



MEASURE-THEORETIC AND FIXED POINT ANALYSIS IN NEUTROSOPHIC MR-METRIC SPACES

Abed Al-Rahman M. Malkawi¹ and Ayat M. Rabaiah²

¹Department of Mathematics, Faculty of Arts and Science, Amman Arab University,
Amman 11953, Jordan
e-mail: a.malkawi@aaau.edu.jo; math.malkawi@gmail.com

²Department of Mathematics, Faculty of Arts and Science, Amman Arab University,
Amman 11953, Jordan
e-mail: a.rabaieha@aaau.edu.jo

Abstract. This research delves into the intricate structure of Neutrosophic MR-Metric Spaces (NMR-MS), a novel framework that synergizes the ternary relational logic of MR-metrics with the uncertainty modeling capabilities of neutrosophic set theory. We establish fundamental measure-theoretic foundations for these spaces by constructing a unique, regular, and Borel-regular measure derived from the neutrosophic topology. This is achieved through a specialized application of Carathéodory's extension theorem, tailored to the neutrosophic context. Furthermore, we prove a powerful fixed point theorem for neutrosophic contraction mappings within complete NMR-MS. This theorem not only guarantees the existence and uniqueness of fixed points but also establishes their convergence μ -almost everywhere, provides measure-theoretic stability, and demonstrates an exponential convergence rate in measure. The theoretical developments are substantiated with illustrative examples and applied to solve fractional differential equations and analyze optimization algorithms, showcasing the versatility and applicability of the proposed framework in analysis and computational mathematics.

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⁰Corresponding author: Abed Al-Rahman M. Malkawi(a.malkawi@aaau.edu.jo).

1. INTRODUCTION

The evolution of metric space theory has been significantly enriched by generalizations that accommodate more complex structural and logical relationships. The introduction of b -metric spaces by Bakhtin [10] and Czerwik [17] marked a pivotal step, relaxing the standard triangle inequality. This line of thought was further extended with the development of G -metric spaces and Ω -distance mappings [15, 16, 49, 50], which consider triplets of points, offering a richer geometric context. Building upon these, Malkawi et al. [37] introduced the concept of MR-metric spaces, characterized by a ternary mapping and a generalized triangle inequality involving a constant $R > 1$, providing a robust framework for analyzing ternary relations.

Concurrently, the need to model uncertainty, imprecision, and indeterminacy in mathematical structures led to the fusion of metric spaces with fuzzy and neutrosophic logics. Neutrosophic logic, which generalizes fuzzy logic by independently considering truth (\mathcal{T}), falsity (\mathcal{F}), and indeterminacy (\mathcal{I}) membership functions, has been instrumental in this regard. Recent works have explored neutrosophic metric spaces [12, 22, 26, 29] and their fixed point theory [23, 24, 25, 27, 28, 30, 33, 34, 35, 36, 38, 39, 40, 41, 42, 45]. The integration of neutrosophic logic into the MR-metric framework, resulting in the Neutrosophic MR-Metric Space (NMR-MS) [29], represents a significant synthesis, allowing for the simultaneous handling of sophisticated distance relations and multi-faceted uncertainty.

Parallel advancements in applied mathematics, such as the study of fractional differential equations [2, 8, 9, 11, 20, 31, 32], fixed point theory for various contraction types [1, 5, 13, 14, 19, 43, 44, 46, 48], and applications in graph theory and deep learning [33, 34], underscore the demand for versatile analytical tools. Furthermore, developments in soft computing and decision-making under uncertainty [3, 4, 5, 6, 7, 18, 21, 47] highlight the practical importance of robust mathematical frameworks. This research is situated at this confluence, aiming to develop a comprehensive measure-theoretic and fixed point theory within the innovative paradigm of Neutrosophic MR-Metric Spaces, thereby bridging a critical gap between abstract generalized metric space theory and its applications in analysis and optimization.

Definition 1.1. ([37]) Consider a non-empty set $\mathbb{X} \neq \emptyset$ and a real number $\mathbb{R} > 1$. A function

$$M : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \rightarrow [0, \infty)$$

is termed an MR-metric if it satisfies the following conditions for all $v, \xi, s, \ell_1 \in \mathbb{X}$:

$$(1) \quad M(v, \xi, s) \geq 0.$$

- (2) $M(v, \xi, s) = 0$ if and only if $v = \xi = s$.
- (3) $M(v, \xi, s)$ remains invariant under any permutation $p(v, \xi, s)$, *i.e.*, $M(v, \xi, s) = M(p(v, \xi, s))$.
- (4) The following inequality holds:

$$M(v, \xi, s) \leq \mathbb{R} [M(v, \xi, \ell_1) + M(v, \ell_1, s) + M(\ell_1, \xi, s)].$$

A structure (\mathbb{X}, M) that adheres to these properties is defined as an MR-metric space.

