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# THE NEUTROSOPHIC POISSON DISTRIBUTION AND THE GENERALIZED BIVARIATE FIBONACCI-LIKE POLYNOMIAL ESTIMATE COEFFICIENTS FOR SUBCLASSES OF BI-UNIVALENT FUNCTIONS

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**Abstract.** In this paper, we introduce and investigate several new subclasses of bi-univalent functions by employing the Neutrosophic Poisson distribution in conjunction with a generalized family of bivariate Fibonacci-like polynomials, which includes the well-known Horadam and Chebyshev polynomials as special cases. By applying analytic and geometric function theory techniques, we obtain explicit upper bounds for the initial TaylorMaclaurin coefficients of functions belonging to these subclasses. Moreover, the classical Fekete-Szegö problem is examined and the corresponding estimates are derived.

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## 1. INTRODUCTION

Orthogonal polynomials were first introduced by Legendre in 1784 [23] and have since become a fundamental tool in mathematical analysis. They are widely used in the solution of ordinary differential equations under various modeling constraints and play a central role in approximation theory and numerical analysis [16, 17].

In recent years, the incorporation of special polynomial families such as Chebyshev, Faber, Horadam, Lucas, and Fibonacci polynomials into geometric function theory has attracted considerable attention. This growing interest has led to the introduction of numerous subclasses of univalent and bi-univalent functions that are holomorphic in the open unit disk.

By utilizing the classical notion of subordination along with fundamental techniques from geometric function theory in complex analysis, several well-known problems, including initial coefficient estimates, the Fekete-Szegö functional, and Hankel determinant problems, have been systematically investigated for these newly defined subclasses of holomorphic functions.

Motivated by the intrinsic connections between special polynomials and classes of holomorphic functions, the present paper introduces a new subfamily of holomorphic and bi-univalent functions. Sharp upper bounds for the first two TaylorMaclaurin coefficients of functions belonging to these subclasses are derived. Furthermore, the Fekete-Szegö problem is addressed within this framework. Several corollaries and remarks are finally presented to illustrate the applicability of the main results and to highlight their relevance to existing literature.

Let $\mathcal{A}$ be the class of regular functions $f\ (\aleph)$ of the form

$$f(\aleph) = \aleph + \rho_2\aleph^2 + \rho_3\aleph^3 + \cdots = \aleph + \sum_{n=2}^{\infty} \rho_n \aleph^n. \qquad (1.1)$$

are univalent in the open unit disk $\mathbb{U} = \{\aleph \in \mathbb{C} : |\aleph| < 1\}$ and satisfy conditions $f(0) = 0$ and $f'(0) = 1$ which are normalized conditions. Let $S$ denote the subclass of $\mathcal{A}$ consisting of functions of the form (1.1).

Due to the well-known Koebe one quarter theorem [18], it is said that $f \in S$ has an inverse $f^{-1}$ defined by $f^{-1}(f(\aleph)) = \aleph$, $(\aleph \in \mathbb{U})$ and $f(f^{-1}(\Finv)) = \Finv$, $(|\Finv| < r_0(f), r_0(f) \geq \frac{1}{4})$, where

$$f^{-1}(\Finv) = \Finv - \rho_2\Finv^2 + \left(2\rho_2^2 - \rho_3\right)\Finv^3 - \left(5\rho_2^3 - 5\rho_2\rho_3 + \rho_4\right)\Finv^4 + \cdots = g(\Finv). \qquad (1.2)$$

It is well known that if a function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ then both $f$ and $f^{-1}$ are univalent in $\mathbb{U}$. Let $\Sigma$ be the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). In fact, very recently Srivastava et al. [27]. In fact, others are reviving the study of holomorphic and bi-univalent functions in recent years, see, for example, [1, 3, 4, 6, 7, 11, 13, 15, 26, 29].

