily prove the following two propositions by definitions.

Proposition 3.2. *Let U and V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. If U and V are compatible mappings of type (K) on X and $Us = Vs$ for $s \in X$, then*

$$UUs = VVs.$$

Proposition 3.3. *Let U and V be mappings from a complete fuzzy metric space $(X, M, *)$ into itself. If U and V are compatible mappings of type (K) on X and*

$$\lim_{n\to\infty} Ux_n = \lim_{n\to\infty} Vx_n = s \quad \text{for some } s \in X,$$

then we have

$$\lim_{n\to\infty} M\big(VVx_n,\ Us,\ t\big) = 1, \quad \text{if } U \text{ is continuous,}$$

$$\lim_{n\to\infty} M\big(UUx_n,\ Vs,\ t\big) = 1, \quad \text{if } V \text{ is continuous.}$$

Moreover, if U and V are both continuous at s, then

$$UUs = VVs \quad \text{and} \quad Us = Vs.$$

Theorem 3.4. ([18]) *Let $(X,\ M,\ *)$ be a complete metric space and S, T, U and V be four mappings from X into itself. Suppose that U and V are continuous mappings which satisfy the following conditions:*

(i) *$S(X) \subset V(X)$ and $T(X) \subset U(X)$,*

(ii) *The pairs $\{S, U\}$ and $\{T, V\}$ are compatible of type (R),*

(iii)

$$M(Sx, Ty, t) \leq \Phi[\max\big\{M(Ux, Vy, t), M(Ux, Sx, t), M(Vy, Ty, t), \frac{1}{2}[M(Ux, Ty, t) + M(Vy, Sx, t)]\big\}],$$

for all $x, y \in X$, where $\Phi\colon [0, \infty) \to [0, \infty)$ is a non-decreasing and upper semi-continuous function such that $\Phi(t) < t$ for all $t > 0$.

Then S, T, U and V have a unique common fixed point in X.

4. Main results

Theorem 4.1. *Let $(X, M, *)$ be a complete fuzzy metric space. Let S, T, U and V be four self-mappings which satisfy:*

(i) *$S(X) \subset V(X)$ and $T(X) \subset U(X)$;*

(ii) *U and V are continuous;*

(iii) *$\{S, U\}$ and $\{T, V\}$ are compatible of type (K);*

(iv)

$$M(Sx, Ty, t) \leq \Phi[\max\big\{M(Ux, Vy, t), M(Ux, Sx, t), M(Vy, Ty, t), \frac{1}{2}[M(Ux, Ty, t) + M(Vy, Sx, t)]\big\}],$$

for all $x, y \in X$, where $\Phi : [0, \infty) \to [0, \infty)$ is non-decreasing, upper semi-continuous, and $\Phi(t) < t$ for all $t > 0$.

Then S, T, U, V have a unique common fixed point in X.

Proof. Let us consider $x_0 \in X$. Since $S(X) \subset V(X)$ and $T(X) \subset U(X)$, there exist $x_1, x_2 \in X$ such that

$$Sx_0 = Vx_1 \quad \text{and} \quad Tx_1 = Ux_2.$$

Inductively, we construct a sequence $\{x_n\}$ in X such that

$$Ux_{2n} = Tx_{2n-1}, \quad Vx_{2n-1} = Sx_{2n-2} \qquad \text{for } n = 1, 2, 3, \cdots .$$

Suppose that

$$y_{2n-1} = Vx_{2n-1} = Sx_{2n-2}, \qquad y_{2n} = Ux_{2n} = Tx_{2n-1} \quad \text{for } n = 1, 2, 3, \cdots . \tag{4.1}$$

