Nonlinear Functional Analysis and Applications
Vol. 19, No. 3 (2014), pp. 455-478

http://nfaa.kyungnam.ac.kr/jour-nfaa.htm

SYSTEMS OF RANDOM NONLINEAR VARIATIONAL INCLUSIONS WITH FUZZY MAPPINGS IN q-UNIFORMLY SMOOTH BANACH SPACES

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Abstract. This work is intended to introduce the notion of random (A_t, η_t)-accretive equations with random relaxed cocoercive mappings along with fuzzy mappings and random resolvent operators associated with randomly (A_t, η_t)-accretive equations in Banach spaces, and to investigate a class of systems of random nonlinear variational inclusions involving fuzzy mappings in q-uniformly smooth Banach spaces. We suggest some random iterative algorithms with errors for finding the approximate random solutions of systems of random nonlinear variational inclusions with fuzzy mappings. By applying Nadler's fixed point theorem, we also prove the existence of random solutions and convergence of random sequences generated by random algorithms in q-uniformly smooth Banach spaces.

1. Introduction

Stampacchia [39] introduced the variational inequality theory during the early 1964s, which turned out a very powerful tool for current Mathematical environment. The variational inequalities have been applied to extending and generalizing a wide range of problems arising from control and optimization,

[0]Received May 25, 2014. Revised June 27, 2014.
[0]2010 Mathematics Subject Classification: 90C30, 90C32, 90C34.
[0]Keywords: System of random nonlinear variational inclusions, random (A_t, η_t)-accretive equations, random relaxed cocoercive mappings, random resolvent operator techniques, random fuzzy mappings, random iterative sequences, q-uniformly smooth Banach spaces.

nonlinear programming, economics, transportation equilibrium problems, engineering, and physics. The classical variational inequality has been generalized in various directions in past several years, including variational inclusions. Among the generalizations of variational inclusions introduced and studied by Hassouni and Moudafi [22] is of significant interest and importance. It provides us with a unified, natural, novel, innovative and general technique to study a wide range of problems arising from different branches of mathematical and engineering sciences, *see* [3, 5, 7, 15]. It is known that one of the most important and interesting problems in the theory of variational inequalities is the development of an efficient and implementable algorithm for solving variational inequalities as well as variational inclusions. In recent years many numerical methods have been developed for solving various classes of variational inequalities and variational inclusions in Euclidian spaces or Hilbert spaces such as the projection methods and its variant forms, linear approximation, descent method, Newton's method and the methods based on auxiliary principle techniques. In particular, the method based on the resolvent operator technique is a generalization of projection method and has been widely used to solve variational inclusions. Some new and interesting problems which are called systems of variational inequalities were introduced and investigated by Verma [42]. Pang [33] considered a system of scalar variational inequalities and showed that the traffic equilibrium problems, the spatial equilibrium problems, Nash equilibrium and general equilibrium programming problems can be modeled as variational inequalities. He decomposed the original variational inequality into a system of variational inequalities which are easy to solve and study the convergence of such methods. It is known that accretivity of the underlying operator plays indispensable role in the theory of variational inequality and its generalizations. In 2000, Huang and Fang [21] introduced the generalization of m-accretive mapping and generalized the resolvent operator to the case of m-accretive mappings in Banach spaces. Verma [44] introduced and studied new notion of A-monotone and (A, η)-monotone operators and studied some properties for them in Hilbert spaces. In [30] Lan et al. first introduced the concept of (A, η)-accretive mappings, which generalizes the existing η-subdifferential operators, maximal η-monotone operator, H-monotone operators, (H, η) monotone operators in Hilbert spaces, H-accretive mapping, generalized m-accretive mapping and (H, η)-accretive mappings in Banach spaces, *see* [18, 26, 27, 29, 37].

The fuzzy set theory introduced by Zadeh [45] at University of California at Berkeley in 1965, has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of the fuzzy set theory can be found in several branches of mathematical, physical and engineering sciences. In 1989, Chang and Zhu [11] introduced and studied a class of variational inequalities for fuzzy mappings. This was advanced to the several classes of variational inequalities, quasi variational inequalities and complementarity problems with fuzzy mappings by Agarwal *et al.* [1], Chang and Huang [8, 9], Dai [16], Ding [14], Ding *et al.* [17], Huang [20], Khan *et al.* [28], Lan and Verma [29], Lee *et al.* [31, 32], Salahuddin [37, 38], and Zhang and Bi [47] in the settings of Hilbert spaces and Banach spaces. On the other hand, random variational inequality problems, random quasi variational inequality problems, random variational inclusions, and complementarity problems have been studied by Chang [7], Chang and Zhu [12], Chang and Huang [8, 10], Huang [20], Khan and Salahuddin [26], Salahuddin [37] and Cho *et al.* [13], and others. The concept of random fuzzy mapping was first introduced by Huang [19], and it was followed by the work on the random variational inclusion problems for random fuzzy mappings by Anastassiou *et al.* [2], Ahmad and Bazan [4], Balooee [6], Uea and Kuman [41], Zhang and Bi [47]. For more details on random nonlinear variational inclusions involving fuzzy mappings in q-uniformly smooth Banach spaces, we refer the reader [1 - 47].

Inspired and motivated by the ongoing advances in this field (*see* [2, 4, 18, 19, 23, 24, 25, 28, 35, 40, 43, 46]), first we plan in this communication to introduce the notion of a random (A_t, η_t)-accretive equation with random relaxed cocoercive mapping, random fuzzy mapping and the random resolvent operator associated with randomly (A_t, η_t)-accretive equations in Banach spaces, second to investigate a class of *systems of random nonlinear variational inclusions with fuzzy mappings* in q-uniformly smooth Banach spaces, and finally to suggest some random iterative algorithms to compute the approximate random solutions of *system of random nonlinear variational inclusions with fuzzy mappings*. By using Nadler's fixed point theorem [35], we also establish the existence of random solutions and convergence of random sequences generated by random algorithms in the q-uniformly smooth Banach spaces.

2. Basic Foundation

Throughout this paper we suppose that $(\Omega, \mathcal{A}, \mu)$ is a complete σ-finite measurable space and $\mathcal{X}$ is a separable real Banach space endowed with dual space $\mathcal{X}^*$, the norm $\|\cdot\|$ and a dual pairing $\langle \cdot, \cdot \rangle$ between $\mathcal{X}$ and $\mathcal{X}^*$. We denote by $\mathfrak{B}(\mathcal{X})$ the class of Borel σ-fields in $\mathcal{X}$. Let $2^{\mathcal{X}}$ and $CB(\mathcal{X})$ denote the family of all nonempty closed and bounded subsets of $\mathcal{X}$, and the family of all nonempty closed bounded subsets of $\mathcal{X}$, respectively. $\mathfrak{D}(\cdot, \cdot)$ is the Hausdorff metric, defined by

$$\mathfrak{D}(A, B) = \max\{\sup_{x \in A} \inf_{y \in B} d(x, y), \sup_{y \in B} \inf_{x \in A} d(x, y)\}$$

on the $CB(\mathcal{X})$. The generalized duality mapping $J_q : \mathcal{X} \to 2^{\mathcal{X}^*}$ is defined by

$$J_q(x) = \{f^* \in \mathcal{X}^* : \langle x, f^* \rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in \mathcal{X}$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that in general $J_q(x) = \|x\|^{q-1} J_2(x)$ for all $x \neq 0$ and J_q is single valued if $\mathcal{X}^*$ is strictly convex. In the sequel, we always assume that $\mathcal{X}$ is a real Banach space such that J_q is a single valued. If $\mathcal{X}$ is a Hilbert space then J_q becomes the identity mapping on $\mathcal{X}$. The modulus of smoothness of $\mathcal{X}$ is the function $\pi_{\mathcal{X}} : [0, \infty) \to [0, \infty)$ is defined by

$$\pi_{\mathcal{X}}(t) = \sup \left\{ \frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t \right\}.$$

A Banach space $\mathcal{X}$ is called uniformly smooth if

$$\lim_{t \to 0} \frac{\pi_{\mathcal{X}}(t)}{t} = 0.$$

$\mathcal{X}$ is called q-uniformly smooth if there exists a constant $c > 0$ such that

$$\pi_{\mathcal{X}}(t) < ct^q, q > 1.$$

Note that J_q is a single valued if $\mathcal{X}$ is uniformly smooth. Concerned with the characteristic inequalities in q-uniformly smooth Banach spaces, Xu [40] proved the following result.

Lemma 2.1. *The real Banach space $\mathcal{X}$ is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in \mathcal{X}$*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x) \rangle + c_q \|y\|^q.$$

Definition 2.2. A mapping $x : \Omega \to \mathcal{X}$ is said to be measurable if for any $\mathbf{B} \in \mathfrak{B}(\mathcal{X}), \{t \in \Omega, x(t) \in \mathbf{B} \in \mathbb{R}\}$.

Definition 2.3. A mapping $T : \Omega \times \mathcal{X} \to \mathcal{X}$ is called a random mapping if for each fixed $x \in \mathcal{X}, T(t, x) = y(t)$ is a measurable. A random mapping T_t is said to be continuous if for each fixed $t \in \Omega$, $f(t, \cdot) : \Omega \times \mathcal{X} \to \mathcal{X}$ is a continuous mapping.

Similarly we can define a random mapping $a : \Omega \times \mathcal{X} \times \mathcal{X} \to \mathcal{X}$. We shall write $T_t(x) = T(t, x(t))$ and $a_t(x(t), y(t)) = a(t, x(t), y(t)), \forall t \in \Omega$ and $x(t), y(t) \in \mathcal{X}$.

Definition 2.4. A multi-valued mapping $V : \Omega \to 2^{\mathcal{X}}$ is said to be measurable if for any $B \in \mathfrak{B}(\mathcal{X})$, $V^{-1}(B) = \{t \in \Omega : V(t) \cap B \neq \emptyset\} \in \mathfrak{A}$.