Generalized bivariate Fibonacci-like polynomial. The Chebyshev, Faber, Horadam, Lucas, and Fibonacci polynomials and their generalizations are of significant importance in applied sciences, including physics, engineering, and related fields. Among these, the Fibonacci polynomial is one of the most prominent special polynomials. Due to its extensive applications in applied sciences, several generalizations of the Fibonacci polynomial have been introduced in the literature (see [22]).

The following recurrence relation defines the Fibonacci numbers:

$$F_n = F_{n-1} + F_{n-2}; \quad F_0 = 0; \quad F_1 = 1,$$

for $n \geq 2$, the reader can find a concise history and comprehensive information about the generalized bivariate Fibonacci polynomial in [25] and its references. Additionally, the authors in [25] introduced a new generalization of the Fibonacci polynomial, referred to as the generalized bivariate Fibonacci-like polynomial.

Let $p, q$ be positive integers and $x, y$ be real numbers. For $n \geq 2$, generalized bivariate Fibonacci-like polynomials are defined by the following recurrence relation.

$$H_n(x,y) = pxH_{n-1}(x,y) + qyH_{n-2}(x,y), \tag{1.3}$$

where $H_0(x,y) = a$; $H_1(x,y) = b$ and $px$, $qy \neq 0$, $p^2x^2$ $+4qy \neq 0$. The generating functions of generalized bivariate Fibonacci-like polynomials is (see [25])

$$H^{(x,y)}(\aleph) = \sum_{n=0}^{\infty} H_0(x,y)\aleph^n = \frac{a + (b - apx)\aleph}{1 - px\aleph - qy\aleph^2}, \tag{1.4}$$

the first three generalized bivariate Fibonacci-like polynomials are expressed as follows:

$$H_0(x,y) = a, \; H_1(x,y) = b, \; H_2(x,y) = pbx + aqy. \tag{1.5}$$

By selecting different values for $p, q, a, b$, and y, various polynomial sequences can be derived using the recurrence relation. These polynomial sequences are summarized in Table 1 below:

| $(p,q)$ | $(a,b)$ | $(x,y)$ | $H_n(x,y)$ |
|---|---|---|---|
| $(1,1)$ | $(0,1)$ | $(x,y)$ | Bivariate Fibonacci, $F_n(x,y)$ |
| $(1,1)$ | $(0,1)$ | $(x,1)$ | Fibonacci, $F_n(x)$ |
| $(2,1)$ | $(0,1)$ | $(x,1)$ | Pell, $P_n(x)$ |
| $(1,1)$ | $(2,x)$ | $(x,y)$ | Bivariate Lucas, $L_n(x,y)$ |
| $(2,1)$ | $(1,2t)$ | $(t,-1)$ | Chebyshev of the second kind, $U_n(x)$ |
| $(p,q)$ | $(a,bx)$ | $(x,1)$ | Horadam, $H_{n+1}(x)$ |

Table 1: Special cases of the generalized bivariate Fibonacci-like polynomials

The Poisson, Pascal, Borel, Mittag-Leffler-type Poisson distributions, among others, have been extensively used in recent studies to explore key aspects of geometric function theory (see [9, 8, 24, 30, 28, 20, 5]). These studies include topics such as coefficient estimates, inclusion relations, and criteria for membership in specific function classes.

In 1995, Smarandache introduced neutropophy as a new branch of philosophy, serving as a generalization of both fuzzy logic and intrinsic fuzzy logic (see [2]). Although the neutrosophic Poisson distribution of a discrete variable $X$ is fundamentally a classical Poisson distribution, its parameter $m$ is not fixed. Instead, $m$ can be specified as a range or interval encompassing two or more elements. This type of distribution is most commonly encountered when $m$ is represented as an interval. Let

$$NP\left(X=d\right)=\frac{(m)^d}{d!}e^{-m},\quad d=0,1,2,..., \tag{1.6}$$

where the expected value and the variance, denoted by the distribution parameter $m$

$$NL(X)=NV(X)=m\ ,$$

a neutrosophic statistical number is $N=O+I$; for more information, see [2] and the sources therein. We are going to provide a new power series, and the coefficients of this series will be the probabilities of the neutrosophic Poisson distribution

$$\mathbb{B}(m,\aleph)=\aleph+\sum_{n=2}^{\infty}\frac{(m)^{n-1}e^{-m}}{(n-1)!}\aleph^n,\qquad \aleph\in\mathbb{U}. \tag{1.7}$$

