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ALMOST STABILITY OF THE ISHIKAWA ITERATION METHOD WITH ERROR TERMS INVOLVING STRICTLY HEMICONTRACTIVE MAPPINGS IN SMOOTH BANACH SPACES

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Abstract. Let K be a nonempty closed bounded convex subset of an arbitrary smooth Banach space X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Under some conditions we obtain that the Ishikawa iteration method with error terms converges strongly to a unique fixed point of T and is almost T-stable on K.

1. Introduction

Chidume [4] established that the Mann iteration sequence converges strongly to the unique fixed point of T in case T is a Lipschitz strongly pseudo-contractive mapping from a bounded closed convex subset of L_p $(or\ l_p)$ into itself. Schu [18] generalized the result in [4] to both uniformly continuous strongly pseudo-contractive mappings and real smooth Banach spaces. Park

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[16] extended the result in [4] to both strongly pseudocontractive mappings and certain smooth Banach spaces. Rhoades [17] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear mappings. Harder and Hicks [7-8] revealed the importance of investigating the stability of various iteration procedures for various classes of nonlinear mappings. Harder [6] established applications of stability results to first order differential equations. Afterwards, several generalizations have been made in various directions (see for example [2-3, 5, 9-15, 19]).

Let K be a nonempty closed bounded convex subset of an arbitrary smooth Banach space X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Under some conditions we obtain that the Ishikawa iteration method with error terms converges strongly to a unique fixed point of T and is almost T-stable on K. The results presented here generalize the corresponding results in [5, 9, 14, 16, 19,20].

2. Preliminaries

Let K be a nonempty subset of an arbitrary Banach space E and E^* be its dual space. The symbols $D(T)$, $R(T)$ and $F(T)$ stand for the domain, the range and the set of fixed points of T (for a single-valued map $T : X \to X$, $x \in X$ is called a fixed point of T iff $T(x) = x$).We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}.$$

Let T be a self-mapping of K.

Definition 2.1. The mapping T is called *Lipshitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \leqslant L \|x - y\|,$$

for all $x, y \in K$. If $L = 1$, then T is called *non-expansive* and if $0 \leqslant L < 1$, T is called *contraction.*

Definition 2.2. ([5, 20])

(1) The mapping T is said to be *pseudocontractive* if the inequality

$$\|x - y\| \leqslant \| x - y + t((I - T)x - (I - T)y \|, \tag{2.1}$$

holds for each $x, y \in K$ and for all $t > 0$.

(2) T is said to be *strongly pseudocontractive* if there exists a $t > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rt(Tx - Ty)\| \tag{2.2}$$

for all $x, y \in D(T)$ and $r > 0$.

(3) T is said to be *local strongly pseudocontractive* if for each $x \in D(T)$ there exists a $t_x > 1$ such that

$$\|x - y\| \le \|(1 + r)(x - y) - rt_x(Tx - Ty)\| \tag{2.3}$$

for all $y \in D(T)$ and $r > 0$.

(4) T is said to be *strictly hemicontractive* if $F(T) \ne \varphi$ and if there exists a $t > 1$ such that

$$\|x - q\| \le \|(1 + r)(x - q) - rt(Tx - q)\| \tag{2.4}$$

for all $x \in D(T)$, $q \in F(T)$ and $r > 0$.

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive.

Definition 2.3. ([6-8]) Let K be a nonempty convex subset of X and $T : K \to K$ be an operator. Assume that $x_o \in K$ and $x_{n+1} = f(T, x_n)$ defines an iteration scheme which produces a sequence $\{x_n\}_{n=0}^{\infty} \subset K$. Suppose, furthermore, that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $q \in F(T) \ne \varphi$. Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in K and put $\varepsilon_n = \|y_{n+1} - f(T, y_n)\|$.

(1) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be T-stable on K if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$,

(2) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be almost T-stable on K if $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$.

It is easy to verify that an iteration scheme $\{x_n\}_{n=0}^{\infty}$ which is T-stable on K is almost T-stable on K.

