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COUPLED RANDOM COINCIDENCE POINT FOR RANDOM OPERATORS SATISFYING RATIONAL TYPE CONTRACTIVE INEQUALITIES

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Abstract. The object of this paper is to establish certain coupled random coincidence point theorems for a pair of random operators satisfying a contractive condition of rational type. We also prove some coupled common random fixed point of two random operators using I-scheme. These results present random versions and extensions of recent results of Chandok et. al. [9] and Singh et. al. [39].

1. Introduction

Fixed point theory is one of the famous and traditional theories in mathematics. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by Banach [5] in 1922. After the classical result, Kannan [24] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been working on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, Branciari [8] analyzed the existence of fixed point for mapping defined on a complete metric space (X, d) satisfying a general contractive condition of rational type(see [25, 27]).

In recent years, the study of random fixed points has attracted much attention. Some random fixed point theorems play an important role in the theory

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of random differential and random integral equations(see [23, 29]). Random fixed point theorems for contractive mappings on separable complete metric spaces have been proved by several authors [1, 3, 6, 13, 18, 26]. Sehgal and Singh [38] have proved different stochastic versions of well-known Schauder fixed point theorem. Fixed point theorems for monotone operators in ordered Banach spaces have been investigated and have found various applications in differential and integral equations (see [4, 11, 19]). Bhaskar and Lakshmikantham [7] introduced the concept of mixed monotone mappings and obtained some coupled fixed point results. Recently Ćirić and Lakshmikantham [14] and Zhu and Xiao [42] proved some coupled random fixed point and coupled random coincidence results in partially ordered complete metric spaces.

Ishikawa [22] and Mann [31] iteration schemes have been successfully applied in linear spaces to fixed point problem of operators and also for obtaining solution of operator equations. In [2, 16, 34, 35, 40], it has shown that for a mapping T satisfying certain conditions, if the sequence of Mann iterates converges, then it converges to a fixed point of T. Singh et al. in [39] defined an iteration scheme called I-scheme to find common fixed point theorem in Hilbert spaces using rational inequality. The purpose of this article is to improve these results for a pair of random operators $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$, where F and g satisfying contractive condition of rational type. Presented results are the extension and improvment of the corresponding results in [9, 10, 39] and many others.

2. Preliminaries

Recall that if $(X, \leq)$ is a partially ordered set and $F : X \to X$ is such that for $x, y \in X$, $x \leq y$ implies $F(x) \leq F(y)$, then a mapping F is said to be a non-decresing. Similarly, a non-increasing map may be defined. Bhaskar and Lakshmikantham [7] intrduced the following notions of a mixed monotone mapping and a coupled fixed point.

Definition 2.1. ([7]) Let $(X, \leq)$ be a partially ordered set and $F : X \times X \to X$. The mapping F is said to has the mixed monotone property if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is , for any $x, y \in X$,

$$x_1, x_2 \in X; x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X; y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2).$$

The concept of the mixed monotone property is generalized in [28].

Definition 2.2. ([28]) Let $(X, \leq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \times X$. The mapping F is said to has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X; g(x_1) \leq g(x_2) \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X; g(y_1) \leq g(y_2) \Rightarrow F(x, y_1) \geq F(x, y_2).$$

Definition 2.3. ([7]) An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$$F(x, y) = x, \qquad F(y, x) = y.$$

Definition 2.4. An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = g(x), \qquad F(y, x) = g(y).$$

Definition 2.5. Let (X, d) be a metric space. The mappings $F : X \to X$ and $g : X \to X$ are said to be commute if

$$F(gx, gy) = g(F(x, y)) \quad \text{for all } x, y \in X.$$

Definition 2.6. Two mappings $F : X \times X \to X$ and $g : X \to X$ are said to be compatible if

$$\lim_{n\to\infty} d(g(F(x_n, y_n)), F(g(x_n), g(y_n))) = 0$$

and

$$\lim_{n\to\infty} d(g(F(y_n, x_n)), F(g(y_n), g(x_n))) = 0,$$

whenever $\{x_n\}$, $\{y_n\}$ are sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x$$

and

$$\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$$

for all $x, y \in X$.

Using the concept of mixed g-monotone property Chandok et al. [9] proved the following theorem.

Theorem 2.7. ([9]) *Let* $(X, \preccurlyeq)$ *ba a partially ordered set and suppose that there exist a metric* d *on* X *such that* (X, d) *is a complete metric space. Suppose that* $F : X \times X \to X$ *and* $g : X \to X$ *are self mappings on* X *such that* F *has the mixed* g*-monotone property on* X *such that there exists two* $x_0, y_0 \in X$ *with* $g(x_0) \preccurlyeq F(x_0, y_0)$ *and* $g(y_0) \succcurlyeq F(y_0, x_0)$*. Suppose that there exists* $\alpha \in [0, 1)$ *such that*

$$\begin{aligned} &d\left(F\left(x, y\right), F\left(u, v\right)\right) \\ &\leq \alpha \max \left\{ d\left(gx, gu\right), d\left(gy, gv\right), \frac{d(gx,F(x,y))d(gu,F(u,v))}{d(gx,gu)}, \right. \\ &\qquad \frac{d(gx,F(u,v))d(gu,F(x,y))}{d(gx,gu)}, \frac{d(gy,F(y,x))d(gv,F(v,u))}{d(gy,gv)}, \\ &\qquad \left. \frac{d(gy,F(v,u))d(gv,F(y,x))}{d(gy,gv)} \right\} \end{aligned}$$

satisfies for all $x, y, u, v \in X$*,* $gx \neq gu$*,*$gy \neq gv$ *with* $gx \succcurlyeq gu$ *and* $gy \preccurlyeq gv$*. Further suppose that* F *is continuous,* $F(X \times X) \subseteq g(X)$*,* g *is continuous non-decreasing and commutes with* F*. Then there exist* $x, y \in X$ *such that either* $gx = F(x, y)$ *or* $gy = F(y, x)$ *or* $gx = F(x, y)$ *and* $gy = F(y, x)$ *i,e.,* F *and* g *have a coupled coincidence point* $(x, y) \in X \times X$*.*

Doric et al. [15] showed that a mixed monotone property in coupled fixed point results for mappings in ordered metric spaces can be replaced by another property. If x, y are elements of a partially ordered set $(X, \leq)$ are comparable (i.e., $x \leq y$ or $y \leq x$ hold) we will write $x \lesseqgtr y$. Let $g : X \to X$ and $F : X \times X \to X$. We will consider the following condition:

if $x, y, u, v \in X$ are such that $gx \lesseqgtr F(x, y) = gu$, then $F(x, y) \lesseqgtr F(u, v)$.

If g is an identity mapping, for all x, y, v if $x \lesseqgtr F(x, y)$, then $F(x, y) \lesseqgtr F(F(x, y), v)$.

Chandok et al. [10] also proved the following theorem without mixed monotonicity.

Theorem 2.8. ([10]) *Let* $(X, d, \leq)$ *be a complete partially ordered metric space and let* $F : X \times X \to X$ *and* $g : X \to X$*. Suppose that the following hold:*

(i) *g is continuous and $g(X)$ is closed;*
(ii) *$F(X \times X) \subseteq g(X)$ and g and F are compatible;*
(iii) *for all $x, y, u, v \in X$, if $g(x) \lesseqgtr F(x, y) = gu$, then $F(x, y) \lesseqgtr F(u, v)$;*
(iv) *there exist $x_0, y_0 \in X$ such that $gx_0 \lesseqgtr F(x_0, y_0)$ and $gy_0 \lesseqgtr F(y_0, x_0)$;*

(v) *there exists* $\alpha \in [0,1)$ *such that for all* $x, y, u, v \in X$ *with* $gx \lesseqgtr gu$ *and* $gy \lesseqgtr gv$*satisfies*

$$
\begin{aligned}
& d\left(F\left(x,y\right), F\left(u,v\right)\right) \\
& \leq \alpha \max \left\{ d\left(gx,gu\right), d\left(gy,gv\right), \tfrac{d(gx,F(x,y))d(gu,F(u,v))}{d(gx,gu)}, \right. \\
& \qquad \tfrac{d(gx,F(u,v))d(gu,F(x,y))}{d(gx,gu)}, \tfrac{d(gy,F(y,x))d(gv,F(v,u))}{d(gy,gv)}, \\
& \qquad \left. \tfrac{d(gy,F(v,u))d(gv,F(y,x))}{d(gy,gv)} \right\};
\end{aligned}
$$

(vi) F *is continuous.*

Then there exist $x, y \in X$ *such that* $F(x,y) = g(x)$ *and* $gy = F(y,x)$, *that is,* F *and* g *have a coupled coincidence point* $(x,y) \in X \times X$.

Let C be a non-empty convex subset of a normed space E and $T : C \to C$ be a mapping. The Mann iteration process is defined by the sequence $\{x_n\}$ in [31],

$$
\begin{aligned}
x_1 &= x \in C, \\
x_{n+1} &= \left(1 - b_n\right) x_n + b_n T x_n, \quad n \in \mathbb{N},
\end{aligned}
$$

where $\mathbb{N}$ denote the set of all positive integers and $\{b_n\}$ is a sequence in $[0,1]$. Liu [30] introduced the concept of Mann iteration process with errors by the sequence $\{x_n\}$ defined as follows:

$$
\begin{aligned}
x_1 &= x \in C, \\
x_{n+1} &= \left(1 - b_n\right) x_n + b_n T x_n + u_n, \quad n \in \mathbb{N},
\end{aligned}
$$

where $\{b_n\}$ is a sequence in $[0,1]$ and $\{u_n\}$ satisfy $\sum\limits_{n=1}^{\infty} \|u_n\| < \infty$. Singh et al. [39] introduced the iteration scheme called I-scheme are defined as follows:

Definition 2.9. Let X be a Banach space and C be a non-empty subset of X. Let $T_1, T_2 : C \to C$ be two mappings. The I-scheme is defined as follows:

$$x_0 \in C, \tag{2.1}$$

$$
\begin{aligned}
y_{2n} &= \beta_{2n} T_1 x_{2n} + \left(1 - \beta_{2n}\right) x_{2n}, \\
x_{2n+1} &= \left(1 - \alpha_{2n}\right) x_{2n} + \alpha_{2n} T_2 y_{2n},
\end{aligned} \tag{2.2}
$$

$$
\begin{aligned}
y_{2n+1} &= \beta_{2n+1} T_1 x_{2n+1} + \left(1 - \beta_{2n+1}\right) x_{2n+1}, \\
x_{2n+2} &= \left(1 - \alpha_{2n+1}\right) x_{2n+1} + \alpha_{2n+1} T_2 y_{2n+1}, \; n \geq 0
\end{aligned} \tag{2.3}
$$

where $\{\alpha_{2n}\}$, $\{\beta_{2n}\}$ satisfying the following conditions

(i) $0 \leq \alpha_{2n} \leq \beta_{2n} \leq 1$, for all n,
(ii) $\lim_{n\to\infty} \alpha_{2n} = \alpha_{2n} > 0$, and
(iii) $\lim_{n\to\infty} \beta_{2n} = \beta_{2n} < 1$.