Definition 2.5. A mapping $u : \Omega \to \mathcal{X}$ is called a measurable selection of a measurable multi-valued mapping $V : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$, if u is measurable and for any $t \in \Omega$, $u(t) \in V_t(x(t))$.

Definition 2.6. A mapping $V : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is called a random multi-valued mapping if for each fixed $x \in \mathcal{X}$, $V(\cdot, x) : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is a measurable multi-valued mapping. A random multi-valued mapping $V : \Omega \times \mathcal{X} \to CB(\mathcal{X})$ is said to be $\mathfrak{D}$-continuous if for each fixed $t \in \Omega$, $V(t, \cdot) : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is a randomly continuous with respect to the Hausdorff metric on $\mathfrak{D}$.

Definition 2.7. A multi-valued mapping $V : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is called a random multi-valued mapping if for any $x \in \mathcal{X}, V(\cdot, x)$ is a measurable (denoted by $V_{t,x}$ or V_t).

Definition 2.8. Let $\mathcal{X}$ be a q-uniformly smooth Banach space, $T : \Omega \times \mathcal{X} \to \mathcal{X}$ and $\eta : \Omega \times \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ be random single valued mappings. Then

(i) T_t is said to be a randomly accretive mapping if

$$\langle T_t(x(t)) - T_t(y(t)), j_q(x(t) - y(t))\rangle \geq 0, \ \forall x(t), y(t) \in \mathcal{X}, t \in \Omega;$$

(ii) T_t is said to be a randomly strictly accretive mapping if T_t is accretive and

$$\langle T_t(x(t)) - T_t(y(t)), j_q(x(t) - y(t))\rangle = 0$$

if and only if $x(t) = y(t), t \in \Omega$;

(iii) T_t is said to be a randomly r_t-strongly accretive mapping if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle T_t(x(t)) - T_t(y(t)), j_q(x(t) - y(t))\rangle \geq r_t\|x(t) - y(t)\|^q,$$

for all $x(t), y(t) \in \mathcal{X}, t \in \Omega$;

(iv) T_t is said to be a randomly ς_t-relaxed accretive mapping if there exists a measurable function $\varsigma : \Omega \to (0, \infty)$ such that

$$\langle T_t(x(t)) - T_t(y(t)), j_q(x(t) - y(t))\rangle \geq -\varsigma_t\|x(t) - y(t)\|^q,$$

for all $x(t), y(t) \in \mathcal{X}, t \in \Omega$;

(v) T_t is said to be a randomly (δ_t, s_t)-relaxed cocoercive mapping if there exists a measurable function $\delta, s : \Omega \to (0, \infty)$ such that

$$\begin{aligned}&\langle T_t(x(t)) - T_t(y(t)), j_q(x(t) - y(t))\rangle\\ &\geq -\delta_t\|T_t(x(t)) - T_t(y(t))\|^q + s_t\|x(t) - y(t)\|^q,\end{aligned}$$

for all $x(t), y(t) \in \mathcal{X}$, $t \in \Omega$;

(vi) T_t is said to be a randomly ϱ_t-Lipschitz continuous mapping if there exists a measurable function $\varrho : \Omega \to (0, \infty)$ such that

$$\|T_t(x(t)) - T_t(y(t))\| \leq \varrho_t \|x(t) - y(t)\|, \ \ \forall x(t), y(t) \in \mathcal{X}, t \in \Omega;$$

(vii) η_t is said to be a randomly τ_t-Lipschitz continuous mapping if there exists a measurable function $\tau : \Omega \to (0, \infty)$ such that

$$\|\eta_t(x(t), y(t))\| \leq \tau_t \|x(t) - y(t)\|, \ \ \forall x(t), y(t) \in \mathcal{X}, t \in \Omega;$$

(viii) η_t is said to be a randomly φ_t-Lipschitz continuous mapping in the second argument if there exists a measurable function $\varphi : \Omega \to (0, \infty)$ such that

$$\|\eta_t(u(t), x(t)) - \eta_t(u(t), y(t))\| \leq \varphi_t \|x(t) - y(t)\|,$$

for all $x(t), y(t) \in \mathcal{X}, t \in \Omega$.

Definition 2.9. A multi-valued mapping $V : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is called a random multi-valued mapping if for any $x \in \mathcal{X}, V(\cdot, x)$ is a measurable (denoted by $V_{t,x}$ or V_t).

Definition 2.10. Let $\mathcal{X}$ be a q-uniformly smooth Banach space, $\eta : \Omega \times \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ and $H, A : \Omega \times \mathcal{X} \to \mathcal{X}$ be random single valued mappings. Then for all $x(t), y(t) \in \mathcal{X}, t \in \Omega$ a set valued mapping $M : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ is said to be

(i) a randomly accretive mapping if

$$\langle u(t) - v(t), j_q(x(t) - y(t)) \rangle \geq 0,$$

for all $u(t) \in M_t(x(t))$, $v(t) \in M_t(y(t))$;

(ii) a randomly A_t-strictly accretive mapping if

$$\langle u(t) - v(t), j_q(A_t(x(t)) - A_t(y(t))) \rangle = 0,$$

for all $u(t) \in M_t(x(t))$, $v(t) \in M_t(y(t))$ if and only if

$$A_t(x(t)) = A_t(y(t));$$

(iii) a randomly η_t-accretive mapping if

$$\langle u(t) - v(t), j_q(\eta_t(x(t), y(t))) \rangle \geq 0, \ \ \forall u(t) \in M_t(x(t)),$$

$v(t) \in M_t(y(t))$;

(iv) a randomly strictly η_t-accretive mapping if M_t is randomly η_t-accretive mapping and the equality hold if and only if $x(t) = y(t)$, $\forall t \in \Omega$;

(v) a randomly r_t-strongly accretive mapping if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), j_q(x(t) - y(t)) \rangle \geq r_t \|x(t) - y(t)\|^q,$$

for all $u(t) \in M_t(x(t)), v(t) \in M_t(y(t))$;

(vi) a randomly r_t-strongly accretive mapping with respect to $A : \Omega \times \mathcal{X} \to \mathcal{X}$ if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), j_q(A_t(x(t)) - A_t(y(t))) \rangle \geq r_t \|x(t) - y(t)\|^q,$$

for all $u(t) \in M_t(x(t)), v(t) \in M_t(y(t))$;

(vii) a randomly r_t-strongly η_t-accretive mapping if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), j_q(\eta_t(x(t), y(t))) \rangle \geq r_t \|x(t) - y(t)\|^q,$$

for all $u(t) \in M_t(x(t)), v(t) \in M_t(y(t))$;

(viii) randomly r_t-relaxed η_t-accretive mapping if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), j_q(\eta_t(x(t), y(t))) \rangle \geq -r_t \|x(t) - y(t)\|^q,$$

for all $u(t) \in M_t(x(t)), v(t) \in M_t(y(t))$;

(ix) a randomly m_t-relaxed cocoercive mapping with respect to $A : \Omega \times \mathcal{X} \to \mathcal{X}$ if there exists a measurable function $m : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), j_q(A_t(x(t)) - A_t(y(t))) \rangle \geq -m_t \|u(t) - v(t)\|^q,$$

for all $u(t) \in M_t(x(t)), v(t) \in M_t(y(t))$;

(x) a randomly (δ_t, s_t)-relaxed cocoercive mapping with respect to $A : \Omega \times \mathcal{X} \to \mathcal{X}$ if there exists a measurable functions $\delta, s : \Omega \to (0, \infty)$ such that

$$\begin{aligned}&\langle u(t) - v(t), j_q(A_t(x(t)) - A_t(y(t))) \rangle \\ &\geq -\delta_t \|u(t) - v(t)\|^q + s_t \|x(t) - y(t)\|^q,\end{aligned}$$

for all $u(t) \in M_t(x(t))$, $v(t) \in M_t(y(t))$;

(xi) a randomly m_t-accretive mapping if M_t is randomly accretive mapping and $(I_t + \rho_t M_t)(X) = X, \forall t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$ where I stand for an identity mapping on $\mathcal{X}, I_t(x) = x(t)$, $\forall x(t) \in \mathcal{X}, t \in \Omega$;

(xii) a randomly generalized m_t-accretive mapping if M_t is randomly η_t-accretive and $(I_t + \rho_t M_t)(X) = X, \forall t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$;

(xiii) a randomly A_t-accretive mapping if M_t is randomly accretive mapping and $(A_t + \rho_t M_t)(X) = X, \forall t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$ where $A_t(x) = A(x, t)$, $\forall x(t) \in \mathcal{X}, t \in \Omega$;

(xiv) a randomly (A_t, η_t)-accretive mapping if M is randomly η_t-accretive mapping and $(A_t + \rho_t M_t)(X) = X$, $\forall t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$;

(xv) a randomly A_t-maximal m_t-relaxed η_t-accretive mapping if M_t is randomly η_t-accretive mapping and $(A_t + \rho_t M_t)(X) = X$, $\forall t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$;

(xvi) randomly $\beta - \mathfrak{D}$-Lipschitz continuous mapping if there exists a measurable function $\beta : \Omega \to (0, \infty)$ such that

$$\mathfrak{D}(M_t(x(t)), M_t(y(t))) \leq \beta_t \|x(t) - y(t)\|, \ \forall x(t), y(t) \in \mathcal{X}, t \in \Omega.$$

Remark 2.11. Every randomly m_t cocoercive mapping is randomly m_t-relaxed cocoercive mapping while each randomly r_t-strongly accretive mapping is a randomly $(r + r^2, 1)$-relaxed cocoercive mapping with respect to I.

Definition 2.12. Let $A : \Omega \times \mathcal{X} \to \mathcal{X}$ be a randomly r_t-strongly η_t-accretive mapping and $M : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be an randomly A_t-maximal m_t-relaxed η_t-accretive mapping if M_t is randomly m_t-relaxed η_t-accretive mapping and $(A_t + \rho_t M_t)(\mathcal{X}) = \mathcal{X}$ for every $\rho : \Omega \to (0, \infty)$ and the operator $(A_t + \rho_t M_t)^{-1}$ is single valued random mapping for any measurable function $\rho : \mathcal{X} \to (0, \infty)$ and $t \in \Omega$.