Consider the linear operator $\mathbb{P}_m:\mathcal{A}\to\mathcal{A}$ defined by the convolution

$$\mathbb{P}_l f(\aleph)=\mathbb{B}(m,\aleph)*f(\aleph)=\aleph+\sum_{n=2}^{\infty}\frac{(m)^{n-1}e^{-m}}{(n-1)!}\rho_n\aleph^n,\qquad \aleph\in\mathbb{U}. \tag{1.8}$$

Nowadays, many researchers have been studying bi-univalent functions associated with orthogonal polynomials, few to mention [10, 12, 14, 21, 31]. It is important to mention that a very little is known about Generalized bivariate Fibonacci-like polynomial in the context of bi-univalent functions as far as we know.

We study some new subclasses of bi-univalent functions subordinate to Generalized bivariate Fibonacci-like polynomial and obtain bounds for the $|\rho_2|$ and $|\rho_3|$ coefficients and Fekete-Szegö functional problems for functions in these new classes.

## 2. Definition and examples

The following new subclasses of bi-univalent functions are presented in this section:

**Definition 2.1.** Suppose that $0 \leq Y \leq 1$: It is claimed that a function $f \in \Sigma$ of the type (1.1) belongs to the class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ if the subordinations listed below are true:

$$(1-Y)\frac{\mathbb{P}_l f(\aleph)}{\aleph} + Y\left(\mathbb{P}_l f(\aleph)\right)' \prec (h(\aleph))$$

and

$$(1-Y)\frac{\mathbb{P}_l f(\daleth)}{\daleth} + Y\left(\mathbb{P}_l f(\daleth)\right)' \prec (h(\daleth)),$$

where the function $g$ has the form (1.2), $\aleph; \daleth \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

**Example 2.2.** Let $Y = 0$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{n,\Sigma,0}(p,q,x,y)(h(\aleph))$ if it satisfies the subordinations:

$$\frac{\mathbb{P}_l f(\aleph)}{\aleph} \prec (h(\aleph))$$

and

$$\frac{\mathbb{P}_l f(\daleth)}{\daleth} \prec (h(\daleth)),$$

where the function $g$ has the form (1.2), $\aleph; \daleth \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

**Example 2.3.** Let $Y = 1$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{n,\Sigma,1}(p,q,x,y)(h(\aleph))$ if it satisfies the subordinations:

$$(\mathbb{P}_l f(\aleph))' \prec (h(\aleph))$$

and

$$(\mathbb{P}_l f(\daleth))' \prec (h(\daleth)),$$

where the function $g$ has the form (1.2), $\aleph; \daleth \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

## 3. COEFFICIENT BOUNDS OF THE CLASS $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ AND SPECIAL CASE

First, we give the coefficient estimates for the class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ given in Definition 2.1.

**Theorem 3.1.** *Let* $f \in \mathcal{A}$ *by given* (1.1) *and* $Y \in \mathbb{R}$. *Then* $f \in \mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$,

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{\left|\left[(1+2Y)b^2 - 2(pbx+aqy)(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}}$$