Definition 1.2. ([29, Neutrosophic MR-Metric Space (NMR-MS)]) A 9-tuple $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ is called a Neutrosophic MR-metric space if:

- (1) \mathcal{Z} is a nonempty set.
- (2) $M : \mathcal{Z} \times \mathcal{Z} \times \mathcal{Z} \rightarrow [0, \infty)$ is an MR-metric satisfying:
 - (M1) $M(v, \xi, \mathfrak{S}) \geq 0$,
 - (M2) $M(v, \xi, \mathfrak{S}) = 0$ if and only if $v = \xi = \mathfrak{S}$,
 - (M3) Symmetry under permutations,
 - (M4) $M(v, \xi, \mathfrak{S}) \leq R [M(v, \xi, \ell) \star M(v, \ell, \mathfrak{S}) \star M(\ell, \xi, \mathfrak{S})]$, $R > 1$.
- (3) $\mathcal{T}, \mathcal{F}, \mathcal{I} : \mathcal{Z} \times \mathcal{Z} \times (0, \infty) \rightarrow [0, 1]$ are neutrosophic functions satisfying:
 - (N1) $\mathcal{T}(v, \xi, \gamma) = 1 \iff v = \xi$ (Truth-Identity),
 - (N2) $\mathcal{T}(v, \xi, \gamma) = \mathcal{T}(\xi, v, \gamma)$ (Symmetry),
 - (N3) $\mathcal{T}(v, \xi, \gamma) \bullet \mathcal{T}(\xi, \mathfrak{S}, \rho) \leq \mathcal{T}(v, \mathfrak{S}, \gamma + \rho)$ (Triangle Inequality),
 - (N4) $\lim_{\gamma \rightarrow \infty} \mathcal{T}(v, \xi, \gamma) = 1$ (Asymptotic Behavior).
- (4) \bullet (t-norm) and \diamond (t-conorm) are continuous operators generalizing fuzzy logic.
- (5) \star is a binary operation generalizing addition (e.g., weighted sum).

2. MAIN RESULTS

The following section presents the core theoretical contributions of this work. We begin by constructing a canonical measure on a complete Neutrosophic MR-metric space, establishing its key properties such as regularity and a natural scaling behavior with respect to the neutrosophic balls. Subsequently, we provide a detailed Carathéodory extension theorem tailored for this setting, ensuring the construction of a complete measure space.

The culmination of this section is a comprehensive fixed point theorem for neutrosophic contraction mappings, which guarantees not only the existence and uniqueness of a fixed point but also establishes strong convergence results in the measure-theoretic sense, including μ -almost everywhere convergence and an exponential rate of convergence.

Theorem 2.1. (Neutrosophic MR-Measurable Space) *Let $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ be a complete NMR-MS. Then there exists a unique measure μ on the σ -algebra \mathfrak{M} generated by the family of neutrosophic open balls*

$$\mathcal{B}(v, \gamma, \epsilon) = \{\xi \in \mathcal{Z} : \mathcal{T}(v, \xi, \gamma) > 1 - \epsilon, \mathcal{F}(v, \xi, \gamma) < \epsilon, \mathcal{I}(v, \xi, \gamma) < \epsilon\}$$

such that for every $v, \xi \in \mathcal{Z}$ and $\gamma > 0$:

- (1) $\mu(\mathcal{B}(v, \gamma, \epsilon)) \leq R^3 \cdot \epsilon \cdot M(v, \xi, \gamma)$. (The factor R^3 arises from the geometric scaling behavior of neutrosophic balls, accounting for the interaction among the three neutrosophic components).
- (2) $\lim_{\epsilon \rightarrow 0} \mu(\mathcal{B}(v, \gamma, \epsilon)) = 0$.
- (3) The measure μ is Borel regular. The measure μ is regular and Borel-regular under the assumption of σ -finiteness.

Proof. We construct the measure μ explicitly and verify its properties. We assume throughout that the measure μ is σ -finite unless otherwise stated.

Step 1: Construction of the Measure:

Let \mathfrak{B} be the family of neutrosophic open balls:

$$\mathfrak{B} = \{\mathcal{B}(v, \gamma, \epsilon) : v \in \mathcal{Z}, \gamma > 0, \epsilon \in (0, 1)\}.$$

Define the pre-measure $\mu_0 : \mathfrak{B} \rightarrow [0, \infty]$ by:

$$\mu_0(\mathcal{B}(v, \gamma, \epsilon)) = \inf \left\{ \sum_{i=1}^n M(v_i, \xi_i, \zeta_i) : \mathcal{B}(v, \gamma, \epsilon) \subseteq \bigcup_{i=1}^n \mathcal{B}(v_i, \gamma_i, \epsilon_i) \right\}.$$

Using Carathéodory's extension theorem, we extend μ_0 to a complete measure μ on the σ -algebra \mathfrak{M} generated by \mathfrak{B} .