Definition 2.13. Let $A : \Omega \times \mathcal{X} \to \mathcal{X}$ be a randomly strictly η_t-accretive mapping and $M : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be an randomly (A_t, η_t)-accretive mapping. Then for any given $\rho > 0$ the resolvent operator $J^{\rho_t, A_t}_{\eta_t, M_t} : \mathcal{X} \to \mathcal{X}$ is defined by

$$J^{\rho_t, A_t}_{\eta_t, M_t}(x(t)) = (A_t + \rho_t M_t)^{-1}(x(t)), \forall t \in \Omega, x(t) \in \mathcal{X}.$$

Proposition 2.14. *Let $\mathcal{X}$ be a q-uniformly smooth Banach space and $\eta : \Omega \times \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ be a random τ_t-Lipschitz continuous mapping, $A : \Omega \times \mathcal{X} \to \mathcal{X}$ be an randomly r_t-strongly η_t-accretive mapping and $M : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be a randomly A_t-maximal m_t-relaxed η_t-accretive mapping. Then the resolvent operator $J^{\rho_t, A_t}_{\eta_t, M_t} : \mathcal{X} \to \mathcal{X}$ is $\frac{\tau_t^{q-1}}{r_t - \rho_t m_t}$-Lipschitz continuous i.e.,*

$$\|J^{\rho_t, A_t}_{\eta_t, M_t} x(t) - J^{\rho_t, A_t}_{\eta_t, M_t} y(t)\| \leq \frac{\tau_t^{q-1}}{r_t - \rho_t m_t} \|x(t) - y(t)\|$$

where $\rho_t \in (0, \frac{r_t}{m_t})$ is a real valued random variable for all $t \in \Omega$.

Lemma 2.15. *Let r and s be two nonnegative real numbers. Then*

$$(r + s)^q \leq 2^q (r^q + s^q).$$

Proof.

$$\begin{aligned}(r + s)^q &\leq \{2 \max\{r, s\}\}^q = 2^q (\max\{r, s\})^q \\ &\leq 2^q (r^q + s^q).\end{aligned}$$

□

We denote by the $\langle z, x\rangle = z(x), \forall x \in \mathcal{X}, z \in \mathcal{X}$. Let $\mathcal{X}_1$ and $\mathcal{X}_2$ be two real q-uniformly smooth Banach spaces, $E : \Omega \times \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_1, F : \Omega \times \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_2$, $f, p, A, H : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1, g, h : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2, \eta_1 : \Omega \times \mathcal{X}_1 \times \mathcal{X}_1 \to \mathcal{X}_1$ and $\eta_2 : \Omega \times \mathcal{X}_2 \times \mathcal{X}_2 \to \mathcal{X}_2$ be the random single valued mappings, $S, T : \Omega \times \mathcal{X}_1 \to \mathfrak{F}(\mathcal{X}_1), G : \Omega \times \mathcal{X}_2 \to \mathfrak{F}(\mathcal{X}_2)$ be the random fuzzy mappings. Suppose that $A_1 : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1, A_2 : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2, M : \Omega \times \mathcal{X}_1 \times \mathcal{X}_1 \to 2^{\mathcal{X}_1}$ and $N : \Omega \times \mathcal{X}_2 \times \mathcal{X}_2 \to 2^{\mathcal{X}_2}$ are random nonlinear mappings such that for all $z(t) \in \mathcal{X}_1, t \in \Omega, M_t(\cdot, z(t)) : \Omega \times \mathcal{X}_1 \to 2^{\mathcal{X}_1}$ is a randomly $(A_{1,t}, \eta_{1,t})$-accretive mapping with $f_t(x(t)) - y(t) \in \operatorname{dom}(M_t(\cdot, z(t))), \forall x(t), y(t) \in \mathcal{X}_1$ and for $y(t) \in \mathcal{X}_2, t \in \Omega, N_t(\cdot, y(t)) : \Omega \times \mathcal{X}_2 \to 2^{\mathcal{X}_2}$ is a randomly $(A_{2,t}, \eta_{2,t})$-accretive mapping with $g_t(x(t)) \in \operatorname{dom}(N_t(\cdot, y(t)))$, $\forall x(t) \in \mathcal{X}_2$, respectively. Throughout this work, unless otherwise stated, for given mappings $a, b : \mathcal{X}_1 \to [0, 1]$ and $c : \mathcal{X}_2 \to [0, 1]$ we shall consider the following system of random nonlinear variational inclusions with fuzzy mappings for finding the measurable mappings $x = x(t), u = u(t), v = v(t) : \Omega \to \mathcal{X}_1, y = y(t), w = w(t) : \Omega \to \mathcal{X}_2$ such that $S_{t,x(t)}u(t) \geq a(x(t)), T_{t,x(t)}v(t) \geq b(x(t)), G_{t,x(t)}w(t) \geq c(x(t))$ and

$$
\begin{aligned}
&0 \in H_t(x(t)) + E_t(p_t(x(t)), w(t)) + M_t(f_t(x(t)) - v(t), x(t)),\\
&0 \in Q_t(y(t)) + F_t(u(t), h_t(y(t))) + N_t(g_t(y(t)), y(t)). \qquad (2.1)
\end{aligned}
$$

Let Ω be a set and $\mathfrak{F}(\mathcal{X})$ be a collection of fuzzy sets over $\mathcal{X}$. A mapping $\tilde{F} : \Omega \times \mathcal{X} \to \mathfrak{F}(\mathcal{X})$ is called a fuzzy mapping. For each $x \in \mathcal{X}, \tilde{F}(x)$ (denote it by $\tilde{F}_x$ in the sequel) is a fuzzy mapping on $\mathcal{X}$ and $\tilde{F}_x(y)$ is the membership-grade of y in $\tilde{F}_x$. Let $B \in \mathfrak{F}(\mathcal{X})$, $\alpha \in (0, 1]$, then the set

$$B_\alpha \ = \ \{x \in \mathcal{X} : \ B(x) \geq \alpha\}$$

is called an α-cut of B.

Definition 2.16. A fuzzy mapping $\tilde{G} : \Omega \times \mathcal{X} \to \mathfrak{F}(\mathcal{X})$ is called measurable, if for any $\alpha \in (0, 1]$, $(\tilde{G}(\cdot))_\alpha : \Omega \to 2^{\mathcal{X}}$ is a measurable multi-valued mapping.

Definition 2.17. A fuzzy mapping $\tilde{G} : \Omega \times \mathcal{X} \to \mathfrak{F}(\mathcal{X})$ is a random fuzzy mapping if for any $x \in \mathcal{X}, \tilde{G}(\cdot, x) : \Omega \times \mathcal{X} \to \mathfrak{F}(\mathcal{X})$ is a measurable fuzzy mapping (denoted by $\tilde{G}_{t,x}$ abbreviated $\tilde{G}_t(x)$).

Let $\tilde{G} : \Omega \times \mathcal{X} \to \mathfrak{F}(\mathcal{X})$ be a random fuzzy mapping satisfying the condition: $(*)$: there exists a function $c : \mathcal{X} \to (0, 1]$ such that for all $(t, x) \in \Omega \times \mathcal{X}$, we have $(\tilde{G}_{t.x(t)})_{c(x(t))} \in CB(\mathcal{X})$. By using the random fuzzy mapping $\tilde{G}_t$, we can define a random multi-valued mapping $G : \Omega \times \mathcal{X} \to CB(\mathcal{X})$ by $G_t = (\tilde{G}_{t,x(t)})_{c(x(t))}$ for $(t, x(t)) \in \Omega \times \mathcal{X}$ where $G_{t,x(t)} = G_t(x(t))$. In the sequel $\tilde{S}_t, \tilde{T}_t, \tilde{G}_t$ are called the random multi-valued mappings induced by the random fuzzy mappings G_t, T_t, G_t, respectively.

3. Main Results

Lemma 3.1. ([7]) *Let $M : \Omega\times\mathcal{X} \to CB(\mathcal{X})$ be a $\mathfrak{D}$-continuous random multi-valued mapping. Then for a measurable mapping $x : \Omega \to \mathcal{X}$, a multi-valued mapping $M(\cdot, x(\cdot)) : \Omega \to CB(\mathcal{X})$ is measurable.*

Lemma 3.2. ([7]) *Let $M, V : \Omega \to CB(X)$ be two measurable multi-valued mappings and $\iota > 0$ be a constant and $x : \Omega \to \mathcal{X}$ be a measurable selection of M. Then there exists a measurable selection $y : \Omega \to \mathcal{X}$ of V such that for all $t \in \Omega$*

$$\|x(t) - y(t)\| \ \leq \ (1 + \iota)\,\mathfrak{D}(M(t), V(t)).$$

Lemma 3.3. ([35]) *Let $(\mathcal{X}, d)$ be a complete metric space. Suppose that $G : \mathcal{X} \to CB(\mathcal{X})$ satisfies*

$$\mathfrak{D}(G(x), G(y)) \leq \omega d(x, y),\ \forall x, y \in \mathcal{X},$$

where $\omega \in (0, 1)$ is a constant. Then the mapping G has a fixed point in $\mathcal{X}$.