Definition 2.10. Let X be a Banach space and C be a non-empty subset of X. Let $T_1, T_2 : C \to C$ be two mappings. The I-scheme is defined as follows:

$$x_0 \in C, \tag{2.4}$$

$$\begin{aligned} y_{2n} &= \beta_{2n} T_1 x_{2n} + (1 - \beta_{2n})\, x_{2n}, \\ x_{2n+1} &= (1 - \alpha_{2n})\, x_{2n} + \alpha_{2n} T_2 y_{2n}, \end{aligned} \tag{2.5}$$

$$\begin{aligned} y_{2n+1} &= \beta_{2n+1} T_1 x_{2n+1} + (1 - \beta_{2n+1})\, x_{2n+1}, \\ x_{2n+2} &= (1 - \alpha_{2n+1})\, x_{2n+1} + \alpha_{2n+1} T_2 y_{2n+1}, \ n \geq 0, \end{aligned} \tag{2.6}$$

where $\{\alpha_{2n}\}$, $\{\beta_{2n}\}$ satisfying the following conditions

(i) $0 \leq \alpha_{2n} \leq \beta_{2n} \leq 1$, for all n,
(ii) $\lim_{n\to\infty} \alpha_{2n} = \alpha_{2n} > 0$, and
(iii) $\lim_{n\to\infty} \beta_{2n} = \beta_{2n} < 1$.

Theorem 2.11. ([39]) *Let X be Hilbert space and C be closed convex subset of X. Let T_1 and T_2 be two sets of mapping satisfying*

$$\begin{aligned} &\|T_1x - T_2y\| \\ &\leq K \max\Bigg\{ \|y - T_2y\|^2, \tfrac{1}{4}\left(\|x - T_2y\|^2 + \|y - T_1x\|^2\right), \\ &\qquad \tfrac{1}{2}\left(\|x - T_1x\|^2 + \|y - T_2y\|^2\right), \tfrac{\|y-T_2y\|^2[1+\|x-T_1x\|^2]}{1+\|x-y\|^2}, \\ &\qquad \tfrac{\|x-T_1x\|^2[1+\|x-y\|^2]}{1+\|y-T_2y\|^2}, \tfrac{\|x-y\|^2[1+\|x-T_1x\|^2]}{1+\|y-T_2y\|^2}, \tfrac{\|x-T_1x\|^2[1+\|y-T_2y\|^2]}{1+\|x-y\|^2}, \\ &\qquad \tfrac{\left(1+\|y-T_2y\|^2\right)[1+\|x-T_1x\|^2]}{1+\|x-y\|^2} \Bigg\}, \end{aligned}$$

where, $o \leq k < \frac{1}{4}$. If there exist a point x_0 such that the I-scheme for point of T_1 and T_2 defined by (2.5) *and* (2.6)*, converges to a point p, then p is a common fixed point of T_1 and T_2.*

3. Main Results

Let (Ω, Σ) be a measurable space with Σ being a sigma algebra of subsets of Ω and let (X, d) be a metric space. A mapping $T : \Omega \to X$ is called Σ-measurable if for any open subset U of X, $T^{-1}(U) = \{\omega : T(\omega) \in U\} \in \Sigma$. In what follows, when we speak of measurability we will mean Σ-measurability. A mapping $T : \Omega \times X \to X$ is called a random operator if for any $x \in X$, $T(\cdot, x)$ is measurable. A measurable mapping $\zeta : \Omega \to X$ is called a random fixed point of a random function $T : \Omega \times X \to X$ if $\zeta(\omega) = T(\omega, \zeta(\omega))$ for every $\omega \in \Omega$. A measurable mapping $\zeta : \Omega \to X$ is called a random coincidence of $T : \Omega \times X \to X$ and $g : \Omega \times X \to X$ if $g(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega))$ for every $\omega \in \Omega$.

Definition 3.1. Let (X,d) be a separable metric space, (Ω,Σ) be a measurable space and $F:\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ be mappings. We say that F and g are said to be commute if

$$F\left(\omega,\left(g\left(\omega,x\right),g\left(\omega,y\right)\right)\right)=g\left(\omega,F\left(\omega,\left(x,y\right)\right)\right)\quad\text{for all } x,y\in X \text{ and } \omega\in\Omega.$$

Definition 3.2. Let (X,d) be a separable metric space, (Ω,Σ) be a measurable space and $F:\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ be mappings. We say F and g are compatible if

$$\lim_{n\to\infty} d\left(g\left(\omega,F\left(\omega,\left(x_n,y_n\right)\right)\right),F\left(\omega,\left(g\left(\omega,x_n\right),g\left(\omega,y_n\right)\right)\right)\right)=0$$

and

$$\lim_{n\to\infty} d\left(g\left(\omega,F\left(\omega,\left(y_n,x_n\right)\right)\right),F\left(\omega,\left(g\left(\omega,y_n\right),g\left(\omega,x_n\right)\right)\right)\right)=0$$

whenever $\{x_n\}$, $\{y_n\}$ are sequences in X, such that

$$\lim_{n\to\infty} F(\omega,(x_n,y_n))=\lim_{n\to\infty} g(\omega,x_n)=x$$

and

$$\lim_{n\to\infty} F(\omega,(y_n,x_n))=\lim_{n\to\infty} g\left(\omega,y_n\right)=y$$

for all $\omega\in\Omega$ and $x,y\in X$ are satisfied.

Theorem 3.3. *Let $(X,\preccurlyeq,d)$ be a complete separable partially ordered metric space, (Ω,Σ) be a measurable space and $F:\Omega\times(X\times X)\to X$ and $g:X\to X$ be mappings such that*

(i) *$g\left(\omega,\cdot\right)$ is continuous for all $\omega\in\Omega$;*
(ii) *$F\left(\cdot,v\right)$ and $g\left(\cdot,x\right)$ are measurable for all $v\in X\times X$ and $x\in X$ respectively;*
(iii) *$F\left(\omega,\cdot\right)$ has the mixed $g\left(\omega,\cdot\right)$-monotone property for each $\omega\in\Omega$ and there exists $\alpha\in[0,1)$ such that*

$$\begin{aligned} &d\left(F\left(\omega,\left(x,y\right)\right),F\left(\omega,\left(u,v\right)\right)\right)\\ &\le\alpha\max\left\{d\left(g\left(\omega,x\right),g\left(\omega,u\right)\right),d\left(g\left(\omega,y\right),g\left(\omega,v\right)\right),\right.\\ &\quad\frac{d(g(\omega,x),F(\omega,(x,y)))d(g(\omega,u),F(\omega,(u,v)))}{d(g(\omega,x),g(\omega,u))},\frac{d(g(\omega,x),F(\omega,(u,v)))d(g(\omega,u),F(\omega,(x,y)))}{d(g(\omega,x),g(\omega,u))},\\ &\quad\left.\frac{d(g(\omega,y),F(\omega,(y,x)))d(g(\omega,v),F(\omega,(v,u)))}{d(g(\omega,y),g(\omega,v))},\frac{d(g(\omega,y),F(\omega,(v,u)))d(g(\omega,v),F(\omega,(y,x)))}{d(g(\omega,y),g(\omega,v))}\right\} \end{aligned}\tag{3.1}$$

holds for all $x,y,u,v\in X$, $g\left(\omega,x\right)\neq g\left(\omega,u\right)$, $g\left(\omega,y\right)\neq g\left(\omega,v\right)$ with $g\left(\omega,x\right)\succcurlyeq g\left(\omega,u\right)$ and $g\left(\omega,y\right)\preccurlyeq g\left(\omega,v\right)$ for all $\omega\in\Omega$.

Further, suppose that F is continuous, $F(\omega, X\times X)\subseteq g(\omega, X)$, g is monotone increasing and commute with F. If there exist measurable mappings $\zeta_0, \eta_0 : \Omega \to X$ such that

$$g(\omega, \zeta_0(\omega)) \preccurlyeq F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$$

and

$$g(\omega, \eta_0(\omega)) \succcurlyeq F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

for all $\omega\in\Omega$, then there are measurable mappings $\zeta, \theta : \Omega \to X$ such that $F(\omega, (\zeta(\omega), \theta(\omega))) = g(\omega, \zeta(\omega))$ and $F(\omega, (\theta(\omega), \zeta(\omega))) = g(\omega, \theta(\omega))$ for all $\omega \in \Omega$, that is, F and g have a coupled random coincidence point.

Proof. Let $\Theta = \{\zeta : \Omega \to X\}$ be a family of measurable mappings. Define a function $h : \Omega \times X \to R^+$ as follows

$$h(\omega, x) = d(x, g(\omega, x)).$$

Since $x \to g(\omega, x)$ is continuous for all $\omega\in\Omega$, we conclude that $h(\omega,\cdot)$ is continuous for all $\omega\in\Omega$. Also, since $x \to g(\omega, x)$ is measurable for all $x \in X$, we conclude that $h(\cdot, x)$ is measurable for all $\omega\in\Omega$ (see Wagner [41], p. 868). Thus, $h(\omega, x)$ is the Caratheodory function. Therefore, if $\zeta : \Omega \to X$ is a measurable mapping, then $\omega \to h(\omega, \zeta(\omega))$ is also measurable(see [36]). Also, for each $\zeta \in \Theta$ the function $\eta : \Omega \to X$ defined by $\eta(\omega) = g(\omega, \zeta(\omega))$ is measurable, that is, $\eta \in \Theta$.