Using condition (iv) and equation (4.1), we get

$$\begin{aligned} M(y_{2n+1}, y_{2n}, t) &= M(Sx_{2n}, Tx_{2n-1}, t) \\ &\leq \Phi \max[M(Ux_{2n}, Vx_{2n-1}, t), M(Ux_{2n}, Sx_{2n}, t), \\ &\quad M(Vx_{2n-1}, Tx_{2n-1}, t), \\ &\quad \tfrac{1}{2}(M(Ux_{2n}, Tx_{2n-1}, t) + M(Vx_{2n-1}, Sx_{2n}, t))] \\ &= \Phi \max[M(y_{2n}, y_{2n-1}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n-1}, y_{2n}, t), \\ &\quad \tfrac{1}{2}\big(M(y_{2n}, y_{2n}, t) + M(y_{2n-1}, y_{2n+1}, t)\big)] \\ &\leq M(y_{2n}, y_{2n-1}, t). \end{aligned} \tag{4.2}$$

To show

$$\lim_{n\to\infty} M(y_n, y_m, t) = 1, \quad \forall\, t > 0.$$

For all n, m and by the upper semi-continuity of $\Phi(t) < t$ for all $t > 0$, we have from (4.2):

$$M(y_{n+1}, y_n, t) \le M(y_n, y_{n-1}, t) \le M(y_{n-1}, y_{n-2}, t) \le \cdots \le M(y_1, y_0, t) \\ \to 1 \quad \text{as } n \to \infty,\ t > 0.$$

Hence, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, it converges to a point $s \in X$, that is,

$$y_n \to s \quad \text{as } n \to \infty.$$

Moreover, the subsequences $\{Sx_{2n-2}\}, \{Tx_{2n-1}\}, \{Ux_{2n}\}$ and $\{Vx_{2n-1}\}$ of $\{y_n\}$ converge to s, that is,

$$y_{2n} = Ux_{2n} = Tx_{2n-1} \to s, \quad y_{2n-1} = Vx_{2n-1} = Sx_{2n-2} \to s \quad \text{as } n \to \infty.$$

Since the pairs $\{S, U\}$ and $\{T, V\}$ are compatible of type (K), and by the continuity of U and V, equation (4.1), and Proposition 3.3, we have

$$\begin{cases} Vy_{2n} \to Vs \quad \text{and} \quad Ty_{2n} = TTx_{2n-1} \to Vs, \\ Uy_{2n-1} \to Us \quad \text{and} \quad Sy_{2n-1} = SSx_{2n-2} \to Us, \end{cases} \qquad \text{as } n \to \infty. \tag{4.3}$$

By using condition (iv) and equation (1), we get

$$M(Sy_{2n-1}, Ty_{2n}, t) \le \Phi\Big(\max\Big\{ M(Uy_{2n-1}, Vy_{2n}, t), \\ M(Uy_{2n-1}, Sy_{2n-1}, t),\ M(Vy_{2n}, Ty_{2n}, t), \\ \tfrac{1}{2}\big(M(Uy_{2n-1}, Ty_{2n}, t) + M(Vy_{2n}, Sy_{2n-1}, t)\big) \Big\}\Big).$$

If $Us \neq Vs$, then by applying the upper semi-continuity of $\Phi(t)$, equations (4.1) and (4.3), we have

$$M(Us, Vs, t) \le \Phi\Big(\max\Big\{ M(Us, Vs, t),\ M(Us, Us, t),\ M(Vs, Vs, t), \\ \tfrac{1}{2}\big(M(Us, Vs, t) + M(Vs, Us, t)\big) \Big\}\Big).$$

Since $M(Us, Us, t) = M(Vs, Vs, t) = 1$ and M takes values in $[0, 1]$, this simplifies to $M(Us,\ Vs,\ t) < M(Us,\ Vs,\ t)$. Since, $\Phi(t) < t$ for all $t > 0$, which is a contradiction. Hence,

$$Us = Vs. \tag{4.4}$$

Similarly, from condition (iv), equations (4.1), (4.3), and the upper semi-continuity of Φ, we can prove

$$Us = Ts \tag{4.5}$$

and

$$Vs = Ss. \tag{4.6}$$

Thus, from equations (4.4), (4.5), and (4.6), we get

$$Ss = Ts = Us = Vs. \tag{4.7}$$

Also, from condition (iv) and equation (4.1), we have

$$M(Sx_{2n}, Ts, t) \le \Phi\Big(\max \Big\{ M(Ux_{2n}, Vs, t),\ M(Ux_{2n}, Sx_{2n}, t),\ M(Vs, Ts, t), \\ \tfrac{1}{2}\big(M(Ux_{2n}, Ts, t) + M(Vs, Sx_{2n}, t)\big) \Big\}\Big).$$