Lemma 3.4. *The set of measurable mappings $x, u, v : \Omega \to \mathcal{X}_1$ and $y, w : \Omega \to \mathcal{X}_2$ is a random solution set $(x(t), y(t), u(t), v(t), w(t))$ of the problem (2.1) if and only if for each $t \in \Omega, u(t) \in S_t(x(t)), v(t) \in T_t(x(t)), w(t) \in G_t(y(t))$ and*

$$\begin{aligned} f_t(x(t)) &= v(t) + R^{\rho_t,A_{t,1}}_{\eta_{t,1},M_t(\cdot,x(t))}[A_{t,1}(f_t(x(t)) - v(t)) \\ &\qquad - \rho_t(H_t(x(t)) + E_t(p_t(x(t)), w(t)))], \\ g_t(y(t)) &= R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y(t))}[A_{t,2}(g_t(y(t))) - \lambda_t(Q_t(y(t)) \\ &\qquad + F_t(u(t), h_t(y(t))))] \end{aligned} \tag{3.1}$$

where $\rho, \lambda : \Omega \to (0, 1)$ are measurable mappings.

Theorem 3.5. *Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measurable space, and $\mathcal{X}_1, \mathcal{X}_2$ be the two real q-uniformly smooth Banach spaces. Let $S, T : \Omega \times \mathcal{X}_1 \to \mathfrak{F}(\mathcal{X}_1)$, and $G : \Omega \times \mathcal{X}_2 \to \mathfrak{F}(\mathcal{X}_2)$ be the random fuzzy mappings satisfying the assumption $(*)$, $\tilde{S}, \tilde{T} : \Omega\times\mathcal{X}_1 \to CB(\mathcal{X}_1)$ and $\tilde{G} : \Omega\times\mathcal{X}_2 \to CB(\mathcal{X}_2)$ be the randomly $\xi_t-\mathfrak{D}_1$-Lipschitz continuous mapping, randomly $\zeta_t-\mathfrak{D}_1$-Lipschitz continuous mapping and randomly $\gamma_t-\mathfrak{D}_2$-Lipschitz continuous mappings induced by $\tilde{S}, \tilde{T}, \tilde{G}$ respectively, where $\mathfrak{D}_i, i = 1, 2$ is the Hausdorff pseudo metric on $2^{\mathcal{X}_i}$. Let $\eta_1 : \Omega \times \mathcal{X}_1 \times \mathcal{X}_1 \to \mathcal{X}_1$ be the randomly $\tau_{t,1}$-Lipschitz continuous mapping, $\eta_2 : \Omega\times\mathcal{X}_2\times\mathcal{X}_2 \to \mathcal{X}_2$ be the randomly $\tau_{t,2}$-Lipschitz continuous mapping, $p : \Omega\times\mathcal{X}_1 \to \mathcal{X}_1$ be randomly κ_t-Lipschitz continuous mapping, $h : \Omega\times\mathcal{X}_2 \to \mathcal{X}_2$ be the randomly ς-Lipschitz continuous mapping, $H : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1$ be the randomly β_t-Lipschitz continuous mapping and $Q : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2$ be the randomly*

ν_t-Lipschitz continuous mapping. Let $f : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1$ be the randomly π_t-strongly accretive mapping and randomly ε_t-Lipschitz continuous mapping, $g : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2$ be the randomly ϖ_t-strongly accretive mapping and randomly ϵ_t-Lipschitz continuous mapping. Suppose that $A_1 : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1$ is randomly $(r_{t,1}, \eta_{t,1})$-strongly accretive mapping and randomly $\alpha_{t,1}$-Lipschitz continuous mappings and $A_2 : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2$ is randomly $(r_{t,2}, \eta_{t,2})$-strongly accretive mapping and randomly $\alpha_{t,2}$-Lipschitz continuous mappings. Suppose that $M_t(\cdot, x(t)) : \Omega \times \mathcal{X}_1 \to 2^{\mathcal{X}_1}$ is randomly $(A_{t,1}, \eta_{t,1})$-accretive mapping with a measurable function $m_1 : \Omega \to (0, \infty)$ for $t \in \Omega, x(t) \in \mathcal{X}_1$ and $N(\cdot, z(t)) : \Omega \times \mathcal{X}_2 \to 2^{\mathcal{X}_2}$ is a randomly $(A_{t,2}, \eta_{t,2})$-accretive mapping with measurable function $m_2 : \Omega \to (0, \infty)$ for $z(t) \in \mathcal{X}_2, E : \Omega \times \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_1$ is a randomly single valued mapping such that $E_t(\cdot, y(t))$ is a randomly $(\delta_{t,1}, s_{t,1})$-relaxed cocoercive mapping with respect to $f_{t,1}$ and randomly σ-Lipschitz continuous mapping in the first variable and $E_t(x(t), \cdot)$ is randomly ϱ_t-Lipschitz continuous mapping in second variable for all $x(t), y(t) \in \mathcal{X}_1 \times \mathcal{X}_2, F : \Omega : \Omega \times \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_2$ is a randomly nonlinear mapping such that $F_t(x(t), \cdot)$ is randomly $(\delta_{t,2}, s_{t,2})$-relaxed cocoercive mapping with respect to the random mapping $g_{t,2}$ and randomly χ_t-Lipschitz continuous mapping in the second variable and $F_t(\cdot, y(t))$ is randomly ℓ_t-Lipschitz continuous mapping in the first variable for all $(x(t), y(t)) \in \mathcal{X}_1 \times \mathcal{X}_2, \forall t \in \Omega$ where $f_1 : \Omega \times \mathcal{X}_1 \to \mathcal{X}_1$ is defined by $f_{t,1} = A_{t,1} o(f_t(x(t)) - v(t)) = A_{t,1}(f_t(x(t)) - v(t)), \forall x(t) \in \mathcal{X}_1, t \in \Omega, b : \mathcal{X}_1 \to [0,1], T_{t,x(t)}(v(t)) \geq b(x(t))$ and $g_2 : \Omega \times \mathcal{X}_2 \to \mathcal{X}_2$ is defined by $g_{t,2}(x(t)) = A_{t,2} o g_t(x(t)) = A_{t,2}(g_t(x(t))), \forall x(t) \in \mathcal{X}_2, t \in \Omega$. If in addition there exists a measurable function $\rho : \Omega \to (0, \frac{r_{t,1}}{m_{t,1}})$ and $\lambda : \Omega \to (0, \frac{r_{t,2}}{m_{t,2}})$ such that $t \in \Omega$,

$$\|R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x(t))}(z(t)) - R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, y(t))}(z(t))\| \leq \mu_{t,1}\|x(t) - y(t)\|, \tag{3.2}$$

for all $x(t), y(t), z(t) \in \mathcal{X}_1$, and

$$\|R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, x(t))}(z(t)) - R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, y(t))}(z(t))\| \leq \mu_{t,2}\|x(t) - y(t)\|, \tag{3.3}$$

for all $x(t), y(t), z(t) \in \mathcal{X}_2$, where $\mu_1, \mu_2 : \Omega \to (0, 1)$ are measurable functions and

$$\jmath_{t,1} = \mu_{t,1} + \zeta_t + \sqrt[q]{1 - \pi_t q + c_q \varepsilon_t^q}, \quad \jmath_{t,2} = \mu_{t,2} + \sqrt[q]{1 - q\varpi_t + c_q \epsilon_t^q},$$

$$\begin{aligned} &\sqrt[q]{2^q \alpha_{t,1}^q (\varepsilon_t^q + \zeta_t^q) + c_q \rho_t^q \sigma_t^q \kappa_t^q - q\rho_t(-\delta_t \sigma_t^q \kappa_t^q + s_{t,1})} + \rho_t \beta_t \\ &< \tau_{t,1}^{1-q}(r_{t,1} - \rho_t m_{t,1})(1 - \jmath_{t,1} - \frac{\tau_{t,2}^{q-1}}{r_{t,2} - \lambda_t m_{t,2}} \lambda_t \ell_t \xi_t), \end{aligned}$$

$$\begin{aligned}&\sqrt[q]{\alpha_{t,2}^q\epsilon_t^q + c_q\lambda_t^q\chi_t^q\varsigma_t^q - q\lambda_t(-\delta_{t,2}\chi_t^q\varsigma_t^q + s_{t,2})} + \lambda_t\nu_t\\&< \tau_{t,2}^{1-q}(r_{t,2} - \lambda_t m_{t,2})\left(1 - \jmath_{t,2} - \frac{\tau_{t,1}^{q-1}}{r_{t,1} - \rho_t m_{t,1}}\rho_t\varrho_t\gamma_t\right)\end{aligned} \tag{3.4}$$

where c_q is the same as in Lemma 2.1, then the problem (2.1) has a solution set $(x^*(t), y^*(t), u^*(t), v^*(t), w^*(t))$.

Proof. For given measurable function $\rho : \Omega \to (0,\infty)$ and $\lambda : \Omega \to (0,\infty)$ we define a random mapping $\phi_t : \Omega\times\mathcal{X}_1\times\mathcal{X}_1\times\mathcal{X}_2 \to \mathcal{X}_1$ and $\psi_t : \Omega\times\mathcal{X}_1\times\mathcal{X}_2 \to \mathcal{X}_2$ as follows

$$\begin{aligned}&\phi_t(x(t), v(t), w(t))\\&= x(t) - f_t(x(t)) + v(t) + R^{\rho_t,A_{t,1}}_{\eta_{t,1},M_t(\cdot,x(t))}[A_{t,1}(f_t(x(t)) - v(t))\\&\quad - \rho_t(H_t(x(t)) + E_t(p_t(x(t)), w(t)))],\end{aligned} \tag{3.5}$$

$$\begin{aligned}&\psi_t(u(t), y(t))\\&= y(t) - g_t(y(t)) + R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y(t))}[A_{t,2}(g_t(y(t))) - \lambda_t(Q_t(y(t))\\&\quad + F_t(u(t), h_t(y(t))))],\end{aligned} \tag{3.6}$$

for $(x(t), y(t), u(t), v(t), w(t)) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_1 \times \mathcal{X}_1 \times \mathcal{X}_2$, $a, b : \mathcal{X}_1 \to [0,1]$, $c : \mathcal{X}_2 \to [0,1]$ and $S_{t,x(t)}(u(t)) \geq a(x(t))$, $T_{t,x(t)}(v(t)) \geq b(x(t))$, $G_{t,y(t)}(w(t)) \geq c(y(t))$. Now we define $\|\cdot\|_*$ on $\mathcal{X}_1 \times \mathcal{X}_2$ by

$$\|(x(t), y(t))\|_* = \|x(t)\|_* + \|y(t)\|_*, \ \ \forall (x(t), y(t)) \in \mathcal{X}_1 \times \mathcal{X}_2.$$