Now we shall construct two sequences of measurable mappings $\{\zeta_n\}$ and $\{\eta_n\}$ in Θ, and two sequences $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ in X as follows. Let $\zeta_0, \eta_0 \in \Theta$ such that $g(\omega, \zeta_0(\omega)) \preccurlyeq F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$ and $g(\omega, \eta_0(\omega)) \succcurlyeq F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$ for all $\omega \in \Omega$. Since

$$F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \in X = g(\omega \times X),$$

by a sort of Filippov measurable implicit function theorem [14, 17, 23, 32], there is $\zeta_1 \in \Theta$ such that $g(\omega, \zeta_1(\omega)) = F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$. Similarly as $F(\omega, (\eta_0(\omega), \zeta_0(\omega))) \in g(\omega \times X)$, there is $\eta_1(\omega) \in \Theta$ such that $g(\omega, \eta_1(\omega)) = F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$. Now $F(\omega, (\zeta_1(\omega), \eta_1(\omega)))$ and $F(\omega, (\eta_1(\omega), \zeta_1(\omega)))$ are well defined. Again from

$$F(\omega, (\zeta_1(\omega), \eta_1(\omega))), \quad F(\omega, (\eta_1(\omega), \zeta_1(\omega))) \in g(\omega \times X),$$

there are $\zeta_2, \eta_2 \in \Theta$ such that

$$g(\omega, \zeta_2(\omega)) = F(\omega, (\zeta_1(\omega), \eta_1(\omega)))$$

and

$$g(\omega, \eta_2(\omega)) = F(\omega, (\eta_1(\omega), \zeta_1(\omega))).$$

Continuing this process we can construct sequences $\{\zeta_n(\omega)\}$ and $\{\eta_n(\omega)\}$ in X such that

$$\begin{aligned} g(\omega,\zeta_{n+1}(\omega)) &= F(\omega,(\zeta_n(\omega),\eta_n(\omega))),\\ g(\omega,\eta_{n+1}(\omega)) &= F(\omega,(\eta_n(\omega),\zeta_n(\omega))) \end{aligned} \tag{3.2}$$

for all $n \geq 0$. We shall prove that

$$g(\omega,\zeta_n(\omega)) \preccurlyeq g(\omega,\zeta_{n+1}(\omega)) \quad \text{for all} \quad n \geq 0 \tag{3.3}$$

and

$$g(\omega,\eta_n(\omega)) \succcurlyeq g(\omega,\eta_{n+1}(\omega)) \quad \text{for all} \quad n \geq 0. \tag{3.4}$$

The proof will be given by the mathematical induction. Let $n = 0$. By assumption we have $g(\omega,\zeta_0(\omega)) \preccurlyeq F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_0(\omega)) \succcurlyeq F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$. Since

$$g(\omega,\zeta_1(\omega)) = F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$$

and

$$g(\omega,\eta_1(\omega)) = F(\omega,(\eta_0(\omega),\zeta_0(\omega))),$$

we have

$$g(\omega,\zeta_0(\omega)) \preccurlyeq g(\omega,\zeta_1(\omega)) \quad \text{and} \quad g(\omega,\eta_0(\omega)) \succcurlyeq g(\omega,\eta_1(\omega)).$$

Therefore (3.3) and (3.4) hold for $n = 0$. Suppose that (3.3) and (3.4) hold for some fixed $n \geq 0$. Then, since $g(\omega,\zeta_n(\omega)) \preccurlyeq g(\omega,\zeta_{n+1}(\omega))$, $g(\omega,\eta_n(\omega)) \succcurlyeq g(\omega,\eta_{n+1}(\omega))$ and F is monotone g-non-decreasing in its first argument, from (3.2), we get

$$\begin{aligned} F(\omega,(\zeta_n(\omega),\eta_n(\omega))) &\preccurlyeq F\left(\omega,\left(\zeta_{n_{+1}}(\omega),\eta_n(\omega)\right)\right)\\ F(\omega,(\eta_{n+1}(\omega),\zeta_n(\omega))) &\preccurlyeq F(\omega,(\eta_n(\omega),\zeta_n(\omega))). \end{aligned} \tag{3.5}$$

Similarly, from (3.2), as

$$\begin{aligned} g(\omega,\eta_{n+1}(\omega)) \preccurlyeq g(\omega,\eta_n(\omega)) \quad &and \quad g(\omega,\zeta_n(\omega)) \preccurlyeq g(\omega,\zeta_{n+1}(\omega)),\\ F(\omega,(\zeta_{n+1}(\omega),\eta_{n+1}(\omega))) &\succcurlyeq F\left(\omega,\left(\zeta_{n_{+1}}(\omega),\eta_n(\omega)\right)\right)\\ F(\omega,(\eta_{n+1}(\omega),\zeta_{n+1}(\omega))) &\succcurlyeq F(\omega,(\eta_{n+1}(\omega),\zeta_{n+1}(\omega))). \end{aligned} \tag{3.6}$$

Now from (3.2),(3.5) and (3.6), we get

$$g(\omega,\zeta_{n+1}(\omega)) \preccurlyeq g(\omega,\zeta_{n+2}(\omega)) \tag{3.7}$$

and

$$g(\omega,\eta_{n+1}(\omega)) \succcurlyeq g(\omega,\eta_{n+2}(\omega)). \tag{3.8}$$

Thus, by the mathematical induction we conclude that (3.3) and (3.4) hold for all $n \geq 0$. From (3.3) and (3.4), we have $g(\omega, \zeta_{n-1}(\omega)) \preccurlyeq g(\omega, \zeta_n(\omega))$ and $g(\omega, \eta_{n-1}(\omega)) \succcurlyeq g(\omega, \eta_n(\omega))$. Therefore, from (3.1) and (3.2), we have

$$\begin{aligned}
& d\left(g\left(\omega, \zeta_{n+1}(\omega)\right), g\left(\omega, \zeta_n(\omega)\right)\right) \\
& = d\left(F\left(\omega, \left(\zeta_n(\omega), \eta_n(\omega)\right)\right), F\left(\omega, \left(\zeta_{n-1}(\omega), \eta_{n-1}(\omega)\right)\right)\right) \\
& \leq \alpha \max \left\{ d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n-1}(\omega)\right)\right), d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n-1}(\omega)\right)\right), \right. \\
& \quad \frac{d(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))d(g(\omega,\zeta_{n-1}(\omega)),F(\omega,(\zeta_{n-1}(\omega),\eta_{n-1}(\omega))))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n-1}(\omega)))}, \\
& \quad \frac{d(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_{n-1}(\omega),\eta_{n-1}(\omega))))d(g(\omega,\zeta_{n-1}(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n-1}(\omega)))}, \\
& \quad \frac{d(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))d(g(\omega,\eta_{n-1}(\omega)),F(\omega,(\eta_{n-1}(\omega),\zeta_{n-1}(\omega))))}{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n-1}(\omega)))}, \\
& \quad \left. \frac{d(g(\omega,\eta_n(\omega)),F(\omega,(\eta_{n-1}(\omega),\zeta_{n-1}(\omega))))d(g(\omega,\eta_{n-1}(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))}{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n-1}(\omega)))} \right\}. \\
& = \alpha \max \left\{ d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n-1}(\omega)\right)\right), d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n-1}(\omega)\right)\right), \right. \\
& \quad \frac{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))d(g(\omega,\zeta_{n-1}(\omega)),g(\omega,\zeta_n(\omega)))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n-1}(\omega)))}, \\
& \quad \frac{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_n(\omega)))d(g(\omega,\zeta_{n-1}(\omega)),g(\omega,\zeta_{n+1}(\omega)))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n-1}(\omega)))}, \\
& \quad \frac{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))d(g(\omega,\eta_{n-1}(\omega)),g(\omega,\eta_n(\omega)))}{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n-1}(\omega)))}, \\
& \quad \left. \frac{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_n(\omega)))d(g(\omega,\eta_{n-1}(\omega)),g(\omega,\eta_{n+1}(\omega)))}{d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n-1}(\omega)))} \right\} \\
& = \alpha \max \left\{ d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n-1}(\omega)\right)\right), d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n-1}(\omega)\right)\right), \right. \\
& \quad \left. d\left(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))\right), d\left(g(\omega, \eta_n(\omega)), g(\omega, \eta_{n+1}(\omega))\right) \right\}.
\end{aligned}$$

Which implies that

$$\begin{aligned}
& d\left(g\left(\omega, \zeta_{n+1}(\omega)\right), g\left(\omega, \zeta_n(\omega)\right)\right) \\
& \leq \alpha \max\{d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n-1}(\omega)\right)\right), \\
& \qquad d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n-1}(\omega)\right)\right), d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n+1}(\omega)\right)\right), \\
& \qquad d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n+1}(\omega)\right)\right)\}.
\end{aligned} \tag{3.9}$$

Similarly, we can prove that

$$\begin{aligned}
& d\left(g\left(\omega, \eta_{n+1}(\omega)\right), g\left(\omega, \eta_n(\omega)\right)\right) \\
& \leq \alpha \max\{d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n-1}(\omega)\right)\right), \\
& \qquad d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n-1}(\omega)\right)\right), d\left(g\left(\omega, \zeta_n(\omega)\right), g\left(\omega, \zeta_{n+1}(\omega)\right)\right), \\
& \qquad d\left(g\left(\omega, \eta_n(\omega)\right), g\left(\omega, \eta_{n+1}(\omega)\right)\right)\}.
\end{aligned} \tag{3.10}$$

Set

$$\begin{aligned}
\rho_n = \max\{ & d\left(g\left(\omega, \zeta_{n+1}(\omega)\right), g\left(\omega, \zeta_n(\omega)\right)\right), \\
& d\left(g\left(\omega, \eta_{n+1}(\omega)\right), g\left(\omega, \eta_n(\omega)\right)\right)\}.
\end{aligned} \tag{3.11}$$