Lemma 2.4. ([16]) *Let X be a smooth Banach space. Suppose one of the following holds:*

(1) *J is uniformly continuous on any bounded subsets of X,*

(2) *$\langle x - y, j(x) - j(y)\rangle \le \|x - y\|^2$, for all x, y in X,*

(3) *for any bounded subset D of X, there is a $c : [0, \infty) \to [0, \infty)$ such that $\mathrm{Re}\,\langle x - y, j(x) - j(y)\rangle \le c(\|x - y\|)$, for all $x, y \in D$, where c satisfies*

$$\lim_{t\to 0^+} \frac{c(t)}{t} = 0. \tag{2.5}$$

Then for any $\epsilon > 0$ and any bounded subset K, there exists $\delta > 0$ such that

$$\|sx + (1 - s)y\|^2 \le (1 - 2s)\|y\|^2 + 2s\mathrm{Re}\,\langle x,\ j(y)\rangle + 2s\epsilon \tag{2.6}$$

for all $x, y \in K$ and $s \in [0, \delta]$.

Lemma 2.5. ([5]) *Let $T : D(T) \subseteq X \to X$ be an operator with $F(T) \neq \varphi$. Then T is strictly hemicontractive if and only if there exists $t > 1$ such that for all $x \in D(T)$ and $q \in F(T)$, there exists $j(x-q) \in J(x-q)$ satisfying*

$$\operatorname{Re}\langle x - Tx,\ j(x-q)\rangle \geq \left(1 - \frac{1}{t}\right)\|x-q\|^2. \tag{2.7}$$

Lemma 2.6. ([14]) *Let X be an arbitrary normed linear space and $T : D(T) \subseteq X \to X$ be an operator.*

(1) *If T is a local strongly pseudocontractive operator and $F(T) \neq \varphi$, then $F(T)$ is a singleton and T is strictly hemicontractive.*
(2) *If T is strictly hemicontractive, then $F(T)$ is a singleton.*

Lemma 2.7. ([14]) *Let $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ be nonnegative real sequences and let $\epsilon' > 0$ be a constant satisfying*

$$\beta_{n+1} \leq (1-\alpha_n)\beta_n + \epsilon'\alpha_n + \gamma_n,\ n \geq 0,$$

where $\sum\limits_{n=0}^{\infty} \alpha_n = \infty$, $\alpha_n \leq 1$ for all $n \geq 0$ and $\sum\limits_{n=0}^{\infty} \gamma_n < \infty$. Then, $\limsup\limits_{n\to\infty} \beta_n \leq \epsilon'$.

Remark 2.8. If $\gamma_n = 0$ for each $n \geq 0$, then Lemma 2.7 reduces to Lemma 1 of Park [16].

3. Main results

We now prove our main results.

Theorem 3.1. *Let X be a smooth Banach space satisfying the axioms (1)-(3) of Lemma 2.4. Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Suppose that $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are arbitrary sequences in K and $\{a'_n\}_{n=0}^{\infty}, \{b'_n\}_{n=0}^{\infty}, \{c'_n\}_{n=0}^{\infty}, \{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying conditions (i) $a'_n + b'_n + c'_n = 1 = a_n + b_n + c_n$, (ii) $c'_n = o(b'_n)$, (iii) $\lim\limits_{n\to\infty} b'_n = 0 = \lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} c_n$, and (iv) $\sum_{n=0}^{\infty} b'_n = \infty$.*

Suppose that $\{x_n\}_{n=0}^{\infty}$ is the sequence generated from an arbitrary $x_0 \in K$ by

$$\begin{aligned} x_{n+1} &= a'_n x_n + b'_n T y_n + c'_n u_n, \\ y_n &= a_n x_n + b_n T x_n + c_n v_n,\ n \geq 0. \end{aligned} \tag{3.1}$$

Let $\{z_n\}_{n=0}^{\infty}$ be any sequence in K and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\varepsilon_n = \|z_{n+1} - p_n\|,\ n \geq 0, \tag{3.2}$$

where

$$\begin{aligned} p_n &= a'_n z_n + b'_n T w_n + c'_n u_n, \\ w_n &= a_n z_n + b_n T z_n + c_n v_n, \ n \geq 0. \end{aligned} \tag{3.3}$$