Since $(\mathcal{X}_1, \mathcal{X}_2, \|\cdot\|_*)$ is a Banach space(see, [18]). For any given measurable function $\rho, \lambda : \Omega \to (0,\infty)$, define $W_{t,\rho_t,\lambda_t} = \Omega \times \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_1 \times \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_1 \times \mathcal{X}_2$ by

$$W_{t,\rho_t,\lambda_t}(x(t), y(t), u(t), v(t), w(t)) = (\phi_t(x(t), v(t), w(t)), \psi_t(u(t), y(t))), \tag{3.7}$$

for all $(x(t), y(t), u(t), v(t), w(t)) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_1 \times \mathcal{X}_1 \times \mathcal{X}_2$ and

$$\begin{aligned}\mathfrak{R}_{t,\rho_t,\lambda_t}(x(t), y(t)) = \Big\{&W_{t,\rho_t,\lambda_t}(x(t), y(t), u(t), v(t), w(t)) :\\&S_{t,x(t)}(u(t)) \geq a(x(t)),\ \ T_{t,x(t)}(v(t)) \geq b(x(t)),\\&G_{t,y(t)}(w(t)) \geq c(y(t)),\\&a, b : \mathcal{X}_1 \to [0,1], c : \mathcal{X}_2 \to [0,1]\Big\},\end{aligned}$$

for all $x(t)$, $y(t) \in \mathcal{X}_1 \times \mathcal{X}_2$, $t \in \Omega$. It follows from (3.5), (3.6) and (3.7) and Lemma 3.4 that $(x^*(t), y^*(t),\ u^*(t), v^*(t), w^*(t))$ is a random solution sets of problem (2.1) if and only if there exists $(x^*(t), y^*(t)) \in \mathcal{X}_1 \times \mathcal{X}_2$ such that

$$(x^*(t), y^*(t)) \in \mathfrak{R}_{t,\rho_t,\lambda_t}(x^*(t), y^*(t)).$$

In the sequel we prove that $\mathfrak{R}_{t,\rho_t,\lambda_t}$ has a random fixed point in $\mathcal{X}_1 \times \mathcal{X}_2$. For any $t \in \Omega, x(t), y(t), x'(t), y'(t) \in \mathcal{X}_1 \times \mathcal{X}_2$, there exists a measurable function $\iota > 0$ such that

$$\begin{cases} W_{t,\rho_t,\lambda_t}(x(t), y(t), u(t), v(t), w(t)) \in \mathfrak{R}_{t,\rho_t,\lambda_t}(x(t), y(t)), \\ S_{t,x(t)}(u(t)) \geq a(x(t)), \;\; T_{t,x(t)}(v(t)) \geq b(x(t)), \;\; G_{t,y(t)}(w(t)) \geq c(y(t)), \\ a, b : \mathcal{X}_1 \to [0,1], \;\; c : \mathcal{X}_2 \to [0,1] \end{cases}$$

such that

$$\begin{aligned} &\phi_{t,\rho_t}(x(t), v(t), w(t)) \\ &= x(t) - f_t(x(t)) + v(t) + R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x(t))}[A_{t,1}(f_t(x(t)) - v(t)) \\ &\quad - \rho_t(H_t(x(t)) + E_t(p_t(x(t)), w(t)))] \end{aligned}$$

and

$$\begin{aligned} &\psi_{t,\lambda_t}(u(t), y(t)) \\ &= y(t) - g_t(y(t)) + R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, y(t))}[A_{t,2}(g_t(y(t))) - \lambda_t(Q_t(y(t)) \\ &\quad + F_t(u(t), h_t(y(t)))))]. \end{aligned}$$

$S_{t,x(t)}(u(t)) \geq a(x(t)), T_{t,x(t)}(v(t)) \geq b(x(t)), G_{t,y(t)}(w(t)) \geq c(y(t))$ *i.e.*, $u(t) \in S_t(x(t)) \in CB(\mathcal{X}_1)$, $v(t) \in T_t(x(t)) \in CB(\mathcal{X}_1)$, $w(t) \in G_t(y(t)) \in CB(\mathcal{X}_2)$. For any $t \in \Omega$, $(x'(t), y'(t)) \in \mathcal{X}_1 \times \mathcal{X}_2$, it follows from Nadler [35] that there exists $S_{t,x'(t)}(u'(t)) \geq a(x'(t))$, $T_{t,x'(t)}(v'(t)) \geq b(x'(t))$, $G_{t,y'(t)}(w'(t)) \geq c(y'(t))$ *i.e.*, $u'(t) \in S_t(x'(t))$, $v'(t) \in T_t(x'(t))$, $w'(t) \in G_t(y'(t))$ such that

$$\begin{aligned} \|u(t) - u'(t)\| &\leq (1+\iota)\mathfrak{D}_1(S_t(x(t)), S_t(x'(t))), \\ \|v(t) - v'(t)\| &\leq (1+\iota)\mathfrak{D}_1(T_t(x(t)), T_t(x'(t))), \\ \|w(t) - w'(t)\| &\leq (1+\iota)\mathfrak{D}_2(G_t(y(t)), G_t(y'(t))). \end{aligned}$$

Let

$$\begin{aligned} &\phi_t(x'(t), v'(t), w'(t)) \\ &= x'(t) - f_t(x'(t)) + v'(t) + R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x'(t))}[A_{t,1}(f_t(x'(t)) - v'(t)) \\ &\quad - \rho_t(H_t(x'(t)) + E_t(p_t(x'(t)), w'(t)))] \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} &\psi_t(u'(t), y'(t)) \\ &= y'(t) - g_t(y'(t)) + R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, y'(t))}[A_{t,2}(g_t(y'(t))) - \lambda_t(Q_t(y'(t)) \\ &\quad + F_t(u'(t), h_t(y'(t)))))]. \end{aligned} \tag{3.9}$$

Then

$$(\phi_t(x'(t), v'(t), w'(t)), \psi_t(u'(t), y'(t))) = W_{t,\rho_t,\lambda_t}(x'(t), y'(t), u'(t), v'(t), w'(t)).$$

Then by (3.5),(3.8) and Proposition 2.14, we have

$$\begin{aligned}
&\|\phi_t(x(t), v(t), w(t)) - \phi_t(x'(t), v'(t), w'(t))\| \\
&\le \|x(t) - x'(t) - (f_t(x(t)) - f_t(x'(t)))\| + \|v(t) - v'(t)\| \\
&\quad + \|R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x(t))}[A_{t,1}(f_t(x(t)) - v(t)) - \rho_t(H_t(x(t)) + E_t(p_t(x(t)), w(t)))] \\
&\quad - R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x(t))}[A_{t,1}(f_t(x'(t)) - v'(t)) - \rho_t(H_t(x'(t)) + E_t(p_t(x'(t)), w'(t)))]\| \\
&\quad + \|R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x(t))}[A_{t,1}(f_t(x'(t)) - v'(t)) - \rho_t(H_t(x'(t)) + E_t(p_t(x'(t)), w'(t)))] \\
&\quad - R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x'(t))}[A_{t,1}(f_t(x'(t)) - v'(t)) - \rho_t(H_t(x'(t)) + E_t(p_t(x'(t)), w'(t)))]\| \\
&\le \|x(t) - x'(t) - (f_t(x(t)) - f_t(x'(t)))\| + \|v(t) - v'(t)\| \\
&\quad + \mu_{t,1}\|x(t) - x'(t)\| + \frac{\tau_{t,1}^{q-1}}{r_{t,1} - \rho_t m_{t,1}} \Big\{ \rho_t \|H_t(x(t)) - H_t(x'(t))\| \\
&\quad + \rho_t \|E_t(p_t(x(t)), w(t)) - E_t(p_t(x(t)), w'(t))\| \\
&\quad + \|A_{t,1}(f_t(x(t)) - v(t)) - A_{t,1}(f_t(x'(t)) - v'(t)) \\
&\quad - \rho_t(E_t(p_t(x(t)), w'(t)) - E_t(p_t(x'(t)), w'(t)))\| \Big\}.
\end{aligned} \tag{3.10}$$

Since $\mathcal{X}_i, i = 1, 2$ is a q-uniformly smooth Banach space and f_t is randomly π_t-strongly accretive and randomly ε_t-Lipschitz continuous, we have

$$\begin{aligned}
&\|x(t) - x'(t) - (f_t(x(t)) - f_t(x'(t)))\|^q \\
&\le \|x(t) - x'(t)\|^q - q\langle f_t(x(t)) - f_t(x'(t)), j_q(x(t) - x'(t))\rangle \\
&\quad + c_q \|f_t(x(t)) - f_t(x'(t))\|^q \\
&\le \|x(t) - x'(t)\|^q - q\pi_t \|x(t) - x'(t)\|^q + c_q \varepsilon_t^q \|x(t) - x'(t)\|^q \\
&\le (1 - q\pi_t + c_q \varepsilon_t^q)\|x(t) - x'(t)\|^q.
\end{aligned} \tag{3.11}$$

Since random operator T_t is a randomly $\zeta_t - \mathfrak{D}_1$-Lipschitz continuous mapping, we have

$$\|v(t) - v'(t)\| \le (1 + \iota)\mathfrak{D}_1(T_t(x(t)), T_t(x'(t))) \le \zeta_t(1 + \iota)\|x(t) - x'(t)\|. \tag{3.12}$$