Hence

$$\begin{aligned}&\max\left\{d\left(g(\omega,\zeta_{n+1}\left(\omega\right)\right),g(\omega,\zeta_{n}\left(\omega\right))\right),d\left(g(\omega,\eta_{n+1}\left(\omega\right)\right),g(\omega,\eta_{n}\left(\omega\right)))\right\}\\ &\le\alpha\max\left\{d\left(g\left(\omega,\zeta_{n}\left(\omega\right)\right),g\left(\omega,\zeta_{n-1}\left(\omega\right)\right)\right),d\left(g\left(\omega,\eta_{n}\left(\omega\right)\right),g\left(\omega,\eta_{n-1}\left(\omega\right)\right)\right)\right\}\\ &=\alpha\rho_{n-1}.\end{aligned} \tag{3.12}$$

By induction, we have

$$\begin{aligned}&\max\left\{d\left(g\left(\omega,\zeta_{n+1}\left(\omega\right)\right),g\left(\omega,\zeta_{n}\left(\omega\right)\right)\right),d\left(g\left(\omega,\eta_{n+1}\left(\omega\right)\right),g\left(\omega,\eta_{n}\left(\omega\right)\right)\right)\right\}\\ &\le\alpha^{n}\rho_{0}.\end{aligned} \tag{3.13}$$

It easily follows that for each $m,n\in\mathbb{N},m<n$, we have

$$d\left(g\left(\omega,\zeta_{m}\left(\omega\right)\right),g\left(\omega,\zeta_{n}\left(\omega\right)\right)\right)\le\tfrac{\alpha^{m}}{1-\alpha}\rho_{0} \tag{3.14}$$

and

$$d\left(g\left(\omega,\eta_{m}\left(\omega\right)\right),g\left(\omega,\eta_{n}\left(\omega\right)\right)\right)\le\tfrac{\alpha^{m}}{1-\alpha}\rho_{0}. \tag{3.15}$$

Therefore $\{g\left(\omega,\zeta_{n}\left(\omega\right)\right)\}$ and $\{g\left(\omega,\eta_{n}\left(\omega\right)\right)\}$ are Cauchy sequences in X. Since X is complete and $g\left(\omega\times X\right)=X$, there exist $\zeta_{0},\theta_{0}\in\Theta$ such that

$$\lim_{n\to\infty}g\left(\omega,\zeta_{n}\left(\omega\right)\right)=g\left(\omega,\zeta_{0}\left(\omega\right)\right)$$

and

$$\lim_{n\to\infty}g\left(\omega,\eta_{n}\left(\omega\right)\right)=g\left(\omega,\theta_{0}\left(\omega\right)\right).$$

Since $g\left(\omega,\zeta_{0}\left(\omega\right)\right)$ and $g\left(\omega,\theta_{0}\left(\omega\right)\right)$ are measurable, the functions $\zeta\left(\omega\right)$ and $\theta\left(\omega\right)$, defined by $\zeta\left(\omega\right)=g\left(\omega,\zeta_{0}\left(\omega\right)\right)$ and $\theta\left(\omega\right)=g\left(\omega,\theta_{0}\left(\omega\right)\right)$ are measurable. Thus

$$\begin{aligned}&\lim_{n\to\infty}F\left(\omega,\left(\zeta_{n}\left(\omega\right),\eta_{n}\left(\omega\right)\right)\right)=\lim_{n\to\infty}g\left(\omega,\zeta_{n}\left(\omega\right)\right)=\zeta\left(\omega\right),\\ &\lim_{n\to\infty}F\left(\omega,\left(\eta_{n}\left(\omega\right),\zeta_{n}\left(\omega\right)\right)\right)=\lim_{n\to\infty}g\left(\omega,\eta_{n}\left(\omega\right)\right)=\theta\left(\omega\right).\end{aligned} \tag{3.16}$$

Since g is continuous, we have

$$\begin{aligned}&\lim_{n\to\infty}g\left(\omega,\left(g\left(\omega,\zeta_{n}\left(\omega\right)\right)\right)\right)=g\left(\omega,\left(g\left(\omega,\zeta_{0}\left(\omega\right)\right)\right)\right)=g\left(\omega,\zeta\left(\omega\right)\right),\\ &\lim_{n\to\infty}g\left(\omega,\left(g\left(\omega,\eta_{n}\left(\omega\right)\right)\right)\right)=g\left(\omega,\left(g\left(\omega,\theta_{0}\left(\omega\right)\right)\right)\right)=g\left(\omega,\theta\left(\omega\right)\right).\end{aligned} \tag{3.17}$$

From (3.2) and commutativity of F and g,

$$\begin{aligned}&g\left(\omega,\left(g\left(\omega,\zeta_{n+1}\left(\omega\right)\right)\right)\right)=g\left(\omega,\left(F\left(\omega,\left(\zeta_{n}\left(\omega\right),\eta_{n}\left(\omega\right)\right)\right)\right)\right)\\ &=F\left(\omega,\left(g\left(\omega,\zeta_{n}\left(\omega\right)\right),g\left(\omega,\eta_{n}\left(\omega\right)\right)\right)\right)\end{aligned} \tag{3.18}$$

and

$$\begin{aligned}&g\left(\omega,\left(g\left(\omega,\eta_{n+1}\left(\omega\right)\right)\right)\right)=g\left(\omega,\left(F\left(\omega,\left(\eta_{n}\left(\omega\right),\zeta_{n}\left(\omega\right)\right)\right)\right)\right)\\ &=F\left(\omega,\left(g\left(\omega,\eta_{n}\left(\omega\right)\right),g\left(\omega,\zeta_{n}\left(\omega\right)\right)\right)\right).\end{aligned} \tag{3.19}$$

Now, we show that

$$g\left(\omega,\zeta\left(\omega\right)\right)=F\left(\omega,\left(\zeta\left(\omega\right),\theta\left(\omega\right)\right)\right)$$

and

$$g\left(\omega,\theta\left(\omega\right)\right)=F\left(\omega,\left(\theta\left(\omega\right),\zeta\left(\omega\right)\right)\right).$$

Taking the limit as $n \to \infty$ in (3.18) and (3.19), by (3.16) and (3.17) and as F is continuous, we have

$$\begin{aligned} g(\omega, \zeta(\omega)) &= \lim_{n\to\infty} g(\omega, (g(\omega, \zeta_{n+1}(\omega)))) \\ &= \lim_{n\to\infty} F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega)))) \\ &= F\left(\omega, \lim_{n\to\infty} (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega)))\right) \\ &= F(\omega, (\zeta(\omega), \theta(\omega))) \end{aligned}$$

and

$$\begin{aligned} g(\omega, \theta(\omega)) &= \lim_{n\to\infty} g(\omega, (g(\omega, \eta_{n+1}(\omega)))) \\ &= \lim_{n\to\infty} F(\omega, (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega)))) \\ &= F\left(\omega, \lim_{n\to\infty} (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega)))\right) \\ &= F(\omega, (\theta(\omega), \zeta(\omega))). \end{aligned}$$

Thus, we proved that F and g have a coupled random coincidence point. □

Theorem 3.4. *In addition to hypotheses of Theorem* 3.3*, if* $g(\omega, \zeta_0(\omega))$ *and* $g(\omega, \eta_0(\omega))$ *are compareable. Then* F *and* g *have a coupled random coincidence point, that is, for measurable mappings* $\zeta, \theta : \Omega \to X$*, there exist* $\zeta(\omega), \theta(\omega)$ *for all* $\omega \in \Omega$ *such that either*

$$g(\omega, \zeta(\omega)) = F(\omega, (\zeta(\omega), \theta(\omega))) \quad \text{or} \quad g(\omega, \theta(\omega)) = F(\omega, (\theta(\omega), \zeta(\omega)))$$

or

$$g(\omega, \zeta(\omega)) = F(\omega, (\zeta(\omega), \theta(\omega))) = F(\omega, (\theta(\omega), \zeta(\omega))) = g(\omega, \theta(\omega)).$$

Proof. As in Theorem 3.3, we can construct two sequences $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ in X such that $g(\omega, \zeta_n(\omega)) \to g(\omega, \zeta(\omega))$ and $g(\omega, \eta_n(\omega)) \to g(\omega, \theta(\omega))$, where $(\zeta(\omega), \theta(\omega))$ is a random coincidence point of F and g. Suppose $g(\omega, \zeta_0(\omega)) \preccurlyeq g(\omega, \eta_0(\omega))$. We shall show that

$$g(\omega, \zeta_n(\omega)) \preccurlyeq g(\omega, \eta_n(\omega)),$$

where

$$\begin{aligned} g(\omega, \zeta_n(\omega)) &= F(\omega, (\zeta_{n-1}(\omega), \eta_{n-1}(\omega))), \\ g(\omega, \eta_n(\omega)) &= F(\omega, (\eta_{n-1}(\omega), \zeta_{n-1}(\omega))), \end{aligned}$$

for all n. Suppose that it holds for some $n \geq 0$. Then by mixed g-monotone property of F, we have

$$\begin{aligned} &g(\omega, \zeta_{n+1}(\omega)) = F(\omega, (\zeta_n(\omega), \eta_n(\omega))) \\ &\preccurlyeq F(\omega, (\eta_n(\omega), \zeta_n(\omega))) = g(\omega, \eta_{n+1}(\omega)). \end{aligned}$$

Now from (3.1), we have

$$\begin{aligned} & d\left(g\left(\omega,\zeta_{n+1}\left(\omega\right)\right),g\left(\omega,\eta_{n+1}\left(\omega\right)\right)\right) \\ & = d\left(F\left(\omega,\left(\zeta_n\left(\omega\right),\eta_n\left(\omega\right)\right)\right),F\left(\omega,\left(\eta_n\left(\omega\right),\zeta_n\left(\omega\right)\right)\right)\right) \\ & \leq \alpha \max\Bigg\{ d\left(g\left(\omega,\zeta_n\left(\omega\right)\right),g\left(\omega,\eta_n\left(\omega\right)\right)\right), d\left(g\left(\omega,\eta_n\left(\omega\right)\right),g\left(\omega,\zeta_n\left(\omega\right)\right)\right), \\ & \quad \frac{d(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))d(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))}, \\ & \quad \frac{d(g(\omega,\zeta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))d(g(\omega,\eta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))}{d(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))}, \\ & \quad \frac{d(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))d(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))}{d(g(\omega,\eta_n(\omega)),g(\omega,\zeta_n(\omega)))}, \\ & \quad \frac{d(g(\omega,\eta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))d(g(\omega,\zeta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))}{d(g(\omega,\eta_n(\omega)),g(\omega,\zeta_n(\omega)))} \Bigg\}. \end{aligned}$$

On taking limt as $n \to \infty$, we obtain

$$d\left(g\left(\omega,\zeta\left(\omega\right)\right),g\left(\omega,\theta\left(\omega\right)\right)\right) \leq \alpha d\left(g\left(\omega,\zeta\left(\omega\right)\right),g\left(\omega,\theta\left(\omega\right)\right)\right).$$

Since $\alpha < 1$, we have $d\left(g\left(\omega,\zeta\left(\omega\right)\right),g\left(\omega,\theta\left(\omega\right)\right)\right) = 0$. Hence

$$F\left(\omega,\left(\zeta\left(\omega\right),\theta\left(\omega\right)\right)\right) = g\left(\omega,\zeta\left(\omega\right)\right) = g\left(\omega,\theta\left(\omega\right)\right) = F\left(\omega,\left(\theta\left(\omega\right),\zeta\left(\omega\right)\right)\right).$$

A similar argument can be used if $g\left(\omega,\eta_0\left(\omega\right)\right) \preccurlyeq g\left(\omega,\zeta_0\left(\omega\right)\right)$. □

If $F : \Omega \times (X \times X) \to X$ and $g : X \to X$ are compatible random operators, we have the following theorem.