Then

(a) *the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to a unique fixed point* q *of* T,

(b) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} z_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* T*-stable on* K,

(c) $\lim_{n\to\infty} z_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. From (ii), we have

$$c'_n = t_n b'_n, \text{ where } t_n \to 0 \text{ as } n \to \infty.$$

It follows from Lemma 2.6 that $F(T)$ is a singleton. That is, $F(T) = \{q\}$ for some $q \in K$.

Set $M = 1 + diamK$. For all $n \geq 0$ it is easy to verify that

$$\begin{aligned} M = &\sup_{n\geq 0} \|x_n - q\| + \sup_{n\geq 0} \|Tx_n - q\| + \sup_{n\geq 0} \|Ty_n - q\| + \sup_{n\geq 0} \|u_n - q\| \\ &+ \sup_{n\geq 0} \|v_n - q\| + \sup_{n\geq 0} \|z_n - q\| + \sup_{n\geq 0} \|p_n - q\|. \end{aligned} \tag{3.4}$$

Consider

$$\begin{aligned} \|y_n - x_n\| &= \|a_n x_n + b_n T x_n + c_n v_n - x_n\| \\ &= \|b_n (T x_n - x_n) + c_n (v_n - x_n)\| \\ &\leq b_n \|T x_n - x_n\| + c_n \|v_n - x_n\| \\ &\leq 2M (b_n + c_n) \\ &\to 0, \end{aligned} \tag{3.5}$$

as n$\to \infty$.

For given any $\epsilon > 0$ and the bounded subset K, there exists a $\delta > 0$ satisfying (2.6). Note that (ii), (iii), $\lim_{n\to\infty} \|y_n - x_n\| = 0$ and the continuity of T ensure that there exists an N such that

$$b'_n < \min\{\delta, \frac{1}{2(1-k)}\}, \ t_n \leq \frac{\epsilon}{16M^2}, \ \ \|Ty_n - Tx_n\| \leq \frac{\epsilon}{4M}, \ n \geq N, \tag{3.6}$$

where $k = \frac{1}{t}$ and t satisfies (2.7). Using (3.3) and Lemma 2.4, we infer that

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \left\|(1-b'_n)(x_n - q) + b'_n(Ty_n - q) + c'_n(u_n - x_n)\right\|^2 \\
&\le \left(\left\|(1-b'_n)(x_n - q) + b'_n(Ty_n - q)\right\| + 2Mc'_n\right)^2 \\
&\le \left\|(1-b'_n)(x_n - q) + b'_n(Ty_n - q)\right\|^2 + 8M^2c'_n \\
&\le (1-2b'_n)\|x_n - q\|^2 + 2b'_n\mathrm{Re}(Ty_n - q, j(x_n - q)) \\
&\qquad + 2\epsilon b'_n + 8M^2c'_n \\
&= (1-2b'_n)\|x_n - q\|^2 + 2b'_n\mathrm{Re}(Tx_n - q, j(x_n - q)) \\
&\qquad + 2b'_n\mathrm{Re}(Ty_n - Tx_n, j(x_n - q)) + 2\epsilon b'_n + 8M^2c'_n \\
&\le (1-2b'_n)\|x_n - q\|^2 + 2kb'_n\|x_n - q\|^2 \\
&\qquad + 2b'_n\|Ty_n - Tx_n\|\,\|x_n - q\| + 2\epsilon b'_n + 8M^2c'_n \\
&\le (1-2(1-k)b'_n)\|x_n - q\|^2 \\
&\qquad + 2Mb'_n\|Ty_n - Tx_n\| + 2\epsilon b'_n + 8M^2c'_n \\
&\le (1-2(1-k)b'_n)\|x_n - q\|^2 + 3\epsilon b'_n,
\end{aligned} \tag{3.7}$$

for all $n \ge N$.