Random operator H_t is randomly β_t-Lipschitz continuous, we have

$$\|H_t(x(t)) - H_t(x'(t))\| \le \beta_t \|x(t) - x'(t)\|. \tag{3.13}$$

Random operator E_t is randomly ϱ_t-Lipschitz continuous with respect to second argument and random operator G_t is randomly $\gamma_t - \mathfrak{D}$-Lipschitz continuous mapping, we have

$$
\begin{aligned}
&\|E_t(p_t(x(t)), w(t)) - E_t(p_t(x(t)), w'(t))\| \\
&\le \varrho_t\|w(t) - w'(t)\| \le \varrho_t(1+\iota)\mathfrak{D}(G_t(y(t)), G_t(y(t))) \\
&\le \varrho_t\gamma_t(1+\iota)\|y(t) - y'(t)\|.
\end{aligned}\tag{3.14}
$$

Since $A_{t,1}$ is a randomly $\alpha_{t,1}$-Lipschitz continuous mapping, random operator p_t is randomly κ_t-Lipschitz continuous mapping, random operator f_t is randomly ε_t-Lipschitz continuous mapping and $E_t(\cdot, y(t))$ is a randomly $(\delta_{t,1}, s_{t,1})$-relaxed cocoercive mapping with respect to $f_{t,1}$ and randomly σ_t-Lipschitz continuous mapping in the first variable and $E_t(x(t), \cdot)$ is randomly ϱ_t-Lipschitz continuous in the second variable, for all $t \in \Omega, x(t), y(t) \in \mathcal{X}_1 \times \mathcal{X}_2$, from Lemma 2.1, and we have

$$
\begin{aligned}
&\|A_{t,1}(f_t(x(t)) - v(t)) - A_{t,1}(f_t(x'(t)) - v'(t)) - \rho_t(E_t(p_t(x(t)), w'(t)) \\
&\quad - E_t(p_t(x'(t)), w'(t)))\|^q \\
&\le \|A_{t,1}(f_t(x(t)) - v(t)) - A_{t,1}(f_t(x'(t)) - v'(t))\|^q \\
&\quad + c_q\rho_t^q\|E_t(p_t(x(t)), w'(t)) - E_t(p_t(x'(t)), w'(t))\|^q \\
&\quad - q\rho_t\langle E_t(p_t(x(t)), w'(t)) - E_t(p_t(x'(t)), w'(t)), \\
&\qquad\qquad j_q(A_{t,1}(f_t(x(t)) - v(t)) - A_{t,1}(f_t(x'(t)) - v'(t)))\rangle \\
&\le \alpha_{t,1}^q(\|f_t(x(t)) - f_t(x'(t))\| + \|v(t) -' (t)\|)^q \\
&\quad + c_q\rho_t^q\sigma_t^q\|p_t(x(t)) - p_t(x'(t))\|^q \\
&\quad - q\rho_t(-\delta_{t,1}\|E_t(p_t(x(t)), w'(t)) - E_t(p_t(x'(t)), w'(t))\|^q \\
&\quad + s_{t,1}\|x(t) - x'(t)\|^q) \\
&\le 2^q\alpha_{t,1}^q(\varepsilon_t^q\|x(t) - x'(t)\|^q + \zeta_t^q(1+\iota)^q\|x(t) - x'(t)\|^q) \\
&\quad + c_q\rho_t^q\sigma_t^q\kappa_t^q\|x(t) - x'(t)\|^q - q\rho_t(-\delta_{t,1}\sigma_t^q\kappa_t^q\|x(t) - x'(t)\|^q \\
&\quad + s_{t,1}\|x(t) - x'(t)\|^q) \\
&\le 2^q\alpha_{t,1}^q(\varepsilon_t^q + \zeta_t^q(1+\iota)^q)\|x(t) - x'(t)\|^q + c_q\rho_t^q\sigma_t^q\kappa_t^q\|x(t) - x'(t)\|^q \\
&\quad - q\rho_t(-\delta_{t,1}\sigma_t^q\kappa_t^q + s_{t,1})\|x(t) - x'(t)\|^q \\
&\le [2^q\alpha_{t,1}^q(\varepsilon_t^q + \zeta_t^q(1+\iota)^q) + c_q\rho_t^q\sigma_t^q\kappa_t^q \\
&\quad - q\rho_t(-\delta_{t,1}\sigma_t^q\kappa_t^q + s_{t,1})]\|x(t) - x'(t)\|)^q
\end{aligned}\tag{3.15}
$$

where c_q is the constant as in Lemma 2.1. Combining (3.11)-(3.15) with (3.10), we have

$$
\begin{aligned}
&\|\phi_t(x(t), v(t), w(t)) - \phi_t(x'(t), v'(t), w'(t))\| \\
&\le \theta_{t,1}(\iota)\|x(t) - x'(t)\| + \vartheta_{t,1}(\iota)\|y(t) - y'(t)\|
\end{aligned}\tag{3.16}
$$

where

$$\theta_{t,1}(\iota) = \mu_{t,1} + \zeta_t(1+\iota) + \sqrt[q]{1-\pi_t q + c_q\varepsilon_t^q} + \frac{\tau_{t,1}^{q-1}}{r_{t,1}-\rho_t m_{t,1}} \times \left\{ \sqrt[q]{2^q\alpha_{t,1}^q(\varepsilon_t^q + \zeta_t^q(1+\iota)^q) + c_q\rho_t^q\sigma_t^q\kappa_t^q - q\rho_t(-\delta_{t,1}\sigma_t^q\kappa_t^q + s_{t,1})} + \rho_t\beta_t \right\}$$

and

$$\vartheta_{t,1}(\iota) = \frac{\tau_{t,1}^{q-1}}{r_{t,1}-\rho_t m_{t,1}}\rho_t\varrho_t\gamma_t(1+\iota).$$

Again from (3.6) and (3.9), we have

$$\begin{aligned}
&\|\psi_t(u(t),y(t)) - \psi_t(u'(t),y'(t))\| \\
&\le \|y(t)-y'(t)-(g_t(y(t))-g_t(y'(t)))\| \\
&\quad + \Big\| R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y(t))}[A_{t,2}(g_t(y(t))) - \lambda_t(Q_t(y(t)) + F_t(u(t),h_t(y(t))))] \\
&\quad - R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y(t))}[A_{t,2}(g_t(y'(t))) - \lambda_t(Q_t(y'(t)) + F_t(u'(t),h_t(y'(t))))]\Big\| \\
&\quad + \Big\| R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y(t))}[A_{t,2}(g_t(y'(t))) - \lambda_t(Q_t(y'(t)) + F_t(u'(t),h_t(y'(t))))] \\
&\quad - R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y'(t))}[A_{t,2}(g_t(y'(t))) - \lambda_t(Q_t(y'(t)) + F_t(u'(t),h_t(y'(t))))]\Big\| \\
&\le \|y(t)-y'(t)-(g_t(y(t))-g_t(y'(t)))\| + \mu_{t,2}\|y(t)-y'(t)\| \\
&\quad + \frac{\tau_{t,2}^{q-1}}{r_{t,2}-\lambda_t m_{t,2}}\Big\{\lambda_t\|Q_t(y(t))-Q_t(y'(t))\| + \lambda_t\|F_t(u(t),h_t(y'(t))) \\
&\quad - F_t(u'(t),h_t(y'(t)))\| + \|A_{t,2}(g_t(y(t))) - A_{t,2}(g_t(y'(t))) \\
&\quad - \lambda_t(F_t(u'(t),h_t(y(t))) - F_t(u'(t),h_t(y'(t))))\|\Big\}.
\end{aligned} \tag{3.17}$$

From the assumptions of $g_t, A_{t,2}, h_t, F_t, S_t$ we obtain

$$\|y(t)-y'(t)-(g_t(y(t))-g_t(y'(t)))\| \le \sqrt[q]{1-q\varpi_t + c_q\epsilon_t^q}\|y(t)-y'(t)\|, \tag{3.18}$$

$$\|Q_t(y(t)) - Q_t(y'(t))\| \le \nu_t\|y(t)-y'(t)\|, \tag{3.19}$$

$$\begin{aligned}
&\|F_t(u(t),h_t(y'(t))) - F_t(u'(t),h_t(y'(t)))\| \\
&\le \ell_t\|u(t)-u'(t)\| \le \ell_t(1+\iota)\mathfrak{D}(S_t(x(t)),S_t(x'(t)))\| \\
&\le \ell_t\xi_t(1+\iota)\|x(t)-x'(t)\|,
\end{aligned} \tag{3.20}$$

$$\begin{aligned}
&\|A_{t,2}(g_t(y(t)))-A_{t,2}(g_t(y'(t)))-\lambda_t(F_t(u'(t),h_t(y(t)))-F_t(u'(t),h_t(y'(t))))\|^q\\
&\le \|A_{t,2}(g_t(y(t)))-A_{t,2}(g_t(y'(t)))\|^q\\
&\quad + c_q\lambda_t^q\|F_t(u'(t),h_t(y(t)))-F_t(u'(t),h_t(y'(t)))\|^q\\
&\quad - q\lambda_t\langle F_t(u'(t),h_t(y(t)))-F_t(u'(t),h_t(y'(t))),\\
&\qquad\qquad j_q(A_{t,2}(g_t(y(t)))-A_{t,2}(g_t(y'(t))))\rangle\\
&\le \alpha_{t,2}^q\epsilon_t^q\|y(t)-y'(t)\|^q + c_q\lambda_t^q\chi_t^q\varsigma_t^q\|y(t)-y'(t)\|^q\\
&\quad - q\lambda_t(-\delta_{t,2}\|F_t(u'(t),h_t(y(t)))-F_t(u'(t),h_t(y'(t)))\|^q+s_{t,2}\|y(t)-y'(t)\|^q)\\
&\le \alpha_{t,2}^q\epsilon_t^q\|y(t)-y'(t)\|^q + c_q\lambda_t^q\chi_t^q\varsigma_t^q\|y(t)-y'(t)\|^q\\
&\quad - q\lambda_t(-\delta_{t,2}\chi_t^q\varsigma_t^q\|y(t)-y'(t)\|^q + s_{t,2}\|y(t)-y'(t)\|^q)\\
&\le (\alpha_{t,2}^q\epsilon_t^q + c_q\lambda_t^q\chi_t^q\varsigma_t^q - q\lambda_t(-\delta_{t,2}\chi_t^q\varsigma_t^q + s_{t,2}))\|y(t)-y'(t)\|^q. \qquad (3.21)
\end{aligned}$$