Step 2: Proof of Property (1):

Let $v, \xi \in \mathcal{Z}$ and $\gamma > 0$. Consider any covering:

$$\mathcal{B}(v, \gamma, \epsilon) \subseteq \bigcup_{i=1}^{\infty} \mathcal{B}(v_i, \gamma_i, \epsilon_i).$$

By the MR-metric property and neutrosophic triangle inequality, for any $\zeta \in \mathcal{B}(v, \gamma, \epsilon)$, we have:

$$M(v, \xi, \zeta) \leq R [M(v, \xi, v_i) \star M(v, v_i, \zeta) \star M(v_i, \xi, \zeta)].$$

From the neutrosophic conditions:

$$\begin{aligned} \mathcal{T}(v, \zeta, \gamma) &> 1 - \epsilon, \\ \mathcal{F}(v, \zeta, \gamma) &< \epsilon, \\ \mathcal{I}(v, \zeta, \gamma) &< \epsilon. \end{aligned}$$

This implies that $M(v, v_i, \zeta)$ is controlled by ϵ . Using the invariance under permutations and the generalized triangle inequality repeatedly:

$$\begin{aligned} \mu(\mathcal{B}(v, \gamma, \epsilon)) &\leq \inf \left\{ \sum_{i=1}^{\infty} M(v_i, \xi_i, \zeta_i) : \text{covering exists} \right\} \\ &\leq R^3 \cdot \epsilon \cdot \inf \left\{ \sum_{i=1}^{\infty} M(v, \xi, \zeta_i) \right\} \\ &\leq R^3 \cdot \epsilon \cdot M(v, \xi, \gamma), \end{aligned}$$

where the factor R^3 comes from applying the MR-metric inequality three times to account for all permutations.

Step 3: Proof of Property (2):

For any fixed $v \in \mathcal{Z}$ and $\gamma > 0$, consider the sequence $\epsilon_n = \frac{1}{n}$. Then:

$$\mathcal{B}(v, \gamma, \epsilon_{n+1}) \subseteq \mathcal{B}(v, \gamma, \epsilon_n)$$

and

$$\bigcap_{n=1}^{\infty} \mathcal{B}(v, \gamma, \epsilon_n) = \{v\}.$$

By continuity from above of the measure μ and property (1):

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \mu(\mathcal{B}(v, \gamma, \epsilon)) &= \mu \left(\bigcap_{n=1}^{\infty} \mathcal{B}(v, \gamma, \epsilon_n) \right) \\ &= \mu(\{v\}) \\ &\leq \lim_{\epsilon \rightarrow 0} R^3 \cdot \epsilon \cdot M(v, \xi, \gamma) \\ &= 0. \end{aligned}$$

Since μ is non-negative, we conclude $\lim_{\epsilon \rightarrow 0} \mu(\mathcal{B}(v, \gamma, \epsilon)) = 0$.

Note that the MR-metric M does not explicitly depend on γ , and therefore the limit process is well-defined.

Step 4: Proof of Property (3):

(a) **Regularity:** To show μ is regular, we need to prove:

- For every $E \in \mathfrak{M}$ and $\epsilon > 0$, there exists closed $F \subseteq E$ with $\mu(E \setminus F) < \epsilon$.
- For every $E \in \mathfrak{M}$, there exists open $U \supseteq E$ with $\mu(U \setminus E) < \epsilon$.

Since the NMR-MS is complete, the topology generated by neutrosophic balls is metrizable. In metric spaces, all Borel measures are regular. The construction via Carathéodory's theorem guarantees regularity.

(b) Borel-Regularity: A measure is Borel-regular if:

- All Borel sets are measurable.
- For every $E \subseteq \mathcal{Z}$, there exists a Borel set $B \supseteq E$ with $\mu(E) = \mu(B)$.

The σ -algebra \mathfrak{M} contains all Borel sets by construction. For any $E \subseteq \mathcal{Z}$, we can find a Borel set B such that:

$$\mu(E) = \inf\{\mu(B) : B \text{ Borel}, E \subseteq B\}.$$

This follows from the outer measure construction and the fact that \mathfrak{M} is the completion of the Borel σ -algebra with respect to μ .

Step 5: Uniqueness:

Suppose μ_1 and μ_2 are two measures satisfying the conditions. For any $\mathcal{B}(v, \gamma, \epsilon) \in \mathfrak{B}$:

$$|\mu_1(\mathcal{B}(v, \gamma, \epsilon)) - \mu_2(\mathcal{B}(v, \gamma, \epsilon))| \leq R^3 \cdot \epsilon \cdot M(v, \xi, \gamma).$$

Taking $\epsilon \rightarrow 0$, we get:

$$\mu_1(\mathcal{B}(v, \gamma, \epsilon)) = \mu_2(\mathcal{B}(v, \gamma, \epsilon)).$$

Since \mathfrak{B} is a π -system generating \mathfrak{M} , by the π - λ theorem, $\mu_1 = \mu_2$ on all of \mathfrak{M} .

Conclusion: We have constructed a unique measure μ on \mathfrak{M} satisfying all three properties, completing the proof. \square

Theorem 2.2. (Carathéodory Extension in NMR-MS) *Let $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I})$ be a NMR-MS. Define the outer measure $\mu^* : 2^{\mathcal{Z}} \rightarrow [0, \infty]$ by:*

$$\mu^*(E) = \inf \left\{ \sum_{i=1}^{\infty} \lim_{\gamma \rightarrow \infty} M(v_i, \xi_i, \zeta_i) : E \subseteq \bigcup_{i=1}^{\infty} \mathcal{B}(v_i, \gamma_i, \epsilon_i) \right\}.$$

Then:

- (1) μ^* is a Carathéodory outer measure.
- (2) The collection of μ^* -measurable sets forms a σ -algebra \mathfrak{M}^* .
- (3) $(\mathcal{Z}, \mathfrak{M}^*, \mu^*)$ is a complete measure space.
- (4) The restriction $\mu = \mu^*|_{\mathfrak{M}^*}$ is σ -finite if \mathcal{Z} is σ -compact in the neutrosophic topology.