Theorem 3.5. *Let $(X, \preccurlyeq, d)$ be a complete separable partially ordered metric space, (Ω, Σ) be a measurable space and $F : \Omega \times (X \times X) \to X$ and $g : X \to X$ be mapping such that*

(i) *$g(\omega, \cdot)$ is continuous for all $\omega \in \Omega$;*
(ii) *$F(\cdot, v)$ and $g(\cdot, x)$ are measurable for all $v \in X \times X$ and $x \in X$ respectivly;*
(iii) *$F(\omega, \cdot)$ has the mixed $g(\omega, \cdot)$-monotone property for each $\omega \in \Omega$ and there exist $\alpha \in [0, 1)$ such that*

$$\begin{aligned} & d\left(F\left(\omega,(x,y)\right),F\left(\omega,(u,v)\right)\right) \\ & \leq \alpha \max\{d\left(g\left(\omega,x\right),g\left(\omega,u\right)\right), d\left(g\left(\omega,y\right),g\left(\omega,v\right)\right), \\ & \frac{d(g(\omega,x),F(\omega,(x,y)))d(g(\omega,u),F(\omega,(u,v)))}{d(g(\omega,x),g(\omega,u))}, \frac{d(g(\omega,x),F(\omega,(u,v)))d(g(\omega,u),F(\omega,(x,y)))}{d(g(\omega,x),g(\omega,u))}, \\ & \frac{d(g(\omega,y),F(\omega,(y,x)))d(g(\omega,v),F(\omega,(v,u)))}{d(g(\omega,y),g(\omega,v))}, \frac{d(g(\omega,y),F(\omega,(v,u)))d(g(\omega,v),F(\omega,(y,x)))}{d(g(\omega,y),g(\omega,v))}\} \end{aligned} \tag{3.20}$$

holds for all $x, y, u, v \in X$, $g(\omega, x) \neq g(\omega, u)$, $g(\omega, y) \neq g(\omega, v)$ with $g(\omega, x) \succcurlyeq g(\omega, u)$ and $g(\omega, y) \preccurlyeq g(\omega, v)$ for all $\omega \in \Omega$.

Further, suppose that F is continuous, $F(\omega, X \times X) \subseteq g(\omega, X)$, g is monotone increasing and F and g are compatible random operators. If there exist *measurable mappings* $\zeta_0, \eta_0 : \Omega \to X$ *such that*

$$g(\omega, \zeta_0(\omega)) \preccurlyeq F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$$

and

$$g(\omega, \eta_0(\omega)) \succcurlyeq F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

for all $\omega \in \Omega$, *then there are measurable mappings* $\zeta, \theta : \Omega \to X$ *such that* $F(\omega, (\zeta(\omega), \theta(\omega))) = g(\omega, \zeta(\omega))$ *and* $F(\omega, (\theta(\omega), \zeta(\omega))) = g(\omega, \theta(\omega))$ *for all* $\omega \in \Omega$*, that is,* F *and* g *have a coupled random coincidence point.*

Proof. We can construct two sequences $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ in X and proved by the same arguments of Theorem 3.3 that $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ are Cauchy sequences in X. Since X is complete and $g(\omega \times X) = X$, there exist $\zeta_0, \theta_0 \in \Theta$ such that $\lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = g(\omega, \zeta_0(\omega))$ and $\lim_{n\to\infty} g(\omega, \eta_n(\omega)) = g(\omega, \theta_0(\omega))$. Since $g(\omega, \zeta_0(\omega))$ and $g(\omega, \theta_0(\omega))$ are measurable, then the functions $\zeta(\omega)$ and $\theta(\omega)$, defined by $\zeta(\omega) = g(\omega, \zeta_0(\omega))$ and $\theta(\omega) = g(\omega, \theta_0(\omega))$ are measurable. Thus

$$\begin{aligned} \lim_{n\to\infty} F(\omega, (\zeta_n(\omega), \eta_n(\omega))) = \lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = \zeta(\omega), \\ \lim_{n\to\infty} F(\omega, (\eta_n(\omega), \zeta_n(\omega))) = \lim_{n\to\infty} g(\omega, \eta_n(\omega)) = \theta(\omega). \end{aligned} \tag{3.21}$$

Since F and g are compatible mappings, by (3.21), we have

$$\lim_{n\to\infty} d(g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega)))), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))) = 0, \tag{3.22}$$

$$\lim_{n\to\infty} d(g(\omega, F(\omega, (\eta_n(\omega), \zeta_n(\omega)))), F(\omega, (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega))))) = 0. \tag{3.23}$$

As F is continuous, we have

$$\begin{aligned} & d(g(\omega, \zeta(\omega)), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))) \\ & \leq d(g(\omega, \zeta(\omega)), g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega))))) \\ & \quad + d(g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega)))), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))). \end{aligned}$$

Taking the limit as $n \to \infty$, using (3.1), (3.21) and (3.22) and the fact that F and g are continuous, we have

$$d(g(\omega, \zeta(\omega)), F(\omega, (\zeta(\omega), \theta(\omega)))) = 0.$$

Similarly, from (3.1), (3.21) and (3.23) and the continuity of F and g, we have

$$d(g(\omega, \theta(\omega)), F(\omega, (\theta(\omega), \zeta(\omega)))) = 0.$$

□

Theorem 3.6. *Let* $(X, \leq, d)$ *be complete separable partially ordered metric space,* (Ω, Σ) *be a measurable space and* $F : \Omega \times (X \times X) \to X$ *and* $g : X \to X$ *be mapping such that*

(i) $F(\omega, \cdot)$ *and* $g(\omega, \cdot)$ *are continuous for all* $\omega \in \Omega$;

(ii) $F(\cdot,v)$ *and* $g(\cdot,x)$ *are measurable for all* $v\in X\times X$ *and* $x\in X$ *respectivly;*

(iii) *for all* $x,y,u,v\in X$*, if* $g(\omega,x)\lesseqqgtr F(\omega,(x,y))=g(\omega,u)$*, then*

$$F(\omega,(x,y))\lesseqqgtr F(\omega,(u,v)).$$

If there exist $\alpha\in[0,1)$ *such that*

$$\begin{aligned}&d\left(F\left(\omega,(x,y)\right),F\left(\omega,(u,v)\right)\right)\\&\leq\alpha\max\{d\left(g\left(\omega,x\right),g\left(\omega,u\right)\right),d\left(g\left(\omega,y\right),g\left(\omega,v\right)\right),\\&\tfrac{d(g(\omega,x),F(\omega,(x,y)))d(g(\omega,u),F(\omega,(u,v)))}{d(g(\omega,x),g(\omega,u))},\tfrac{d(g(\omega,x),F(\omega,(u,v)))d(g(\omega,u),F(\omega,(x,y)))}{d(g(\omega,x),g(\omega,u))},\\&\tfrac{d(g(\omega,y),F(\omega,(y,x)))d(g(\omega,v),F(\omega,(v,u)))}{d(g(\omega,y),g(\omega,v))},\tfrac{d(g(\omega,y),F(\omega,(v,u)))d(g(\omega,v),F(\omega,(y,x)))}{d(g(\omega,y),g(\omega,v))}\}\end{aligned}\tag{3.24}$$

such that for all $x,y,u,v\in X$ *with* $g(\omega,x)\lesseqqgtr g(\omega,u)$ *and* $g(\omega,y)\lesseqqgtr g(\omega,v)$ *for all* $\omega\in\Omega$*. Suppose* $g(\omega\times X)=X$ *for each* $\omega\in\Omega$ *and* F *and* g *are compatible random operators. If there exist measurable mappings* $\zeta_0,\eta_0:\Omega\to X$ *such that* $g(\omega,\zeta_0(\omega))\lesseqqgtr F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ *and* $g(\omega,\eta_0(\omega))\lesseqqgtr F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ *for all* $\omega\in\Omega$*, then there are measurable mappings* $\zeta,\theta:\Omega\to X$ *such that* $F(\omega,(\zeta(\omega),\theta(\omega)))=g(\omega,\zeta(\omega))$ *and* $F(\omega,(\theta(\omega),\zeta(\omega)))=g(\omega,\theta(\omega))$ *for all* $\omega\in\Omega$*, that is,* F *and* g *have a coupled random coincidence point.*

Proof. Let $\Theta=\{\zeta:\Omega\to X\}$ be a family of measurable mappings. Define a function $h:\Omega\times X\to R^+$ as follows

$$h(\omega,x)=d(x,g(\omega,x)).$$

Since $x\to g(\omega,x)$ is continuous for all $\omega\in\Omega$, we conclude that $h(\omega,\cdot)$ is continuous for all $\omega\in\Omega$. Also, since $x\to g(\omega,x)$ is measurable for all $x\in X$, we conclude that $h(\cdot,x)$ is measurable for all $\omega\in\Omega$(see Wagner [41], p. 868). Thus, $h(\omega,x)$ is the Caratheodory function. Therefore, if $\zeta:\Omega\to X$ is a measurable mapping, then $\omega\to h(\omega,\zeta(\omega))$ is also measurable (see [36]). Also, for each $\zeta\in\Theta$ the function $\eta:\Omega\to X$ defined by $\eta(\omega)=g(\omega,\zeta(\omega))$ is measurable, that is, $\eta\in\Theta$.