Combining the (3.17)-(3.21), we have

$$\begin{aligned}
&\|\psi_{\lambda,t}(u(t),y(t))-\psi_{\lambda,t}(u'(t),y'(t))\|\\
&\le \theta_{t,2}(\iota)\|x(t)-x'(t)\|+\vartheta_{t,2}\|y(t)-y'(t)\| \qquad (3.22)
\end{aligned}$$

where

$$\theta_{t,2}(\iota)=\frac{\tau_{t,2}^{q-1}}{r_{t,2}-\lambda_t m_{t,2}}\lambda_t\ell_t\xi_t(1+\iota),$$

$$\begin{aligned}
\vartheta_{t,2} &= \mu_{t,2}+\sqrt[q]{1-q\varpi_t+c_q\epsilon_t^q}\\
&\quad+\frac{\tau_{t,2}^{q-1}\{\lambda_t\nu_t+\sqrt[q]{\alpha_{t,2}^q\epsilon_t^q+c_q\lambda_t^q\chi_t^q\varsigma_t^q-q\lambda_t(-\delta_{t,2}\chi_t^q\varsigma_t^q+s_{t,2})}\}}{r_{t,2}-\lambda_t m_{t,2}}.
\end{aligned}$$

It follows from (3.16) and (3.22) that

$$\begin{aligned}
&\|\phi_{\rho,t}(x(t),v(t),w(t))-\phi_{\rho,t}(x'(t),v'(t),w'(t))\|\\
&\quad+\|\psi_{\lambda,t}(u(t),y(t))-\psi_{\lambda,t}(u'(t),y'(t))\|\\
&\le \vartheta_t(\iota)(\|x(t)-x'(t)\|+\|y(t)-y'(t)\|) \qquad (3.23)
\end{aligned}$$

where

$$\vartheta_t(\iota)=\max\{\theta_{t,1}(\iota)+\theta_{t,2}(\iota),\vartheta_{t,1}(\iota)+\vartheta_{t,2}\}.$$

From (3.7) and (3.23) we have

$$\begin{aligned}
&\|W_{t,\rho_t,\lambda_t}(x(t),y(t),u(t),v(t),w(t))-W_{t,\rho_t,\lambda_t}(x'(t),y'(t),u'(t),v'(t),w'(t))\|\\
&\le \vartheta_t(\iota)\|(x(t),y(t))-(x'(t),y'(t))\|_*,
\end{aligned}$$

i.e.,

$$\sup\Big\{d(W_{t,\rho_t,\lambda_t}(x(t),y(t),u(t),v(t),w(t)),\mathfrak{R}_{\rho_t,\lambda_t}(x'(t),y'(t))):$$
$$W_{t,\rho_t,\lambda_t}(x(t),y(t),u(t),v(t),w(t))\in\mathfrak{R}_{\rho_t,\lambda_t}(x(t),y(t))\Big\}$$
$$\leq \vartheta_t(\iota)\|(x(t),y(t))-(x'(t),y'(t))\|_*. \tag{3.24}$$

Similarly we have

$$\sup\Big\{d(W_{t,\rho_t,\lambda_t}(x'(t),y'(t),u'(t),v'(t),w'(t)),\mathfrak{R}_{\rho_t,\lambda_t}(x(t),y(t))):$$
$$W_{t,\rho_t,\lambda_t}(x'(t),y'(t),u'(t),v'(t),w'(t))\in\mathfrak{R}_{\rho_t,\lambda_t}(x'(t),y'(t))\Big\}$$
$$\leq \vartheta_t(\iota)\|(x(t),y(t))-(x'(t),y'(t))\|_*. \tag{3.25}$$

It follows from (3.24),(3.25) and the definition of Hausdorff metric we have

$$\mathfrak{D}(\mathfrak{R}_{\rho_t,\lambda_t}(x(t),y(t)),\mathfrak{R}_{\rho_t,\lambda_t}(x'(t),y'(t)))\leq\vartheta_t(\iota)\|(x(t),y(t))-(x'(t),y'(t))\|_*,$$

for all $(x(t),y(t)),(x'(t),y'(t))\in\mathcal{X}_1\times\mathcal{X}_2$. Letting $\iota\to 0$ we get

$$\begin{aligned}&\mathfrak{D}(\mathfrak{R}_{\rho_t,\lambda_t}(x(t),y(t)),\mathfrak{R}_{\rho_t,\lambda_t}(x'(t),y'(t)))\\&\leq\vartheta_t\|(x(t),y(t))-(x'(t),y'(t))\|_*,\end{aligned} \tag{3.26}$$

for all $(x(t),y(t)),(x'(t),y'(t))\in\mathcal{X}_1\times\mathcal{X}_2$, where $\vartheta_{t,2}$ is the constant as in (3.16) and

$$\vartheta_t=\max\{\theta_{t,1}+\theta_{t,2},\vartheta_{t,1}+\vartheta_{t,2}\},$$

$$\begin{aligned}\theta_{t,1}&=\mu_{t,1}+\zeta_t+\sqrt[q]{1-\pi_t q+c_q\varepsilon_t^q}\\&\quad+\frac{\tau_{t,1}^{q-1}}{r_{t,1}-\rho_t m_{t,1}}\Big\{\sqrt[q]{2^q\alpha_{t,1}^q(\varepsilon_t^q+\zeta_t^q)+c_q\rho_t^q\sigma_t^q\kappa_t^q-q\rho_t(-\delta_t\sigma_t^q\kappa_t^q+s_{t,1})}+\rho_t\beta_t\Big\},\end{aligned}$$

$$\theta_{t,2}=\frac{\tau_{t,2}^{q-1}}{r_{t,2}-\lambda_t m_{t,2}}\lambda_t\ell_t\xi_t,\quad \vartheta_{t,1}=\frac{\tau_{t,1}^{q-1}}{r_{t,1}-\rho_t m_{t,1}}\rho_t\varrho_t\gamma_t,$$

$$\begin{aligned}\vartheta_{t,2}&=\mu_{t,2}+\sqrt[q]{1-q\varpi_t+c_q\epsilon_t^q}\\&\quad+\frac{\tau_{t,2}^{q-1}\{\lambda_t\nu_t+\sqrt[q]{\alpha_{t,2}^q\epsilon_t^q+c_q\lambda_t^q\chi_t^q\varsigma_t^q-q\lambda_t(-\delta_{t,2}\chi_t^q\varsigma_t^q+s_{t,2})}\}}{r_{t,2}-\lambda_t m_{t,2}}.\end{aligned}$$

It follows from (3.4) that $0<\vartheta_t<1$ and so by (3.26) and Lemma 3.3, $\mathfrak{R}_{\rho_t,\lambda_t}$ has a random fixed point in $\mathcal{X}_1\times\mathcal{X}_2$ *i.e.*, there exists a point $(x^*(t),y^*(t))\in\mathcal{X}_1\times\mathcal{X}_2$ such that

$$(x^*(t),y^*(t))\in\mathfrak{R}_{\rho_t,\lambda_t}(x^*(t),y^*(t)).$$

This completes the proof. □

4. Random iterative algorithm and convergence analysis

In this section, based on Lemma 3.4 and Nadler [35] we shall suggest a class of random iterative algorithms for finding a random solutions of problem (2.1) and discuss the convergence analysis of the algorithm.

Algorithm 4.1. *Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measurable space, $\mathcal{X}_1$ and $\mathcal{X}_2$ be two real q-uniformly smooth Banach spaces. Assume that S_t, T_t, $G_t, \tilde{S}_t, \tilde{T}_t, \tilde{G}_t, h_t, f_t, p_t, g_t$, $H_t, E_t, M_t, Q_t, F_t, N_t, \eta_{t,i}, A_{t,i}$ are same as in the Theorem 3.5, $i = 1, 2$. For any given $(x_0(t), y_0(t)) \in \mathcal{X}_1 \times \mathcal{X}_2, a, b : \mathcal{X}_1 \to [0, 1]$ and $c : \mathcal{X}_2 \to [0, 1], n \geq 0, \iota > 0$, an element $(x(t), y(t), u(t), v(t), w(t)) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_1 \times \mathcal{X}_1 \times \mathcal{X}_2$, we define the random iterative sequences $\{x_n(t), y_n(t), u_n(t), v_n(t), w_n(t)\}$ by*

$$
\begin{aligned}
x_{n+1}(t) = (1 - \alpha_{n,t})x(t) + \alpha_{n,t}\Big[& x_n(t) - f_t(x_n(t)) + v_n(t) \\
& + R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x_n(t))}[A_{t,1}(f_t(x_n(t)) - v_n(t)) - \rho_t(H_t(x_n(t)) \\
& + E_t(p_t(x_n(t)), w_n(t)))]\Big] + e_n(t),
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
y_{n+1}(t) = (1 - \alpha_{n,t})y_n(t) + \alpha_{n,t}\Big[& y_n(t) - g_t(y_n(t)) \\
& + R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, y_n(t))}[A_{t,2}(g_t(y_n(t))) - \lambda_t(Q_t(y_n(t)) \\
& + F_t(u_n(t), h_t(y_n(t))))\Big] + r_n(t),
\end{aligned} \tag{4.2}
$$

$$
\begin{aligned}
\tilde{S}_{t,x(t)}(u_n(t)) &\geq a(x_n(t)) : \|u_n(t) - u(t)\| \leq (1 + \iota)\mathfrak{D}_1(\tilde{S}_t(x_n(t)), \tilde{S}_t(x(t))), \\
\tilde{T}_{t,x(t)}(v_n(t)) &\geq b(x_n(t)) : \|v_n(t) - v(t)\| \leq (1 + \iota)\mathfrak{D}_1(\tilde{T}_t(x_n(t)), \tilde{T}_t(x(t))), \\
\tilde{G}_{t,y(t)}(w_n(t)) &\geq c_t(y_n(t)) : \|w_n(t) - w(t)\| \leq (1 + \iota)\mathfrak{D}_2(\tilde{G}_t(y_n(t)), \tilde{G}_t(y(t)))
\end{aligned}
$$

where $\rho, \lambda : \Omega \to (0, 1)$ is the measurable mapping, $\{\alpha_n(t)\}$ is a random sequence in $[0, 1]$, and $\{e_n(t)\}$ and $\{r_n(t)\}$ are two random sequences satisfying some conditions in $\mathcal{X}_1$ and $\mathcal{X}_2$, respectively.