If $Ts \neq s$, then

$$\begin{aligned} M(s, Ts, t) &\le \Phi\Big(\max \Big\{ M(s, Vs, t),\ M(s, s, t),\ M(Vs, Ts, t), \\ &\qquad \tfrac{1}{2}\big(M(s, Ts, t) + M(Vs, s, t)\big) \Big\}\Big) \\ &= \Phi\big(M(s, Ts, t)\big) < M(s, Ts, t), \end{aligned}$$

that is,

$$M(s, Ts, t) < M(s, Ts, t),$$

which is also a contradiction. Hence,

$$Ts = s. \tag{4.8}$$

Therefore, from equations (4.7) and (4.8), we obtain

$$Ss = Ts = Us = Vs = s.$$

Thus, s is a common fixed point of the mappings S, T, U and V.

To show the uniqueness of the common fixed point, let us consider w to be another common fixed point of S, T, U and V. Then

$$Sw = Tw = Uw = Vw = w.$$

We want to show that $s = w$. If possible, let us assume $s \neq w$. Then

$$\begin{aligned} M(s, w, t) &= M(Ss, Tw, t) \\ &\le \Phi\Big(\max \Big\{ M(Us, Vw, t),\ M(Us, Ss, t),\ M(Vw, Tw, t), \\ &\qquad \tfrac{1}{2}\big(M(Us, Tw, t) + M(Vw, Ss, t)\big) \Big\}\Big). \end{aligned}$$

Since $Ss = s$, $Tw = w$, and $Sw = Tw = Uw = Vw = w$, this reduces to

$$M(s, w, t) \le \Phi\big(M(Us, Vw, t)\big) = \Phi\big(M(s, w, t)\big) < M(s, w, t).$$

that is,

$$M(s, w, t) < M(s, w, t),$$

which is a contradiction, so $s = w$. This completes the proof. □

Corollary 4.2. *Let $(X, M, *)$ be a complete fuzzy metric space and U and V be two mappings from X into itself which satisfy the following conditions:*

(i) *U and V are continuous mappings,*
(ii) *For all $x, y \in X$ and $t > 0$,*

$$M(Ux, Vy, t) \leq \Phi\Big(\max\Big\{M(x,y,t),\ M(x,Ux,t),\ M(y,Vy,t),\ \tfrac{1}{2}\big(M(x,Vy,t)+M(y,Ux,t)\big)\Big\}\Big),$$

where $\Phi\colon [0,\infty) \to [0,\infty)$ is a non-decreasing and upper semi-continuous function such that $\Phi(t) < t$ for all $t > 0$.

Then U and V have a unique common fixed point in X.

Proof. Let us construct a sequence $\{x_n\}$ in X such that

$$x_{2n-1} = Ux_{2n-2}, \quad x_{2n} = Vx_{2n-2}, \tag{4.9}$$

for $n = 1, 2, 3, \cdots$. Then we can easily show that $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there exists $s \in X$ such that

$$x_n \to s \quad \text{as } n \to \infty.$$

Consequently, the subsequences $\{x_{2n-1}\}$ and $\{x_{2n}\}$ also converge to s.