Now we shall construct two sequences of measurable mappings $\{\zeta_n\}$ and $\{\eta_n\}$ in Θ, and two sequences $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ in X as follows. Let $\zeta_0,\eta_0\in\Theta$ such that $g(\omega,\zeta_0(\omega))\lesseqqgtr F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_0(\omega))\lesseqqgtr F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ for all $\omega\in\Omega$. Since $F(\omega,(\zeta_0(\omega),\eta_0(\omega)))\in X=g(\omega\times X)$, by a sort of Filippov measurable implicit function theorem [14, 17, 23, 32], there is $\zeta_1\in\Theta$ such that $g(\omega,\zeta_1(\omega))=F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$. Similarly as $F(\omega,(\eta_0(\omega),\zeta_0(\omega)))\in g(\omega\times X)$, there is $\eta_1(\omega)\in\Theta$ such that $g(\omega,\eta_1(\omega))=F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$. Now $F(\omega,(\zeta_1(\omega),\eta_1(\omega)))$ and $F(\omega,(\eta_1(\omega),\zeta_1(\omega)))$ are well defined. Again from $F(\omega,(\zeta_1(\omega),\eta_1(\omega)))$,

$F(\omega,(\eta_1(\omega),\zeta_1(\omega))) \in g(\omega \times X)$, there are $\zeta_2, \eta_2 \in \Theta$ such that $g(\omega,\zeta_2(\omega)) = F(\omega,(\zeta_1(\omega),\eta_1(\omega)))$ and $g(\omega,\eta_2(\omega)) = F(\omega,(\eta_1(\omega),\zeta_1(\omega)))$. Continuing this process we can construct sequences $\{\zeta_n(\omega)\}$ and $\{\eta_n(\omega)\}$ in X such that

$$\begin{aligned} g(\omega,\zeta_{n+1}(\omega)) &= F(\omega,(\zeta_n(\omega),\eta_n(\omega))), \\ g(\omega,\eta_{n+1}(\omega)) &= F(\omega,(\eta_n(\omega),\zeta_n(\omega))), \end{aligned} \tag{3.25}$$

for all $n \geq 0$.

Now, we shall prove that

$$g(\omega,\zeta_n(\omega)) \lesseqqgtr g(\omega,\zeta_{n+1}(\omega)) \quad \text{for all } n \geq 0 \tag{3.26}$$

and

$$g(\omega,\eta_n(\omega)) \lesseqqgtr g(\omega,\eta_{n+1}(\omega)) \quad \text{for all } n \geq 0. \tag{3.27}$$

The proof will be given by the mathematical induction. Let $n = 0$. By assumption we have

$$g(\omega,\zeta_0(\omega)) \lesseqqgtr F(\omega,(\zeta_0(\omega),\eta_0(\omega))) = g(\omega,\zeta_1(\omega))$$

and

$$g(\omega,\eta_0(\omega)) \lesseqqgtr F(\omega,(\eta_0(\omega),\zeta_0(\omega))) = g(\omega,\eta_1(\omega)).$$

Therefore (3.26) and (3.27) hold for $n = 0$. Since

$$g(\omega,\zeta_0(\omega)) \lesseqqgtr F(\omega,(\zeta_0(\omega),\eta_0(\omega))) = g(\omega,\zeta_1(\omega))$$

and condition (iii) implies that

$$g(\omega,\zeta_1(\omega)) = F(\omega,(\zeta_0(\omega),\eta_0(\omega))) \lesseqqgtr F(\omega,(\zeta_1(\omega),\eta_1(\omega))) = g(\omega,\zeta_2(\omega)).$$

Similarly, if

$$g(\omega,\eta_0(\omega)) \lesseqqgtr F(\omega,(\eta_0(\omega),\zeta_0(\omega))) = g(\omega,\eta_1(\omega)),$$

then condition (iii) implies that

$$g(\omega,\eta_1(\omega)) = F(\omega,(\eta_0(\omega),\zeta_0(\omega))) \lesseqqgtr F(\omega,(\eta_1(\omega),\zeta_1(\omega))) = g(\omega,\eta_2(\omega)).$$

Thus, we have

$$g(\omega,\zeta_1(\omega)) \lesseqqgtr g(\omega,\zeta_2(\omega))$$

and

$$g(\omega,\eta_1(\omega)) \lesseqqgtr g(\omega,\eta_2(\omega)).$$

Proceeding by induction, we get

$$g(\omega,\zeta_n(\omega)) \lesseqqgtr g(\omega,\zeta_{n+1}(\omega)) \tag{3.28}$$

and

$$g(\omega, \eta_n(\omega)) \lesseqqgtr g(\omega, \eta_{n+1}(\omega)). \tag{3.29}$$

Using (3.24), (3.25) and the same arguments of Theorem 3.3, we can prove that $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ are Cauchy sequences in X. Since X is complete and $g(\omega \times X) = X$, there exist $\zeta_0, \theta_0 \in \Theta$ such that

$$\lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = g(\omega, \zeta_0(\omega))$$

and

$$\lim_{n\to\infty} g(\omega, \eta_n(\omega)) = g(\omega, \theta_0(\omega)).$$

Since $g(\omega, \zeta_0(\omega))$ and $g(\omega, \theta_0(\omega))$ are measurable, then the functions $\zeta(\omega)$ and $\theta(\omega)$, defined by $\zeta(\omega) = g(\omega, \zeta_0(\omega))$ and $\theta(\omega) = g(\omega, \theta_0(\omega))$ are measurable. Thus

$$\begin{aligned} &\lim_{n\to\infty} F(\omega, (\zeta_n(\omega), \eta_n(\omega))) = \lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = \zeta(\omega), \\ &\lim_{n\to\infty} F(\omega, (\eta_n(\omega), \zeta_n(\omega))) = \lim_{n\to\infty} g(\omega, \eta_n(\omega)) = \theta(\omega). \end{aligned} \tag{3.30}$$

Since F and g are compatible mappings, we have by (3.30)

$$\lim_{n\to\infty} d(g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega)))), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))) = 0, \tag{3.31}$$

$$\lim_{n\to\infty} d(g(\omega, F(\omega, (\eta_n(\omega), \zeta_n(\omega)))), F(\omega, (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega))))) = 0. \tag{3.32}$$

As F is continuous, we have

$$\begin{aligned} &d(g(\omega, \zeta(\omega)), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))) \\ &\le d(g(\omega, \zeta(\omega)), g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega))))) \\ &\quad + d(g(\omega, F(\omega, (\zeta_n(\omega), \eta_n(\omega)))), F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))))). \end{aligned}$$

Taking the limit as $n \to \infty$, using (3.25), (3.30) and (3.31) and the fact that F and g are continuous, we have

$$d(g(\omega, \zeta(\omega)), F(\omega, (\zeta(\omega), \theta(\omega)))) = 0.$$

Similarly, from (3.25), (3.30) and (3.32) and the continuity of F and g, we have

$$d(g(\omega, \theta(\omega)), F(\omega, (\theta(\omega), \zeta(\omega)))) = 0.$$

Thus, we have F and g have a coupled random coincidence point. □

If g is the identity mapping in above theorem we have the following result.

Corollary 3.7. *Let $(X, \leq, d)$ be complete separable partially ordered metric space, (Ω, Σ) be a measurable space and $F : \Omega \times (X \times X) \to X$ be mapping such that*

(i) *$F(\omega, \cdot)$ is continuous for all $\omega \in \Omega$;*
(ii) *$F(\cdot, v)$ is measurable for all $v \in X \times X$ and $x \in X$ respectivly;*
(iii) *for all $x, y, u, v \in X$, if $x \lesseqqgtr F(\omega, (x, y)) = u$, then*

$$F(\omega, (x, y)) \lesseqqgtr F(\omega, (u, v)).$$

If there exist $\alpha \in [0, 1)$ such that

$$\begin{aligned} &d(F(\omega, (x, y)), F(\omega, (u, v))) \\ &\leq \alpha \max \Big\{ d(x, u), d(y, v), \tfrac{d(x,F(\omega,(x,y)))d(u,F(\omega,(u,v)))}{d(x,u)}, \\ &\qquad \tfrac{d(x,F(\omega,(u,v)))d(u,F(\omega,(x,y)))}{d(x,u)}, \tfrac{d(x,F(\omega,(x,y)))d(u,F(\omega,(u,v)))}{d((\omega,x),(\omega,u))}, \\ &\qquad \tfrac{d(x,F(\omega,(x,y)))d(u,F(\omega,(u,v)))}{d((\omega,x),(\omega,u))}, \tfrac{d(y,F(\omega,(v,u)))d(v,F(\omega,(y,x)))}{d(y,v)} \Big\} \end{aligned}$$

for all $x, y, u, v \in X$ with $x \lesseqqgtr u$ and $y \lesseqqgtr v$. If there exist measurable mappings $\zeta_0, \eta_0 : \Omega \to X$ such that

$$\zeta_0(\omega) \lesseqqgtr F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \quad \text{and} \quad \eta_0(\omega) \lesseqqgtr F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

for all $\omega \in \Omega$, then there are measurable mappings $\zeta, \theta : \Omega \to X$ such that

$$F(\omega, (\zeta(\omega), \eta(\omega))) = g(\omega, \zeta(\omega))$$

and

$$F(\omega, (\eta(\omega), \zeta(\omega))) = g(\omega, \eta(\omega))$$

for all $\omega \in \Omega$, that is, F has a coupled random fixed point.