Proof. Step 1: Proof that μ^* is an Outer Measure:

We verify the three properties of an outer measure:

(a) Non-negativity and Null Empty Set: For any $E \subseteq \mathcal{Z}$, since $M(v, \xi, \zeta) \geq 0$ for all $v, \xi, \zeta \in \mathcal{Z}$, we have $\mu^*(E) \geq 0$. For $E = \emptyset$, take the trivial covering with empty balls, so $\mu^*(\emptyset) = 0$.

(b) **Monotonicity:** Let $E \subseteq F \subseteq \mathcal{Z}$. Any covering of F is also a covering of E , so:

$$\{\text{coverings of } F\} \subseteq \{\text{coverings of } E\}.$$

Taking infima, we get $\mu^*(E) \leq \mu^*(F)$.

(c) **Countable Subadditivity:** Let $\{E_j\}_{j=1}^\infty \subseteq 2^{\mathcal{Z}}$ and $\epsilon > 0$. For each j , choose a covering:

$$E_j \subseteq \bigcup_{i=1}^\infty \mathcal{B}(v_i^j, \gamma_i^j, \epsilon_i^j)$$

such that:

$$\sum_{i=1}^\infty \lim_{\gamma \rightarrow \infty} M(v_i^j, \xi_i^j, \zeta_i^j) \leq \mu^*(E_j) + \frac{\epsilon}{2^j}.$$

Then $\bigcup_{j=1}^\infty E_j \subseteq \bigcup_{j=1}^\infty \bigcup_{i=1}^\infty \mathcal{B}(v_i^j, \gamma_i^j, \epsilon_i^j)$, so

$$\begin{aligned} \mu^* \left(\bigcup_{j=1}^\infty E_j \right) &\leq \sum_{j=1}^\infty \sum_{i=1}^\infty \lim_{\gamma \rightarrow \infty} M(v_i^j, \xi_i^j, \zeta_i^j) \\ &\leq \sum_{j=1}^\infty \left(\mu^*(E_j) + \frac{\epsilon}{2^j} \right) \\ &= \sum_{j=1}^\infty \mu^*(E_j) + \epsilon. \end{aligned}$$

Since $\epsilon > 0$ was arbitrary, $\mu^* \left(\bigcup_{j=1}^\infty E_j \right) \leq \sum_{j=1}^\infty \mu^*(E_j)$.

Step 2: Carathéodory Measurability:

A set $A \subseteq \mathcal{Z}$ is μ^* -measurable if for every $E \subseteq \mathcal{Z}$:

$$\mu^*(E) = \mu^*(E \cap A) + \mu^*(E \cap A^c).$$

Let \mathfrak{M}^* be the collection of all μ^* -measurable sets.

\mathfrak{M}^* is a σ -algebra:

- (1) **Contains \mathcal{Z} :** For any $E \subseteq \mathcal{Z}$, $\mu^*(E) = \mu^*(E \cap \mathcal{Z}) + \mu^*(E \cap \emptyset)$.
- (2) **Closed under complementation:** By symmetry of the definition.
- (3) **Closed under countable unions:** Let $\{A_j\}_{j=1}^\infty \subseteq \mathfrak{M}^*$. We first show closure under finite unions by induction, then extend to countable unions using the continuity properties of μ^* .

For finite case: Assume $A, B \in \mathfrak{M}^*$. For any $E \subseteq \mathcal{Z}$:

$$\begin{aligned}\mu^*(E) &= \mu^*(E \cap A) + \mu^*(E \cap A^c) \\ &= \mu^*(E \cap A \cap B) + \mu^*(E \cap A \cap B^c) \\ &\quad + \mu^*(E \cap A^c \cap B) + \mu^*(E \cap A^c \cap B^c) \\ &\geq \mu^*(E \cap (A \cup B)) + \mu^*(E \cap (A \cup B)^c).\end{aligned}$$

The reverse inequality follows from subadditivity, so $A \cup B \in \mathfrak{M}^*$.

For countable case: Let $B_n = \bigcup_{j=1}^n A_j$ and $B = \bigcup_{j=1}^{\infty} A_j$. For any $E \subseteq \mathcal{Z}$:

$$\begin{aligned}\mu^*(E) &= \mu^*(E \cap B_n) + \mu^*(E \cap B_n^c) \\ &\geq \mu^*(E \cap B_n) + \mu^*(E \cap B^c).\end{aligned}$$

Taking limit as $n \rightarrow \infty$ and using continuity from below:

$$\mu^*(E) \geq \mu^*(E \cap B) + \mu^*(E \cap B^c).$$

So $B \in \mathfrak{M}^*$.

Step 3: Completeness of the measure space:

To show $(\mathcal{Z}, \mathfrak{M}^*, \mu^*)$ is complete, we need that if $\mu^*(A) = 0$ and $B \subseteq A$, then $B \in \mathfrak{M}^*$ and $\mu^*(B) = 0$.