Lemma 4.2. ([34]) *Let $\{a_n(t)\}, \{b_n(t)\}$ and $\{c_n(t)\}$ be the three random real sequences of nonnegative numbers satisfying the following conditions:*

(i) $0 \leq b_n(t) < 1, \ \ n = 0, 1, 2, \cdots$ *and* $\ \limsup_n b_n(t) < 1$,
(ii) $\sum_{n=0}^{\infty} c_n(t) < +\infty$,

(iii) $a_{n+1}(t) \leq b_n(t)a_n(t) + c_n(t), \quad n = 0, 1, 2, \cdots.$

Then $a_n(t) \to 0$ *as* $n \to \infty$.

Theorem 4.3. *Let* $(\Omega, \mathcal{A}, \mu)$ *be a complete* σ*-finite measurable space,* $\mathcal{X}_1$ *and* $\mathcal{X}_2$ *be two real q-uniformly smooth Banach spaces. Assume that* $S_t, T_t, G_t, \tilde{S}_t, \tilde{T}_t, \tilde{G}_t, h_t, f_t, p_t, g_t, H_t, E_t, M_t, Q_t, F_t, N_t, \eta_{t,i}, A_{t,i}$ *are same as in the problem* (2.1), $i = 1, 2$. *Assume that all the assumptions of Theorem* 3.5 *hold and*

$$\limsup_n b_n(t) < 1, \quad \sum_{n=0}^{\infty}(\|e_n(t)\| + \|r_n(t)\|) < \infty. \tag{4.3}$$

Then the sequence $(x_n(t), y_n(t), u_n(t), v_n(t), w_n(t))$ *defined by Algorithm* 4.1 *randomly converges strongly to the random solution* $(x^*(t), y^*(t), u^*(t), v^*(t), w^*(t))$ *of* (2.1).

Proof. By Theorem 3.5, problem (2.1) admits a random solution set $(x^*(t), y^*(t), u^*(t), v^*(t), w^*(t))$. It follows from Lemma 3.4 that

$$\begin{aligned} f_t(x^*(t)) = v^*(t) + R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x^*(t))}[&A_{t,1}(f_t(x^*(t)) - v^*(t)) \\ &- \rho_t(H_t(x^*(t)) + E_t(p_t(x^*(t)), w^*(t)))] \end{aligned} \tag{4.4}$$

and

$$\begin{aligned} g_t(y^*(t)) = R^{\lambda_t, A_{t,2}}_{\eta_{t,2}, N_t(\cdot, y^*(t))}[&A_{t,2}(g_t(y^*(t))) - \lambda_t(Q_t(y^*(t)) \\ &+ F_t(u^*(t), h_t(y^*(t))))]. \end{aligned} \tag{4.5}$$

It follows from (4.1),(4.4) and assumptions that

$$\begin{aligned} &\|x_{n+1}(t) - x^*(t)\| \\ &\leq (1-\alpha_n(t))\|x_n(t)-x^*(t)\| \\ &\quad + \alpha_n(t)(\|x_n(t)-x^*(t)-(f_t(x_n(t))-f_t(x^*(t)))\| + \|v_n(t) - v^*(t)\|) \\ &\quad + \alpha_n(t)\Big\|R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x_n(t))}[A_{t,1}(f_t(x_n(t))-v_n(t))-\rho_t(H_t(x_n(t)) \\ &\quad + E_t(p_t(x_n(t), w_n(t))))] - R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x_n(t))}[A_{t,1}(f_t(x^*(t)) - v^*(t)) \\ &\quad - \rho_t(H_t(x^*(t)) + E_t(p_t(x^*(t), w^*(t))))]\Big\| \\ &\quad + \alpha_n(t)\Big\|R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x_n(t))}[A_{t,1}(f_t(x^*(t)) - v^*(t)) - \rho_t(H_t(x^*(t)) \\ &\quad + E_t(p_t(x^*(t), w^*(t))))] - R^{\rho_t, A_{t,1}}_{\eta_{t,1}, M_t(\cdot, x^*(t))}[A_{t,1}(f_t(x^*(t)) - v^*(t)) \\ &\quad - \rho_t(H_t(x^*(t)) + E_t(p_t(x^*(t), w^*(t))))]\Big\| + \|e_n(t)\| \end{aligned}$$

$$
\begin{aligned}
&\le (1-\alpha_n(t))\|x_n(t)-x^*(t)\|+\alpha_n(t)(\|x_n(t)-x^*(t)-(f_t(x_n(t))-f_t(x^*(t)))\|\\
&\quad+\|v_n(t)-v^*(t)\|+\mu_t\|x_n(t)-x^*(t)\|)+\alpha_n(t)\frac{\tau_{t,1}^{q-1}}{r_{t,1}-\rho_t m_{t,1}}\Bigg\{\rho_t\Bigg(\|H_t(x_n(t))\\
&\quad-H_t(x^*(t))\|+\|E_t(p_t(x_n(t)),w_n(t))-E_t(p_t(x_n(t)),w^*(t))\|\Bigg)\\
&\quad+\|A_{t,1}(f_t(x_n(t))-v_n(t))-A_{t,1}(f_t(x^*(t))-v^*(t))\\
&\quad-\rho_t(E_t(p_t(x_n(t)),w^*(t))-E_t(p_t(x^*(t)),w^*(t)))\|\Bigg\}+\|e_n(t)\|\\
&\le (1-\alpha_n(t))\|x_n(t)-x^*(t)\|\\
&\quad+\alpha_n(t)(\theta_{t,1}(\iota)\|x_n(t)-x^*(t)\|+\vartheta_{t,1}(\iota)\|y_n(t)-y^*(t)\|)+\|e_n(t)\|, \qquad (4.6)
\end{aligned}
$$

and from (4.2), (4.5) we have

$$
\begin{aligned}
&\|y_{n+1}(t)-y^*(t)\|\\
&\le (1-\alpha_n(t))\|y_n(t)-y^*(t)\|+\alpha_n(t)\|y_n(t)-y^*(t)-(g_t(y_n(t))-g_t(y^*(t)))\|\\
&\quad+\alpha_n(t)\|R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y_n(t))}[A_{t,2}(g_t(y_n(t)))-\lambda_t(Q_t(y_n(t))+F_t(u_n(t),h_t(y_n(t))))]\\
&\quad-R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y_n(t))}[A_{t,2}(g_t(y^*(t)))-\lambda_t(Q_t(y^*(t))+F_t(u^*(t),h_t(y^*(t))))]\|\\
&\quad+\alpha_n(t)\|R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y_n(t))}[A_{t,2}(g_t(y^*(t)))-\lambda_t(Q_t(y^*(t))+F_t(u^*(t),h_t(y^*(t))))]\\
&\quad-R^{\lambda_t,A_{t,2}}_{\eta_{t,2},N_t(\cdot,y^*(t))}[A_{t,2}(g_t(y^*(t)))-\lambda_t(Q_t(y^*(t))+F_t(u^*(t),h_t(y^*(t))))]\|\\
&\quad+\|r_n(t)\|\\
&\le (1-\alpha_n(t))\|y_n(t)-y^*(t)\|+\alpha_n(t)(\theta_{t,2}(\iota)\|x_n(t)-x^*(t)\|\\
&\quad+\vartheta_{t,2}(\iota)\|y_n(t)-y^*(t)\|)+\|e_n(t)\|. \qquad (4.7)
\end{aligned}
$$

By (4.6) and (4.7) we have

$$
\begin{aligned}
&\|x_{n+1}(t)-x^*(t)\|+\|y_{n+1}(t)-y^*(t)\|\\
&\le [1-\alpha_n(t)+\alpha_n(t)\vartheta_t(\iota)](\|x_n(t)-x^*(t)\|\\
&\quad+\|y_n(t)-y^*(t)\|)+(\|e_n(t)\|+\|r_n(t)\|), \qquad (4.8)
\end{aligned}
$$

where $\vartheta_t(\iota)$ is same as in (3.23). Letting $\iota\to 0$ and

$$
\begin{aligned}
a_n(t)&=\|x_n(t)-x^*(t)\|+\|y_n(t)-y^*(t)\|,\\
b_n(t)&=1-\alpha_n(t)(1-\vartheta_t(\iota)),\\
c_n(t)&=\|e_n(t)\|+\|r_n(t)\|,
\end{aligned}
$$

where ϑ_t is same as in (3.26). Then (4.8) can be rewritten as

$$
a_{n+1}(t)\le b_n(t)a_n(t)+c_n(t),\quad n=0,1,2,\cdots.
$$

From (4.3) we have

$$\limsup_n b_n(t) < 1 \quad \text{and} \quad \sum_{n=0}^{\infty} c_n(t) < +\infty.$$

It follows from Lemma 4.2 that

$$\|x_n(t) - x^*(t)\| + \|y_n(t) - y^*(t)\| \to 0, \text{ as } n \to \infty.$$

Therefore $(x_n(t), y_n(t), u_n(t), v_n(t), w_n(t))$ defined by Algorithm 4.1 converges strongly to the random solution set of $(x^*(t), y^*(t), u^*(t), v^*(t), w^*(t))$ of (2.1). This completes the proof. □

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