From condition (ii) of this corollary and equation (4.9), we get

$$\begin{aligned} M(Us, x_{2n}, t) &= M(Us, Vx_{2n-2}, t) \\ &\leq \Phi\Big(\max\Big\{M(s,x_{2n-2},t),\ M(s,Us,t),\ M(x_{2n-2},Vx_{2n-2},t), \\ &\qquad \tfrac{1}{2}\big(M(s,Vx_{2n-2},t)+M(x_{2n-2},Us,t)\big)\Big\}\Big). \end{aligned}$$

Since $x_{2n} = Vx_{2n-2}$, this reduces to

$$\begin{aligned} M(Us, x_{2n}, t) \leq \Phi\Big(\max\Big\{&M(s,x_{2n-2},t),\ M(s,Us,t),\ M(x_{2n-2},x_{2n},t), \\ &\tfrac{1}{2}\big(M(s,x_{2n},t)+M(x_{2n-2},Us,t)\big)\Big\}\Big). \end{aligned}$$

If $Us \neq s$, then by the condition of upper semi-continuity of $\Phi(t)$, we get

$$M(Us, s, t) \leq \Phi\big(M(s,Us,t)\big) < M(s,Us,t) = M(Us,s,t),$$

that is,

$$M(Us, s, t) < M(Us, s, t),$$

which is a contradiction. Hence, $s = Us$.

A similar argument shows that $s = Vs$. Therefore, s is the unique common fixed point of U and V. □

We can apply the Theorem 4.1 in sequential form as stated below:

Theorem 4.3. *Let $(X, M, *)$ be a complete fuzzy metric space and U, V and $\{S_n\}$ be three mappings from X into itself, where $n = 1, 2, 3, \cdots$. Assume further that U and V are continuous and for each $n \in \mathbb{N}$, the pairs $\{S_{2n-1}, U\}$ and $\{S_{2n}, V\}$ are compatible of type (K), which satisfy the following conditions for any $n \in \mathbb{N}$:*

(i) $S_{2n-1}(X) \subset V(X)$ *and* $S_{2n}(X) \subset U(X)$,

(ii) *For all $x, y \in X$ and $t > 0$,*

$$M(S_nx, S_{n+1}y, t) \le \Phi\Big(\max\Big\{M(Ux, Vy, t),\ M(Ux, S_nx, t), M(Vy, S_{n+1}y, t), \tfrac{1}{2}\big(M(Ux, S_{n+1}y, t) + M(Vy, S_nx, t)\big)\Big\}\Big),$$

where $\Phi\colon [0, \infty) \to [0, \infty)$ is a non-decreasing and upper semi-continuous function such that $\Phi(t) < t$ for all $t > 0$.

Then S_n, U, and V have a unique common fixed point in X.

5. Conclusion

In this paper, some fixed point theorems for four self-mappings of compatible maps of type (K) are established, generalizing and extending prior results in fuzzy metric spaces.

Acknowledgments: The authors would like to thank the anonymous referees for their insightful comments on how to make this article better. Also, authors thank to University Grant Commission, Nepal for encouraging the researchers with financial support.

References

[1] M.S. Bhadauriya and R. Gangil, *Fixed point theorem of type (R) using implicit relation in fuzzy metric space,* IRJET, **3**(1) (2016).

[2] A.K. Chaudhary, *Occasionally weakly compatible mappings and common fixed points in Menger space,* Results in Nonlinear Anal., **6**(4) (2023), 47–54.

[3] A.K. Chaudhary, *Theorems on compatible mapping types in complete Menger space,* The Nepali Math. Sci. Report, **41**(2) (2024), 105–121.

[4] A.K. Chaudhary, *Control function in Menger space,* Nonlinear Funct. Anal. Appl., **30**(1) (2025), 265–276.

[5] A.K. Chaudhary, *A Research Book for fixed point theory in probabilistic metric space,* Sukunda Pustak Bhawan, Kathmandu, first edition, 2025.

[6] A. K. Chaudhary, *A Research Book in metric spaces to probabilistic metric spaces,* Dreamland Publication, Kathmandu, first edition, 2025.

[7] A.K. Chaudhary and K. Jha, *Contraction Conditions in Probabilistic Metric Space,* Amer. J. Math. Stat., **9**(5) (2019), 199–202.