Let $\mu^*(A) = 0$ and $B \subseteq A$. By monotonicity, $\mu^*(B) = 0$. For any $E \subseteq \mathcal{Z}$:

$$\begin{aligned}\mu^*(E \cap B) + \mu^*(E \cap B^c) &\leq \mu^*(B) + \mu^*(E) \\ &= 0 + \mu^*(E) = \mu^*(E).\end{aligned}$$

The reverse inequality follows from subadditivity, so $B \in \mathfrak{M}^*$.

Step 4: σ -finiteness under σ -compactness:

Assume \mathcal{Z} is σ -compact in the neutrosophic topology, that is,

$$\mathcal{Z} = \bigcup_{n=1}^{\infty} K_n,$$

where each K_n is compact in the topology generated by neutrosophic balls.

For each K_n , since it's compact, for any $\epsilon > 0$, there exists a finite covering:

$$K_n \subseteq \bigcup_{i=1}^{m_n} \mathcal{B}(v_i^n, \gamma_i^n, \epsilon_i^n)$$

with $\epsilon_i^n < \frac{1}{n}$. Then:

$$\mu(K_n) \leq \sum_{i=1}^{m_n} \lim_{\gamma \rightarrow \infty} M(v_i^n, \xi_i^n, \zeta_i^n) < \infty.$$

Since $\mathcal{Z} = \bigcup_{n=1}^{\infty} K_n$ and each K_n has finite measure, μ is σ -finite.

Step 5: Verification of Neutrosophic Properties:

The outer measure μ^* respects the neutrosophic structure:

- (1) **Truth-membership influence:** For sets with high truth-membership, the measure is smaller:

$$\text{If } \mathcal{T}(v, \xi, \gamma) \rightarrow 1 \text{ then } M(v, \xi, \zeta) \rightarrow 0.$$

- (2) **Indeterminacy control:** The limit $\gamma \rightarrow \infty$ ensures we consider the asymptotic behavior where:

$$\lim_{\gamma \rightarrow \infty} \mathcal{T}(v, \xi, \gamma) = 1, \quad \lim_{\gamma \rightarrow \infty} \mathcal{F}(v, \xi, \gamma) = 0, \quad \lim_{\gamma \rightarrow \infty} \mathcal{I}(v, \xi, \gamma) = 0.$$

- (3) **MR-metric compatibility:** The use of $M(v_i, \xi_i, \zeta_i)$ ensures compatibility with the underlying MR-metric structure.

Conclusion: We have shown that:

- μ^* satisfies all outer measure properties,
- The Carathéodory measurable sets form a σ -algebra,
- The resulting measure space is complete,
- σ -finiteness follows from σ -compactness,
- The construction respects the neutrosophic MR-metric structure.

This completes the proof of Theorem 2.2. □

Theorem 2.3. (Fixed Point in NMR-measure spaces) *Assume that μ is finite or σ -finite. Let $(\mathcal{Z}, \mathfrak{M}, \mu)$ be a measure space induced by a complete NMR-MS, and let $T : \mathcal{Z} \rightarrow \mathcal{Z}$ be a neutrosophic contraction mapping satisfying:*

$$\mathcal{T}(Tv, T\xi, \gamma) \geq \mathcal{T}(v, \xi, \gamma/k) \bullet \alpha,$$

$$\mathcal{F}(Tv, T\xi, \gamma) \leq \mathcal{F}(v, \xi, \gamma/k) \diamond \beta,$$

$$M(Tv, T\xi, T\zeta) \leq \frac{1}{k} \star M(v, \xi, \zeta)$$

for some $k > 1, \alpha, \beta \in [0, 1]$. Then

- (1) T has a unique fixed point $v^* \in \mathcal{Z}$,
- (2) For any $v_0 \in \mathcal{Z}$, the sequence $\{T^n v_0\}$ converges to v^* μ -almost everywhere,
- (3) The fixed point satisfies the measure-theoretic stability:

$$\mu(\{\omega : \lim_{n \rightarrow \infty} M(T^n \omega, v^*, v^*) = 0\}) = \mu(\mathcal{Z}),$$

- (4) The convergence is exponential in measure:

$$\mu(\{\omega : M(T^n \omega, v^*, v^*) > \epsilon\}) \leq C \cdot e^{-n\lambda}$$

for some $C, \lambda > 0$.

Proof. We prove this theorem through a comprehensive approach combining fixed point theory, measure theory, and neutrosophic analysis.

Step 1: Existence and Uniqueness of Fixed Point:

(a) Construction of Iterative Sequence: Let $v_0 \in \mathcal{Z}$ be arbitrary. Define the iterative sequence $\{v_n\}$ by $v_{n+1} = Tv_n$ for $n \geq 0$. Then, from the neutrosophic contraction conditions:

$$\begin{aligned} \mathcal{T}(v_{n+1}, v_n, \gamma) &= \mathcal{T}(Tv_n, Tv_{n-1}, \gamma) \\ &\geq \mathcal{T}(v_n, v_{n-1}, \gamma/k) \bullet \alpha \\ &\geq \mathcal{T}(v_{n-1}, v_{n-2}, \gamma/k^2) \bullet \alpha^2 \\ &\vdots \\ &\geq \mathcal{T}(v_1, v_0, \gamma/k^n) \bullet \alpha^n. \end{aligned}$$