*and*

$$|\rho_3| \leq \frac{2b}{(1+2Y)m^2e^{-m}} + \frac{b^2}{(1+Y)^2m^2e^{-2m}}.$$

*Proof.* Let $f \in \mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$. Then there are two analytic functions $\phi$, $\varrho$ given by

$$\phi(\aleph) = c_1\aleph + c_2\aleph^2 + c_3\aleph^3 + \cdots \quad (\aleph \in \mathbb{U}) \tag{3.1}$$

and

$$\varrho(\daleth) = d_1\daleth + d_2\daleth^2 + d_3\daleth^3 + \cdots \quad (\daleth \in \mathbb{U}), \tag{3.2}$$

with $\phi(0) = \varrho(0) = 0$, $|\phi(\aleph)| < 1$, $|\varrho(\daleth)| < 1$, $\aleph, \daleth \in \mathbb{U}$ such that

$$(1-Y)\frac{\mathbb{P}_l f(\aleph)}{\aleph} + Y\left(\mathbb{P}_l f(\aleph)\right)' = h(x,\phi(\aleph)) \tag{3.3}$$

and

$$(1-Y)\frac{\mathbb{P}_l f(\daleth)}{\daleth} + Y\left(\mathbb{P}_l f(\daleth)\right)' = h(x,\varrho(\daleth)). \tag{3.4}$$

Combining (3.1), (3.2) and (3.3) yields

$$\begin{aligned}&(1-Y)\frac{\mathbb{P}_l f(\aleph)}{\aleph} + Y\left(\mathbb{P}_l f(\aleph)\right)' \\ &= 1 + H_1(x,y)c_1\aleph + \left[H_1(x,y)c_2 + H_2(x,y)c_1^2\right]\aleph^2 + \cdots\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}&(1-Y)\frac{\mathbb{P}_l f(\daleth)}{\daleth} + Y\left(\mathbb{P}_l f(\daleth)\right)' \\ &= 1 + H_1(x,y)d_1\daleth + \left[H_1(x,y)d_2 + H_2(x,y)d_1^2\right])\daleth^2 + \cdots .\end{aligned} \tag{3.6}$$

It is quite well known that if $|\phi(\aleph)| < 1$ and $|\varrho(\daleth)| < 1$, $\aleph, \daleth \in \mathbb{U}$, then

$$|c_i| \leq 1 \quad \text{and} \quad |d_i| \leq 1 \text{ for all } i \in \mathbb{N}. \tag{3.7}$$

Comparing the corresponding coefficients in (3.5) and (3.6), after simplifying, we have

$$(1+Y)me^{-m}\rho_2 = H_1(x,y)c_1, \tag{3.8}$$

$$\frac{1}{2}(1+2Y)m^2e^{-m}\rho_3 = H_1(x,y)c_2 + H_2(x,y)c_1^2, \tag{3.9}$$

$$-(1+Y)me^{-m}\rho_2 = H_1(x,y)d_1, \tag{3.10}$$

and

$$\frac{1}{2}(1+2Y)m^2e^{-m}\left[2\rho_2^2-\rho_3\right] = H_1(x,y)d_2 + H_2(x,y)d_1^2. \tag{3.11}$$

It follows from (3.8) and (3.10) that

$$c_1 = -d_1 \tag{3.12}$$

and

$$2(1+Y)^2m^2e^{-2m}\rho_2^2 = [H_1(x,y)]^2\left(c_1^2+d_1^2\right). \tag{3.13}$$

If we add (3.9) to (3.11), we find that

$$(1+2Y)m^2e^{-m}\rho_2^2 = H_1(x,y)\left(c_2+d_2\right) + H_1(x,y)\left(c_1^2+d_1^2\right). \tag{3.14}$$

Substituting the value of $c_1^2+d_1^2$ from (3.13) in the right hand side of (3.14), we deduce that

$$\rho_2^2 = \frac{[H_1(x,y)]^3\left(c_2+d_2\right)}{\left[\left(H_1(x,y)\right)^2(1+2Y) - 2H_2(x,y)(1+Y)^2e^{-m}\right]m^2e^{-m}}. \tag{3.15}$$

Further computations using (1.5), (3.7) and (3.15), we obtain

$$|\rho_2| \le \frac{|b|\sqrt{|2b|}}{\sqrt{\left|\left[(1+2Y)b^2 - 2(pbx+aqy)(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}}.$$

Next, if we subtract (3.11) from (3.9), we can easily see that

$$(1+2Y)m^2e^{-m}(\rho_3-\rho_2^2) = H_1(x,y)(c_2-d_2) + H_2(x,y)(c_1^2-d_1^2). \tag{3.16}$$