Put

$$\begin{aligned}
\beta_n &= \|x_n - q\|, \\
\alpha_n &= 2(1-k)b'_n, \\
\epsilon' &= \frac{3\epsilon}{2(1-k)}, \\
\gamma_n &= 0,
\end{aligned}$$

we have from (3.7)

$$\beta_{n+1} \le (1-\alpha_n)\beta_n + \epsilon'\alpha_n + \gamma_n, \ n \ge 0.$$

Observe that $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\alpha_n < 1$ for all $n \ge 0$. It follows from Lemma 2.7 that

$$\lim_{n\to\infty} \sup \|x_n - q\|^2 \le \epsilon'.$$

Letting $\epsilon' \to 0^+$, we obtain that $\lim_{n\to\infty} \sup \|x_n - q\|^2 = 0$, which implies that $x_n \to q$ as $n \to \infty$.

Similarly we also have

$$
\begin{aligned}
\|p_n - q\|^2 &= \left\|(1-b'_n)(z_n - q) + b'_n(Tw_n - q) + c'_n(u_n - z_n)\right\|^2 \\
&\leq \left(\left\|(1-b'_n)(z_n - q) + b'_n(Tw_n - q)\right\| + 2Mc'_n\right)^2 \\
&\leq \left\|(1-b'_n)(z_n - q) + b'_n(Tw_n - q)\right\|^2 + 8M^2 c'_n \\
&\leq (1-2b'_n)\|z_n - q\|^2 + 2b'_n \mathrm{Re}(Tw_n - q, j(z_n - q)) \\
&\quad + 2\epsilon b'_n + 8M^2 c'_n \\
&= (1-2b'_n)\|z_n - q\|^2 + 2b'_n \mathrm{Re}(Tz_n - q, j(z_n - q)) \\
&\quad + 2b'_n \mathrm{Re}(Tw_n - Tz_n, j(z_n - q)) + 2\epsilon b'_n + 8M^2 c'_n \\
&\leq (1-2b'_n)\|z_n - q\|^2 + 2kb'_n\|z_n - q\|^2 \\
&\quad + 2b'_n\|Tw_n - Tz_n\|\|z_n - q\| + 2\epsilon b'_n + 8M^2 c'_n \\
&\leq (1-2(1-k)b'_n)\|z_n - q\|^2 \\
&\quad + 2Mb'_n\|Tw_n - Tz_n\| + 2\epsilon b'_n + 8M^2 c'_n \\
&\leq (1-2(1-k)b'_n)\|z_n - q\|^2 + 3\epsilon b'_n,
\end{aligned}
\tag{3.8}
$$

for all $n \geq N$.

Suppose that $\sum_{n=0}^{\infty} \varepsilon_n < \infty$. In view of (3.4) and (3.8), we infer that

$$
\begin{aligned}
\|z_{n+1} - q\|^2 &\leq (\|z_{n+1} - p_n\| + \|p_n - q\|)^2 \\
&\leq \|p_n - q\|^2 + 3M\varepsilon_n \\
&\leq \left[1 - 2b'_n(1-k)\right]\|z_n - q\|^2 + 3\epsilon b'_n + 3M\varepsilon_n,
\end{aligned}
\tag{3.9}
$$

for all $n \geq N$.

Now put

$$
\begin{aligned}
\beta_n &= \|z_n - q\|, \\
\alpha_n &= 2(1-k)b'_n, \\
\epsilon' &= \frac{3\epsilon}{2(1-k)}, \\
\gamma_n &= 3M\varepsilon_n,
\end{aligned}
$$

and we have from (3.9)

$$
\beta_{n+1} \leq (1-\alpha_n)\beta_n + \epsilon'\alpha_n + \gamma_n, \ n \geq 0.
$$

Observe that $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\alpha_n < 1$ and $\sum_{n=0}^{\infty} \gamma_n < \infty$ for all $n \geq 0$. It follows from Lemma 2.7 that

$$
\lim_{n\to\infty} \sup \|z_n - q\|^2 \leq \epsilon'.
$$

Letting $\epsilon' \to 0^+$, we obtain that $\lim_{n\to\infty} \sup \|z_n - q\|^2 = 0$, which implies that $z_n \to q$ as $n \to \infty$.