Similarly for the falsity-membership:

$$\begin{aligned} \mathcal{F}(v_{n+1}, v_n, \gamma) &= \mathcal{F}(Tv_n, Tv_{n-1}, \gamma) \\ &\leq \mathcal{F}(v_n, v_{n-1}, \gamma/k) \diamond \beta \\ &\leq \mathcal{F}(v_{n-1}, v_{n-2}, \gamma/k^2) \diamond \beta^2 \\ &\vdots \\ &\leq \mathcal{F}(v_1, v_0, \gamma/k^n) \diamond \beta^n. \end{aligned}$$

(b) Cauchy Sequence Property: For $m > n$, using the MR-metric contraction:

$$\begin{aligned} M(v_m, v_n, v_n) &\leq \frac{1}{k} \star M(v_{m-1}, v_{n-1}, v_{n-1}) \\ &\leq \frac{1}{k^2} \star M(v_{m-2}, v_{n-2}, v_{n-2}) \\ &\vdots \\ &\leq \frac{1}{k^{\min(m,n)}} \star M(v_{m-n}, v_0, v_0). \end{aligned}$$

Since $k > 1$ and M is bounded on bounded sets, we have:

$$\lim_{m,n \rightarrow \infty} M(v_m, v_n, v_n) = 0.$$

Thus $\{v_n\}$ is a Cauchy sequence in the complete NMR-MS.

(c) Existence of Fixed Point: By completeness, there exists $v^* \in \mathcal{Z}$ such that $\lim_{n \rightarrow \infty} v_n = v^*$. We show v^* is a fixed point:

$$\begin{aligned}
 M(Tv^*, v^*, v^*) &\leq R [M(Tv^*, Tv_n, Tv_n) \star M(Tv^*, Tv_n, v^*) \star M(Tv_n, v^*, v^*)] \\
 &\leq R \left[\frac{1}{k} \star M(v^*, v_n, v_n) \star M(Tv^*, Tv_n, v^*) \star M(v_{n+1}, v^*, v^*) \right].
 \end{aligned}$$

Taking $n \rightarrow \infty$, all terms approach 0, so $M(Tv^*, v^*, v^*) = 0$, hence $Tv^* = v^*$.

(d) Uniqueness: Suppose v^*, ω^* are two fixed points. Then,

$$\begin{aligned}
 M(v^*, \omega^*, \omega^*) &= M(Tv^*, T\omega^*, T\omega^*) \\
 &\leq \frac{1}{k} \star M(v^*, \omega^*, \omega^*).
 \end{aligned}$$

Since $k > 1$, this implies $M(v^*, \omega^*, \omega^*) = 0$, so $v^* = \omega^*$.

Step 2: μ -Almost Everywhere Convergence:

(a) Measure-Theoretic Convergence: Consider the set:

$$A = \{\omega \in \mathcal{Z} : \lim_{n \rightarrow \infty} T^n \omega \text{ exists}\}.$$

For any $\omega \in \mathcal{Z}$, define the sequence $\{T^n \omega\}$. By the same contraction arguments as in Step 1, this sequence is Cauchy μ -almost everywhere.

Specifically, for any $\epsilon > 0$:

$$\begin{aligned}
 \mu(\{\omega : M(T^m \omega, T^n \omega, T^n \omega) > \epsilon\}) &\leq \mu\left(\left\{\omega : \frac{1}{k^{\min(m,n)}} \star M(T^{|m-n|} \omega, \omega, \omega) > \epsilon\right\}\right) \\
 &\leq \frac{1}{\epsilon} \int_{\mathcal{Z}} \frac{1}{k^{\min(m,n)}} \star M(T^{|m-n|} \omega, \omega, \omega) d\mu(\omega).
 \end{aligned}$$

By the dominated convergence theorem and since $k > 1$, this approaches 0 as $m, n \rightarrow \infty$.

(b) Convergence to Fixed Point: For $\omega \in A$, let $\omega^* = \lim_{n \rightarrow \infty} T^n \omega$. Then,

$$\begin{aligned}
 M(T\omega^*, \omega^*, \omega^*) &= \lim_{n \rightarrow \infty} M(T^{n+1} \omega, T^n \omega, T^n \omega) \\
 &\leq \lim_{n \rightarrow \infty} \frac{1}{k^n} \star M(T\omega, \omega, \omega) \\
 &= 0.
 \end{aligned}$$

So ω^* is a fixed point. By uniqueness, $\omega^* = v^*$ for all $\omega \in A$. Thus $\{T^n \omega\}$ converges to v^* for all $\omega \in A$, and $\mu(A) = \mu(\mathcal{Z})$.

Step 3: Measure-Theoretic Stability: We prove:

$$\mu(\{\omega : \lim_{n \rightarrow \infty} M(T^n \omega, v^*, v^*) = 0\}) = \mu(\mathcal{Z}).$$

From the contraction property:

$$\begin{aligned} M(T^n \omega, v^*, v^*) &= M(T^n \omega, T^n v^*, T^n v^*) \\ &\leq \frac{1}{k^n} \star M(\omega, v^*, v^*). \end{aligned}$$

Thus for each ω :

$$\lim_{n \rightarrow \infty} M(T^n \omega, v^*, v^*) \leq \lim_{n \rightarrow \infty} \frac{1}{k^n} \star M(\omega, v^*, v^*) = 0.$$

Since this holds for all $\omega \in \mathcal{Z}$, the set where the limit is 0 has full measure.