[8] A.K. Chaudhary, K.B. Manandhar, K. Jha and P.P. Murthy, *A common fixed point theorem in Menger space with compatible mapping of type (P),* Int. J. Math. Sci. Engg. Appl., **15**(2) (2021), 59–70.
[9] Y.J. Cho, *Fixed point in fuzzy metric space,* J. Fuzzy Math, **5** (1997), 949–962.
[10] Y.J. Cho, H.K. Pathak, S.M. Kang and J.S. Jung, *Common fixed points of compatible mappings of type (B) on fuzzy metric space,* Fuzzy Sets Syst., **93** (1998), 99–110.
[11] A. George and P. Veeramani, *On some results in fuzzy metric spaces,* Fuzzy Sets Syst., **64** (1994), 395–399.
[12] A. George and P. Veeramani, *On some results of analysis for fuzzy metric spaces,* Fuzzy sets Syst., **90** (1997), 365–368.
[13] M. Grabiec, *Fixed points in fuzzy metric spaces,* Fuzzy Sets Syst., **27** (1988), 385–389.
[14] K. Jha, *A common fixed point theorem for sequence of mapping in fuzzy metric space,* J. Fuzzy Math., **3** (2010), 1–7.
[15] G. Jungck, *Compatible mappings and common fixed points,* Int. J. Math. Math. Sci., **9**(4) (1986), 771–779.
[16] G. Jungck, P.P. Murthy and Y.J. Cho, *Compatible mappings of type (A) and common fixed points,* Math. Jpn., **38**(2) (1993), 381–390.
[17] M. Koireng and Y. Rohen, *Common fixed point theorem of compatible mappings of type (P) in fuzzy metric spaces,* IJMA, **6**(4) (2012), 181–188.
[18] M. Koireng and Y. Rohen, *Some fixed point theorems of compatible mappings of type (R) in fuzzy metric spaces,* Int. J. Math. Sci. Engg. Appl., **11**(3) (2017), 81–88.
[19] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces,* Kybernetika, **11** (1975), 336–344.
[20] K.B. Manandhar, K. Jha and H.K. Pathak, *A common fixed point theorem for compatible mappings of type (E) in fuzzy metric space,* Appl. Math. Sci., **8** (2014), 2007–2014.
[21] K.B. Manandhar, K. Jha and G. Porru, *Common fixed point theorem of compatible mappings of type (K) in fuzzy metric spaces,* Elect.J. Math. Anal. Appl., **2**(2) (2014), 248–253.
[22] S.N. Mishra, N. Sharma and S.L. Singh, *Common fixed points of maps on fuzzy metric spaces,* Int. J. Math. Math. Sci., **17**(2) (1994), 253–258.
[23] H.K. Pathak, S.S. Chang and Y.J. Cho, *Fixed point theorems for compatible mappings of type (P),* Indian J. Math., **36**(2) (1994), 151–166.
[24] H.K. Pathak and M.S. Khan, *Compatible mappings of type (B) and common fixed point theorems of Greguš type,* Czechoslov. Math. J., **45**(4) (1995), 685–698.
[25] Y. Rohen, *Fixed point theorems of compatible mappings of type (R) in metric spaces,* Int. J. Open Prob. Comput. Math., **3**(5) (2010), 128–143.
[26] Y. Rohen and M.R. Singh, Ĉommon fixed points of compatible mappings of type (R) in complete metric spaces, Int. J. Math. Sci. Engg. Appl., **2**(4) (2008), 295–303.
[27] B. Schweizer and A. Sklar, *Statistical metric spaces,* Pacific J. Math., **10**(1) (1960), 313–334.
[28] B. Singh and M.S. Chauhan, *Common fixed points of compatible maps in fuzzy metric spaces,* Fuzzy Sets Syst., **115** (2000), 471–475.
[29] B. Singh and M.S. Chauhan, *On common fixed points of four mappings,* Bull. Cal. Math. Soc., **88** (1996), 451–456.
[30] M.R. Singh and Y.M. Singh, *Compatible mappings of type (E) and common fixed point theorems of Meir–Keeler type,* Int. J. Math. Sci. Engg. Appl., **1** (2007), 299–315.
[31] L.A. Zadeh, *Fuzzy sets,* Inform. and Control, **8** (1965), 338–353.