Conversely, suppose that $\lim_{n\to\infty} z_n = q$, then (iii) and (3.8) implies that

$$\begin{aligned} \varepsilon_n &\leq \|z_{n+1} - q\| + \|p_n - q\| \\ &\leq \|z_{n+1} - q\| + \left[\left[1 - 2(1-k)b'_n\right] \|z_n - q\|^2 + 3\epsilon b'_n\right]^{\frac{1}{2}} \\ &\to 0, \end{aligned}$$

as $n \to \infty$, that is, $\varepsilon_n \to 0$ as $n \to \infty$. □

Using the method of Theorem 3.1, we can similarly prove the following.

Theorem 3.2. *Let* $X, K, T, \{u_n\}_{n=0}^{\infty}$ *and* $\{v_n\}_{n=0}^{\infty}$ *be as in Theorem* 3.1. *Suppose that* $\{a'_n\}_{n=0}^{\infty}, \{b'_n\}_{n=0}^{\infty}, \{c'_n\}_{n=0}^{\infty}, \{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ *and* $\{c_n\}_{n=0}^{\infty}$ *are any sequences in* $[0,1]$ *satisfying conditions* (i), $(iii) - (iv)$ *and*

$$\sum_{n=0}^{\infty} c'_n < \infty.$$

If $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ *and* $\{\varepsilon_n\}_{n=0}^{\infty}$ *are as in Theorem* 3.1*, Then the conclusions of Theorem* 3.1 *hold.*

Corollary 3.3. *Let X be a smooth Banach space satisfying the axioms (1)-(3) of Lemma* 2.4. *Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Suppose that* $\{u_n\}_{n=0}^{\infty}$ *is an arbitrary sequence in K and* $\{a'_n\}_{n=0}^{\infty}, \{b'_n\}_{n=0}^{\infty}$ *and* $\{c'_n\}_{n=0}^{\infty}$ *are any sequences in* $[0,1]$ *satisfying conditions* (i) $a'_n + b'_n + c'_n = 1$, (ii) $c'_n = o(b'_n)$, (iii) $\lim_{n\to\infty} b'_n = 0$, *and* (iv) $\sum_{n=0}^{\infty} b'_n = \infty$.

Suppose that $\{x_n\}_{n=0}^{\infty}$ *is the sequence generated from an arbitrary* $x_0 \in K$ *by*

$$x_{n+1} = a'_n x_n + b'_n T x_n + c'_n u_n, \ n \geq 0.$$

Let $\{z_n\}_{n=0}^{\infty}$ *be any sequence in K and define* $\{\varepsilon_n\}_{n=0}^{\infty}$ *by*

$$\varepsilon_n = \|z_{n+1} - p_n\|, \ n \geq 0,$$

where

$$p_n = a'_n z_n + b'_n T z_n + c'_n u_n, \ n \geq 0.$$

Then

(a) *the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to a unique fixed point q of T,*

(b) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} z_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost T-stable on K,*

(c) $\lim_{n\to\infty} z_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Corollary 3.4. *Let* X, K, T *and* $\{u_n\}_{n=0}^{\infty}$ *be as in Corollary* 3.3. *Suppose that* $\{a'_n\}_{n=0}^{\infty}, \{b'_n\}_{n=0}^{\infty}$ *and* $\{c'_n\}_{n=0}^{\infty}$ *are any sequences in* $[0,1]$ *satisfying conditions* (i), $(iii)-(iv)$ *and*

$$\sum_{n=0}^{\infty} c'_n < \infty.$$

If $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$ *and* $\{\varepsilon_n\}_{n=0}^{\infty}$ *are as in Corollary* 3.3, *then the conclusions of Corollary* 3.3 *hold.*

Remark 3.5. All of the above results are also valid for Lipschitz strictly hemicontractive mappings.

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