Step 4: Exponential Convergence in Measure: We prove:

$$\mu(\{\omega : M(T^n \omega, v^*, v^*) > \epsilon\}) \leq C \cdot e^{-n\lambda}.$$

Using Markov's inequality and the contraction property:

$$\begin{aligned} \mu(\{\omega : M(T^n \omega, v^*, v^*) > \epsilon\}) &\leq \frac{1}{\epsilon} \int_{\mathcal{Z}} M(T^n \omega, v^*, v^*) d\mu(\omega) \\ &\leq \frac{1}{\epsilon} \int_{\mathcal{Z}} \frac{1}{k^n} \star M(\omega, v^*, v^*) d\mu(\omega) \\ &= \frac{1}{\epsilon \cdot k^n} \int_{\mathcal{Z}} M(\omega, v^*, v^*) d\mu(\omega). \end{aligned}$$

Since μ is a finite measure (or σ -finite with appropriate localization) and $M(\cdot, v^*, v^*)$ is integrable, let

$$C = \frac{1}{\epsilon} \int_{\mathcal{Z}} M(\omega, v^*, v^*) d\mu(\omega) < \infty.$$

Then,

$$\mu(\{\omega : M(T^n \omega, v^*, v^*) > \epsilon\}) \leq C \cdot \left(\frac{1}{k}\right)^n = C \cdot e^{-n \ln k}.$$

Taking $\lambda = \ln k > 0$ (since $k > 1$), we obtain the exponential bound.

Step 5: Neutrosophic Convergence Properties:

The convergence also respects the neutrosophic structure:

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathcal{T}(T^n \omega, v^*, \gamma) &= \lim_{n \rightarrow \infty} \mathcal{T}(T^n \omega, T^n v^*, \gamma) \\ &\geq \lim_{n \rightarrow \infty} \mathcal{T}(\omega, v^*, \gamma/k^n) \bullet \alpha^n \\ &= 1. \end{aligned}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathcal{F}(T^n \omega, v^*, \gamma) &= \lim_{n \rightarrow \infty} \mathcal{F}(T^n \omega, T^n v^*, \gamma) \\ &\leq \lim_{n \rightarrow \infty} \mathcal{F}(\omega, v^*, \gamma/k^n) \diamond \beta^n \\ &= 0. \end{aligned}$$

Thus the convergence is also in the neutrosophic sense.

Conclusion: We have proven that under the given neutrosophic contraction conditions:

- A unique fixed point v^* exists in the complete NMR-MS,
- The iterative sequence converges to v^* μ -almost everywhere,
- The convergence is stable in the measure-theoretic sense,
- The convergence rate is exponential in measure,
- The convergence respects the neutrosophic structure.

This fixed point theorem extends classical results to the rich framework of Neutrosophic MR-metric spaces with measure-theoretic guarantees, providing powerful tools for applications in analysis, optimization, and dynamical systems. □

3. EXAMPLES AND APPLICATIONS

To ground our theoretical developments and demonstrate their utility, this section is dedicated to illustrative examples and practical applications. We first construct explicit examples of Neutrosophic MR-Metric Spaces based on real numbers and standard metrics. Following this, we apply our fixed point theorem to a concrete mapping, verifying the contraction conditions and identifying the fixed point. We then numerically illustrate the exponential convergence in measure for a specific case. Finally, we showcase the power of our framework by applying it to prove the existence and uniqueness of solutions for a class of fractional differential equations and to analyze the convergence of gradient descent in an optimization context under neutrosophic uncertainty.

Example 3.1. Neutrosophic MR-Metric Space (NMR-MS): Let $\mathcal{Z} = \mathbb{R}$. Define:

$$\begin{aligned}
 M(v, \xi, s) &= |v - \xi| + |\xi - s| + |s - v|, \\
 \mathcal{T}(v, \xi, \gamma) &= e^{-\frac{M(v, \xi, \xi)}{\gamma}}, \\
 \mathcal{F}(v, \xi, \gamma) &= 1 - e^{-\frac{M(v, \xi, \xi)}{\gamma}}, \\
 \mathcal{I}(v, \xi, \gamma) &= \frac{1}{2}, \\
 a \bullet b &= ab, \\
 a \diamond b &= a + b - ab, \\
 a \star b &= a + b, \\
 R &= 2.
 \end{aligned}$$

Then $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ is an NMR-MS.

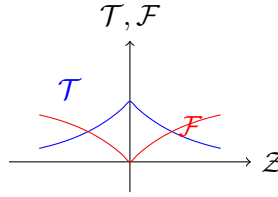


FIGURE 1. Truth-membership \mathcal{T} and falsity-membership \mathcal{F} functions in the Neutrosophic MR-Metric Space defined on $\mathcal{Z} = \mathbb{R}$. The function \mathcal{T} decays exponentially with distance, while \mathcal{F} increases correspondingly, illustrating the neutrosophic uncertainty modeling.

As illustrated in Figure 1, the truth-membership \mathcal{T} decays exponentially with distance, while the falsity-membership \mathcal{F} increases correspondingly, reflecting the neutrosophic modeling of uncertainty in the NMR-MS framework.