In view of (3.12) and (3.13), we get from (3.16)

$$\rho_3 = \frac{H_1(x,y)(c_2-d_2)}{(1+2Y)m^2e^{-m}} + \frac{[H_1(x,y)]^2\left(c_1^2+d_1^2\right)}{2(1+Y)^2m^2e^{-2m}}.$$

Thus applying (1.5), we obtain

$$|\rho_3| \le \frac{b}{(1+2Y)m^2e^{-m}} + \frac{b^2}{(1+Y)^2m^2e^{-2m}}.$$

This completes the proof of Theorem 3.1. □

**Remark 3.2.** By giving different values to the parameters in Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the generalized bivariate Fibonacci-like polynomials, respectively.

(1) Assume that $Y = 0$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,0}(p,q,x,y)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{|[b^2 - 2e^{-m}(pbx + aqy)]\, m^2 e^{-m}|}}$$

and

$$|\rho_3| \leq \frac{|2b|}{m^2 e^{-m}} + \frac{b^2}{m^2 e^{-2m}}.$$

(2) Assume that $Y = 1$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,1}(p,q,x,y)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{|[3b^2 - 8e^{-m}(pbx + aqy)]\, m^2 e^{-m}|}}$$

and

$$|\rho_3| \leq \frac{|2b|}{3m^2 e^{-m}} + \frac{b^2}{4m^2 e^{-2m}}.$$

**Remark 3.3.** By giving $b = bx$, $y = 1$ in class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Horadam polynomials, respectively.

(1) Assume that $Y = 0$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,0}(p,q,x,1)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{|bx|\sqrt{|2bx|}}{\sqrt{|[(bx)^2 - 2e^{-m}(pbx^2 + aq)]\, m^2 e^{-m}|}}$$

and

$$|\rho_3| \leq \frac{|2bx|}{m^2 e^{-m}} + \frac{(bx)^2}{m^2 e^{-2m}}.$$

(2) Assume that $Y = 1$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,1}(p,q,x,1)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{|bx|\sqrt{|2bx|}}{\sqrt{|[3(bx)^2 - 8e^{-m}(pbx^2 + aq)]\, m^2 e^{-m}|}}$$

and

$$|\rho_3| \leq \frac{|2bx|}{3m^2 e^{-m}} + \frac{(bx)^2}{4m^2 e^{-2m}}.$$

**Remark 3.4.** By giving $p = q = 1, a = 0, b = 1, y = 1$ in class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Fibonacci polynomials, respectively.

(1) Assume that $Y = 0$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,0}(1,1,x,1)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{\left|\left[1-2e^{-m}x\right] m^2 e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{2}{m^2 e^{-m}} + \frac{1}{m^2 e^{-2m}}.$$

(2) Assume that $Y = 1$, let (1.1) be in the class $\mathfrak{G}_{n,\Sigma,1}(1,1,x,1)(h(\aleph))$. Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{\left|\left[3-8e^{-m}x\right] m^2 e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{2}{3m^2 e^{-m}} + \frac{1}{4m^2 e^{-2m}}.$$

## 4. FEKETE–SZEGÖ INEQUALITY FOR THE CLASS $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$

Fekete-Szegö inequality is one of the famous problems related to coefficients of univalent analytic functions [19]. In this section, our aim is to study Fekete-Szegö type inequality for the function in the class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$.