4. Random common fixed point in Hilbert space using rational inequality

We define the random I scheme in an analogous manner as follows: Let $T_1, T_2 : \Omega \times C \to C$ be two operators on a nonempty convex subset C of a separable Hilbert space X. Then the sequence $\{x_n\}$ of random I-scheme associated with T_1 and T_2 is defined as follows:

$$\text{Let } x_0 : \Omega \to C \text{ by any given measurable mapping.} \tag{4.1}$$

$$\begin{aligned} y_{2n}(t) &= \beta_{2n} T_1(t, x_{2n}) + (1 - \beta_{2n}) x_{2n}(t), \\ x_{2n+1}(t) &= (1 - \alpha_{2n}) x_{2n}(t) + \alpha_{2n} T_2(t, y_{2n}), \end{aligned} \tag{4.2}$$

$$\begin{aligned} y_{2n+1}(t) &= \beta_{2n+1} T_1(t, x_{2n+1}) + (1 - \beta_{2n+1}) x_{2n+1}(t), \\ x_{2n+2}(t) &= (1 - \alpha_{2n+1}) x_{2n+1}(t) + \alpha_{2n+1} T_2(t, y_{2n+1}), \end{aligned} \tag{4.3}$$

for $n \geq 0$, $t \in \Omega$, where $\{\alpha_{2n}\}$, $\{\beta_{2n}\}$ satisfying the following conditions

(i) $0 \leq \alpha_{2n} \leq \beta_{2n} \leq 1$, for all n,
(ii) $\lim_{n\to\infty} \alpha_{2n} = \alpha_{2n} > 0$, and
(iii) $\lim_{n\to\infty} \beta_{2n} = \beta_{2n} < 1$.

We know that Banach space is Hilbert if and only if its norm satisfies the parallelogram law i.e., for every $x, y \in X$(:Hilbert space),

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

which implies

$$\|x+y\|^2 \leq 2\|x\|^2 + 2\|y\|^2 .$$

We often use this inequality through the result.

Motivated by [33], we prove the following random fixed point theorem.

Theorem 4.1. *Let C be nonempty, closed and convex subset of a seperable Hilbert space X. Let $T_1, T_2 : \Omega \times C \to C$ be two random operators defined on C satisfying the contractive condition*

$$\begin{aligned}
&\|T_1(\omega,x) - T_2(\omega,y)\| \\
&\leq K \max\Bigg\{ \|y - T_2(\omega,y)\|^2, \tfrac{1}{4}\left(\|x - T_2(\omega,y)\|^2 + \|y - T_1(\omega,x)\|^2\right), \\
&\quad \tfrac{1}{2}\left(\|x - T_1(\omega,x)\|^2 + \|y - T_2(\omega,y)\|^2\right), \tfrac{\|y-T_2(\omega,y)\|^2[1+\|x-T_1(\omega,x)\|^2]}{1+\|x-y\|^2}, \\
&\quad \tfrac{\|x-T_1(\omega,x)\|^2[1+\|x-y\|^2]}{1+\|y-T_2(\omega,y)\|^2}, \tfrac{\|x-y\|^2[1+\|x-T_1(\omega,x)\|^2]}{1+\|y-T_2(\omega,y)\|^2}, \tfrac{\|x-T_1(\omega,x)\|^2[1+\|y-T_2(\omega,y)\|^2]}{1+\|x-y\|^2}, \\
&\quad \tfrac{\left(1+\|y-T_2(\omega,y)\|^2\right)[1+\|x-T_1(\omega,x)\|^2]}{1+\|x-y\|^2} \Bigg\}
\end{aligned} \tag{4.4}$$

where, $\omega \in \Omega$ and $0 \leq K < \frac{1}{4}$. If there exist a point ζ_0 such that the random I-scheme for point of T_1 and T_2 defined by (4.2) *and* (4.3), *converges to a point λ, then $\lambda(\omega)$ is a common random fixed point of T_1 and T_2.*

Proof. We may assume that the sequence $\{x_n\}$ defind by (4.2) is a pointwise convergent, that is, for all $\omega \in \Omega$,

$$\lim_{n\to\infty} x_n(\omega) = \lambda(\omega).$$

Since X is seperable Hilberet space, for any random operator $A : \Omega \to X$ and measurable mapping $f : \Omega \to C$, the mapping $x(\omega) = A(\omega, f(\omega))$, is measuable mapping [17]. Since $x(\omega)$ is measurable and C is convex, then the sequence $\{x_n\}$ constructed in the random iteration form (4.2), (4.3) is a sequence of measurable mappings. Hence $x : \Omega \to C$ being limit of measurable mapping sequence is also measurable. Now, let $\{x_n\}$ be a sequence of I-scheme associated with T_2 such that $\lim_{n\to\infty} x_n(\omega) = u(\omega)$. From (4.2),

$$x_{2n+1}(\omega) - x_{2n}(\omega) = \alpha_{2n}\left(T_2(\omega, y_{2n}) - x_{2n}(\omega)\right).$$

Since $x_{2n}(\omega) \to \lambda(\omega)$,

$$\|x_{2n+1}(\omega) - x_{2n}(\omega)\| \to 0$$

and $\{\alpha_{2n}\}$ is bounded away from zero, $\|T_2(\omega, y_{2n}) - x_{2n}(\omega)\| \to 0$. It follows that $\|\lambda(\omega) - T_2(\omega, y_{2n})\| \to 0$. Since T_1 and T_2 satisfies (4.4), we have

$$\begin{aligned}
&\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega))\| \\
&\leq K \max\left\{ \|y_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))\|^2, \right. \\
&\quad \tfrac{1}{4}\left(\|x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))\|^2 + \|y_{2n}(\omega) - T_1(\omega, x_{2n}(\omega))\|^2\right), \\
&\quad \tfrac{1}{2}\left(\|x_{2n}(\omega) - T_1(\omega, x_{2n}(\omega))\|^2 + \|y_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))\|^2\right), \\
&\quad \tfrac{\|y_{2n}(\omega)-T_2(\omega,y_{2n}(\omega))\|^2[1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2]}{1+\|x_{2n}(\omega)-y_{2n}(\omega)\|^2}, \\
&\quad \tfrac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2[1+\|x_{2n}(\omega)-y_{2n}(\omega)\|^2]}{1+\|y_{2n}(\omega)-T_2(\omega,y_{2n}(\omega)\|^2}, \\
&\quad \tfrac{\|x_{2n}(\omega)-y_{2n}(\omega)\|^2[1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2]}{1+\|y_{2n}(\omega)-T_2(\omega,y_{2n}(\omega))\|^2}, \\
&\quad \tfrac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2[1+\|y_{2n}(\omega)-T_2(\omega,y_{2n}(\omega))\|^2]}{1+\|x_{2n}(\omega)-y_{2n}(\omega)\|^2}, \\
&\quad \left.\tfrac{\left(1+\|y_{2n}(\omega)-T_2(\omega,y_{2n}(\omega))\|^2\right)[1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2]}{1+\|x_{2n}(\omega)-y_{2n}(\omega)\|^2}\right\}.
\end{aligned} \tag{4.5}$$

Now consider,

$$\begin{aligned}
&\|y_{2n}(\omega) - x_{2n}(\omega)\|^2 \\
&= \|\beta_{2n}T_1(\omega, x_{2n}(\omega)) + (1-\beta_{2n})x_{2n}(\omega) - x_{2n}(\omega)\|^2 \\
&= \|\beta_{2n}T_1(\omega, x_{2n}(\omega)) + x_{2n}(\omega) - \beta_{2n}x_{2n}(\omega) - x_{2n}(\omega)\|^2 \\
&= \|\beta_{2n}(T_1(\omega, x_{2n}(\omega)) - x_{2n}(\omega))\|^2 \\
&= \beta_{2n}^2 \|(T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega))) \\
&\quad + (T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega))\|^2 \\
&\leq 2\beta_{2n}^2 \|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega))\|^2 \\
&\quad + 2\beta_{2n}^2 \|T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega)\|^2 \\
&\leq 2\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega))\|^2 \\
&\quad + 2\|T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega)\|^2,
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
&\|y_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))\|^2 \\
&= \|\beta_{2n}T_1(\omega, x_{2n}(\omega)) + (1-\beta_{2n})x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))\|^2 \\
&= \left\| \begin{array}{l} \beta_{2n}T_1(\omega, x_{2n}(\omega)) + (1-\beta_{2n})x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega)) \\ +\beta_{2n}T_2(\omega, y_{2n}(\omega)) - \beta_{2n}T_2(\omega, y_{2n}(\omega)) \end{array} \right\|^2
\end{aligned}$$

$$
\begin{aligned}
&= \left\| \begin{array}{l} \beta_{2n}(T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega))) \\ +(1-\beta_{2n})(x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega)) \end{array} \right\|^2 \\
&\leq 2\beta_{2n}^2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&\quad +2(1-\beta_{2n}) \left\| x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&\leq 2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&\quad +2 \left\| x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega)) \right\|^2
\end{aligned} \tag{4.7}
$$

and

$$
\begin{aligned}
&\left\| y_{2n}(\omega) - T_1(\omega, x_{2n}(\omega)) \right\|^2 \\
&= \left\| \beta_{2n} T_1(\omega, x_{2n}(\omega)) + (1-\beta_{2n}) x_{2n}(\omega) - T_1(\omega, y_{2n}(\omega)) \right\|^2 \\
&= \left\| \beta_{2n} T_1(\omega, x_{2n}(\omega)) + x_{2n}(\omega) - \beta_{2n} x_{2n}(\omega) - T_1(\omega, y_{2n}(\omega)) \right\|^2 \\
&= \left\| (1-\beta_{2n})(x_{2n}(\omega)) - T_1 x_{2n}(\omega)) \right\|^2 \\
&= (1-\beta_{2n})^2 \left\| x_{2n}(\omega) - T_1 x_{2n}(\omega) \right\|^2 \\
&= (1-\beta_{2n})^2 \left\| (x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega))) \right. \\
&\quad \left. + (T_2(\omega, y_{2n}(\omega)) - T_1 x_{2n}(\omega)) \right\|^2 \\
&\leq 2(1-\beta_{2n})^2 \left\| (x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&\quad +2(1-\beta_{2n})^2 \left\| T_2(\omega, y_{2n}(\omega)) - T_1 x_{2n}(\omega) \right\|^2 \\
&\leq 2 \left\| x_{2n}(\omega) - T_2(\omega, y_{2n}(\omega)) \right\|^2 + 2 \left\| T_2(\omega, y_{2n}(\omega)) - T_1 x_{2n}(\omega) \right\|^2 .
\end{aligned} \tag{4.8}
$$