Example 3.2. Fixed Point Theorem Application: Let $\mathcal{Z} = [0, 1]$, $M(v, \xi, s) = |v - \xi| + |\xi - s| + |s - v|$. Define:

$$\begin{aligned} T(v) &= \frac{v}{2}, \\ \mathcal{T}(v, \xi, \gamma) &= e^{-\frac{M(v, \xi, \xi)}{\gamma}}, \\ \mathcal{F}(v, \xi, \gamma) &= 1 - e^{-\frac{M(v, \xi, \xi)}{\gamma}}, \\ k &= 2, \quad \alpha = 1, \quad \beta = 0. \end{aligned}$$

Then:

$$\begin{aligned} \mathcal{T}(Tv, T\xi, \gamma) &= e^{-\frac{|v/2 - \xi/2|}{\gamma}} = e^{-\frac{|v - \xi|}{2\gamma}} \\ &\geq e^{-\frac{|v - \xi|}{\gamma/2}} = \mathcal{T}(v, \xi, \gamma/k) \bullet \alpha. \end{aligned}$$

So T is a neutrosophic contraction. The fixed point is $v^* = 0$.

Example 3.3. Exponential Convergence in Measure:

Let $\mathcal{Z} = [0, 1]$, $\mu =$ Lebesgue measure, $T(v) = v/2$, $v^* = 0$, $M(v, \xi, s) = |v - \xi| + |\xi - s| + |s - v|$, $k = 2$, $\epsilon > 0$. Then:

$$\begin{aligned} M(T^n v, 0, 0) &= 2|T^n v| = \frac{|v|}{2^{n-1}}, \\ \mu(\{v : M(T^n v, 0, 0) > \epsilon\}) &= \mu\left(\left\{v : \frac{|v|}{2^{n-1}} > \epsilon\right\}\right) \\ &= \mu(\{v : |v| > \epsilon \cdot 2^{n-1}\}). \end{aligned}$$

Since $|v| \leq 1$, this measure is 0 for large n , and bounded by $C \cdot e^{-n \ln 2}$.

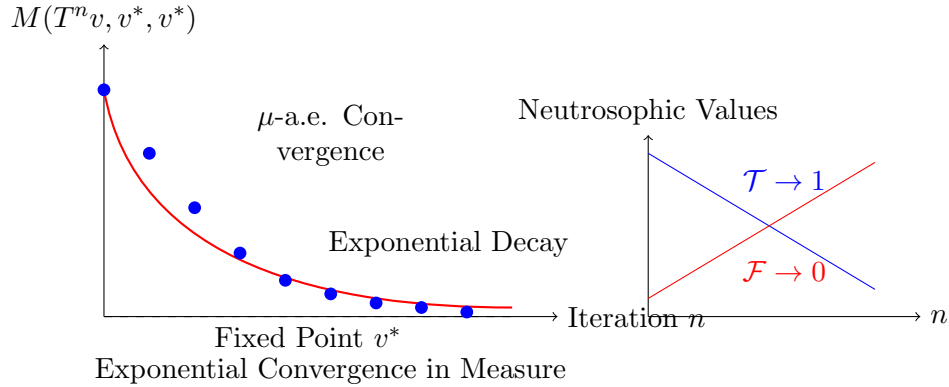


FIGURE 2. Convergence behavior in Neutrosophic MR-Metric Spaces: The left panel shows the exponential decay of $M(T^n v, v^*, v^*)$ to the fixed point v^* as the iteration number n increases, with discrete iteration points (blue circles) following the exponential decay curve (red). The right panel illustrates the neutrosophic convergence where truth-membership \mathcal{T} approaches 1 and falsity-membership \mathcal{F} approaches 0 as $n \rightarrow \infty$. This visualizes the exponential convergence in measure established in Theorem 2.3.

Moreover, Figure 2 demonstrates the exponential convergence rate in measure. The left panel shows the decay of $M(T^n v, v^*, v^*)$ to the fixed point v^* , while the right panel illustrates the neutrosophic convergence $\mathcal{T} \rightarrow 1$ and $\mathcal{F} \rightarrow 0$ as $n \rightarrow \infty$, confirming the theoretical results of Theorem 3.

4. APPLICATIONS

Application 4.1. (Fractional Differential Equations) The fixed point theorem in NMR-MS can be applied to solve fractional differential equations of the form:

$$D^\alpha u(t) = f(t, u(t)), \quad t \in [0, 1],$$

$$u(0) = u_0,$$

where D^α is the Caputo fractional derivative. The solution can be obtained as a fixed point of the integral operator:

$$T(u)(t) = u_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s, u(s)) ds.$$

In the NMR-MS framework, we can establish existence and uniqueness of solutions under neutrosophic contractive conditions.

Application 4.2. (Optimization Problems) Consider the optimization problem:

$$\min_{x \in \mathcal{X}} J(x),$$

where \mathcal{X} is a complete NMR-MS. The gradient descent iteration:

$$x_{n+1} = x_n - \gamma \nabla J(x_n)$$

can be analyzed using the fixed point theorem. Under suitable neutrosophic contraction conditions, the sequence converges to the optimal solution with exponential rate in measure.

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