**Theorem 4.1.** *Let $f \in \mathcal{A}$ by given* (1.1) *and $Y \in \mathbb{R}$. Then*
*$f \in \mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$ and*

$$\left|\rho_3 - \eta\rho_2^2\right| \leq \begin{cases} \frac{|2b|}{(1+2Y)m^2e^{-m}}, & |\eta-1| \leq \ell, \\ \frac{2|b|^3|1-\eta|}{\left|\left[(1+2Y)b^2-2(pbx+aqy)(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}, & |\eta-1| \geq \ell, \end{cases}$$

*where*

$$\ell = \left|1 - \frac{\left[2(1+Y)^2(pbx+aqy)\right]e^{-m}}{(1+2Y)\,b^2}\right|.$$

*Proof.* It follows from (3.15) and (3.16) that

$$\begin{aligned}\rho_3-\eta\rho_2^2 &= \frac{H_1(x,y)(c_2-d_2)}{(1+2Y)\,m^2e^{-m}}+(1-\eta)\,\rho_2^2\\ &= \frac{H_1(x,y)(c_2-d_2)}{(1+2Y)\,m^2e^{-m}}\\ &\quad+\frac{[H_1(x,y)]^3\,(c_2+d_2)\,(1-\eta)}{\left[(H_1(x,y))^2\,(1+2Y)-2H_2(x,y)(1+Y)^2e^{-m}\right]m^2e^{-m}}\\ &= H_1(x,y)\Big[\Big(\top(n)+\frac{1}{(1+2Y)m^2e^{-m}}\Big)c_2\\ &\quad+\Big(\top(n)-\frac{1}{(1+2Y)m^2e^{-m}}\Big)d_2\Big],\end{aligned}$$

where

$$\top(n)=\frac{[H_1(x,y)]^2\,(1-\eta)}{\Big([H_1(x,y)]^2\,(1+2Y)-2H_2(x,y)(1+Y)^2e^{-m}\Big)\,m^2e^{-m}}.$$

According to (1.5), we find that

$$\left|\rho_3-\eta\rho_2^2\right|\leq\begin{cases}\frac{2|H_1(x,y)|}{(1+2Y)m^2e^{-m}} & |\top(n)|\leq\frac{1}{|(1+2Y)m^2e^{-m}|},\\ 2\,|H_1(x,y)|\,|\top(n)| & |\top(n)|\geq\frac{1}{|(1+2Y)m^2e^{-m}|}.\end{cases}$$

This completes the proof of Theorem 4.1. □

**Remark 4.2.** Let (3.1) be in the class $\mathfrak{G}_{n,\Sigma,Y}(p,q,x,y)(h(\aleph))$. By taking some special values in the parameters in Theorem (4.1) we obtain the followings:

(1) Assume that $\eta=1$,

$$\left|\rho_3-\rho_2^2\right|\leq\frac{2b}{(1+2Y)m^2e^{-m}}.$$

(2) Assume that $Y=0$,

$$\left|\rho_3-\eta\rho_2^2\right|\leq\begin{cases}\frac{|2b|}{m^2e^{-m}}, & |\eta-1|\leq\left|1-\frac{[2(pbx+aqy)]e^{-m}}{b^2}\right|,\\ \frac{2|b|^3|1-\eta|}{|[b^2-2(pbx+aqy)e^{-m}]m^2e^{-m}|}, & |\eta-1|\geq\left|1-\frac{[2(pbx+aqy)]e^{-m}}{b^2}\right|.\end{cases}$$

(3) Assume that $Y=1$,

$$\left|\rho_3-\eta\rho_2^2\right|\leq\begin{cases}\frac{|2b|}{3m^2e^{-m}}, & |\eta-1|\leq\left|1-\frac{[8(pbx+aqy)]e^{-m}}{3b^2}\right|,\\ \frac{2|b|^3|1-\eta|}{|[3b^2-8(pbx+aqy)e^{-m}]m^2e^{-m}|}, & |\eta-1|\geq\left|1-\frac{[8(pbx+aqy)]e^{-m}}{3b^2}\right|.\end{cases}$$

## 5. Concluding remark

In this paper, by making use of the generalized bivariate Fibonacci-like polynomials two new subclasses of analytic bi-univalent functions are introduced. Firstly, initial coefficients estimates are discussed and then the well-known Fekete-Szego problem for these subclasses are solved. Note that, if we take$p; q; x; y; a$ and b be as certain values, our evaluations cover most studies in the literature.

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