Using (4.6), (4.7) and (4.8) in (4.5), we have

$$
\begin{aligned}
&\left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&\leq K \max \Bigg\{ \left(2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 + 2 \left\| T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega) \right\|^2 \right), \\
&\frac{1}{4} \left(2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 + 3 \left\| T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega) \right\|^2 \right), \\
&\frac{1}{2} \Bigg(\left\| x_{2n}(\omega) - T_1(\omega, x_{2n}(\omega)) \right\|^2 + 2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 \\
&+ 2 \left\| T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega) \right\|^2 \Bigg), \\
&\frac{(2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|x_{2n}(\omega)-T_2(\omega,y_{2n}(\omega))\|^2)(1+\|T_1(\omega,x_{2n}(\omega))-x_{2n}(\omega)\|^2)}{\left(1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)}, \\
&\frac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2[1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2]}{\left(1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)}, \\
&\frac{\left(2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)[1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2]}{\left(1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)}, \\
&\frac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2[1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2]}{\left(1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)}, \\
&\frac{\left(1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2\right)\left(1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2\right)}{[1+2\|T_1(\omega,x_{2n}(\omega))-T_2(\omega,y_{2n}(\omega))\|^2+2\|T_2(\omega,y_{2n}(\omega))-x_{2n}(\omega)\|^2]} \Bigg\} \\
&\leq K \left(2 \left\| T_1(\omega, x_{2n}(\omega)) - T_2(\omega, y_{2n}(\omega)) \right\|^2 + 2 \left\| T_2(\omega, y_{2n}(\omega)) - x_{2n}(\omega) \right\|^2 \right).
\end{aligned}
$$

Taking limit as $n \to \infty$, we get

$$\left\| T_1\left(\omega, x_{2n}\left(\omega\right)\right) - T_2\left(\omega, y_{2n}\left(\omega\right)\right) \right\|^2 \to 0. \tag{4.9}$$

It follows that

$$\begin{aligned}
&\left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \\
&\leq 2\left\| x_{2n}\left(\omega\right) - T_2\left(\omega, y_{2n}\left(\omega\right)\right) \right\|^2 + 2\left\| T_2\left(\omega, y_{2n}\left(\omega\right)\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \\
&\to 0
\end{aligned}$$

and

$$\begin{aligned}
&\left\| \lambda\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \\
&\leq 2\left\| \lambda\left(\omega\right) - x_{2n}\left(\omega\right) \right\|^2 + 2\left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \\
&\to 0 \quad \text{as} \quad n \to \infty.
\end{aligned} \tag{4.10}$$

If $x_{2n}\left(\omega\right), \lambda\left(\omega\right)$ satisfies (4.4), we have

$$\begin{aligned}
&\left\| T_1\left(\omega, x_{2n}\left(\omega\right)\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2 \\
&\leq K \max\Bigg\{ \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2, \\
&\qquad \frac{1}{4}\left(\left\| x_{2n}\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2 + \left\| \lambda\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \right), \\
&\qquad \frac{1}{2}\left(\left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 + \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2 \right), \\
&\qquad \frac{\left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2 \left[1 + \left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2\right]}{1 + \left\| x_{2n}\left(\omega\right) - \lambda\left(\omega\right) \right\|^2}, \\
&\qquad \frac{\left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \left[1 + \left\| x_{2n}\left(\omega\right) - \lambda\left(\omega\right) \right\|^2\right]}{1 + \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2}, \\
&\qquad \frac{\left\| x_{2n}\left(\omega\right) - \lambda\left(\omega\right) \right\|^2 \left[1 + \left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2\right]}{1 + \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2}, \\
&\qquad \frac{\left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2 \left[1 + \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2\right]}{1 + \left\| x_{2n}\left(\omega\right) - \lambda\left(\omega\right) \right\|^2}, \\
&\qquad \frac{\left(1 + \left\| \lambda\left(\omega\right) - T_2\left(\omega, \lambda\left(\omega\right)\right) \right\|^2\right) \left[1 + \left\| x_{2n}\left(\omega\right) - T_1\left(\omega, x_{2n}\left(\omega\right)\right) \right\|^2\right]}{1 + \left\| x_{2n}\left(\omega\right) - \lambda\left(\omega\right) \right\|^2} \Bigg\}.
\end{aligned}$$

Using parallelograme, we have

$$
\begin{aligned}
&\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, \lambda(\omega))\|^2 \\
&\leq K \max \Bigg\{ \left(2\|\lambda(\omega) - x_{2n}(\omega)\|^2 + 2\|x_{2n}(\omega) - T_2\lambda(\omega)\|^2\right), \\
&\quad \tfrac{1}{4}(2\|x_{2n}(\omega) - T_1(\omega, x_{2n}(\omega))\|^2 + 2\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, \lambda(\omega))\|^2) \\
&\quad + \|\lambda(\omega) - T_1(\omega, x_{2n}(\omega))\|^2, \tfrac{1}{2}(2\|x_{2n}(\omega) - T_1(\omega, x_{2n}(\omega))\|^2 \\
&\quad +2\|\lambda(\omega) - x_{2n}(\omega)\|^2 + 2\|T_2(\omega, \lambda(\omega)) - x_{2n}(\omega)\|^2), \\
&\quad \tfrac{\left(2\|\lambda(\omega)-x_{2n}(\omega)\|^2+2\|T_2(\omega,\lambda(\omega))-x_{2n}(\omega)\|^2\right)\left(1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2\right)}{\left(1+\|\lambda(\omega)-x_{2n}(\omega)\|^2\right)}, \\
&\quad \tfrac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2\left(1+\|x_{2n}(\omega)-\lambda(\omega)\|^2\right)}{1+2\|\lambda(\omega)-x_{2n}(\omega)\|^2+2\|T_2(\omega,\lambda(\omega))-x_{2n}(\omega)\|^2}, \\
&\quad \tfrac{\|x_{2n}(\omega)-\lambda(\omega)\|^2\left(1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2\right)}{1+2\|\lambda(\omega)-x_{2n}(\omega)\|^2+2\|T_2(\omega,\lambda(\omega))-x_{2n}(\omega)\|^2}, \\
&\quad \tfrac{\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2(1+2\|\lambda(\omega)-x_{2n}(\omega)\|^2+2\|x_{2n}(\omega)-T_2(\omega,\lambda(\omega))\|^2]}{1+\|x_{2n}(\omega)-\lambda(\omega)\|^2}, \\
&\quad \tfrac{[1+2\|\lambda(\omega)-x_{2n}(\omega)\|^2+2\|T_2(\omega,\lambda(\omega))-x_{2n}(\omega)\|^2\left(1+\|x_{2n}(\omega)-T_1(\omega,x_{2n}(\omega))\|^2\right)]}{1+\|x_{2n}(\omega)-\lambda(\omega)\|^2} \Bigg\}.
\end{aligned}
$$

Taking the limit as $n \to \infty$, we get $\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, \lambda(\omega))\| \to 0$. Finally, we have

$$
\begin{aligned}
&\|\lambda(\omega) - T_2(\omega, \lambda(\omega))\|^2 \\
&= \|\lambda(\omega) - T_1(\omega, x_{2n}(\omega)) + T_1(\omega, x_{2n}(\omega)) - T_2(\omega, \lambda(v))\|^2 \\
&\leq 2\|\lambda(\omega) - T_1(\omega, x_{2n}(\omega))\|^2 + 2\|T_1(\omega, x_{2n}(\omega)) - T_2(\omega, \lambda(\omega))\|^2 \\
&\to 0 \quad \text{as} \quad n \to \infty.
\end{aligned}
$$

This showing that

$$\lambda(\omega) = T_2(\omega, \lambda(\omega)).$$

Similarly, we can prove that

$$\lambda(\omega) = T_1(\omega, \lambda(\omega)).$$

Thus, $\lambda(\omega)$ is a common random fixed point of T_1 and T_2. If $T_1 = T_2$ in obtain theorem, we obtain the following result. □

Corollary 4.2. *Let C be nonempty, closed and convex subset of a seperable Hilbert space X. Let $T : \Omega \times C \to C$ be random operator defined on C satisfying* *the contractive condition*

$$\begin{aligned}
&\|T(\omega,x)-T(\omega,y)\| \\
&\leq K\max\Bigg\{\|y-T(\omega,y)\|^2,\frac{1}{4}\left(\|x-T(\omega,y)\|^2+\|y-T(\omega,x)\|^2\right), \\
&\frac{1}{2}\left(\|x-T(\omega,x)\|^2+\|y-T(\omega,y)\|^2\right),\frac{\|y-T(\omega,y)\|^2\left[1+\|x-T(\omega,x)\|^2\right]}{1+\|x-y\|^2}, \\
&\frac{\|x-T(\omega,x)\|^2\left[1+\|x-y\|^2\right]}{1+\|y-T(\omega,y)\|^2},\frac{\|x-y\|^2\left[1+\|x-T(\omega,x)\|^2\right]}{1+\|y-T(\omega,y)\|^2}, \\
&\frac{\|x-T(\omega,x)\|^2\left[1+\|y-T(\omega,y)\|^2\right]}{1+\|x-y\|^2}, \\
&\frac{\left(1+\|y-T(\omega,y)\|^2\right)\left[1+\|x-T(\omega,x)\|^2\right]}{1+\|x-y\|^2}\Bigg\}
\end{aligned}$$

where, $\omega \in \Omega$ *and* $0 \leq K < \frac{1}{4}$*. If there exist a point* ζ_0 *such that the random I-scheme for point of* T *defined by*

$$y_n(t) = \beta_n T(t, x_n) + (1-\beta_n)\, x_n(t),$$
$$x_{n+1}(t) = (1-\alpha_n)\, x_n(t) + \alpha_n T(t, y_{2n}), \quad n \geq 0$$

converges to a point λ*, then* $\lambda(\omega)$ *is a common random fixed point of* T*.*

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