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THE EXISTENCE OF DETERMINISTIC RANDOM GENERALIZED VECTOR EQUILIBRIUM PROBLEMS

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Abstract. This paper is concerned with the existence of random solutions for the deterministic generalized vector equilibrium problems. As applications, we also discussed the random generalized vector optimization problems, random generalized variational inequality problems, random generalized vector best approximation problems and random fixed point problems.

1. Introduction

The vector equilibrium problems provide a very general model for a wide range of problems, for example, the vector optimization problems, the vector variational inequality problems, the vector complementarity problems, vector saddle point problems and vector best approximation problems (see [3, 4, 6, 9, 12, 13, 15, 19]). Many interesting and sophisticated problems in applied mathematics, engineering sciences and technology, economics and decision sciences can be cast into the form of vector equilibrium problems, as in the fields of optimizations, mathematical economics, financial sciences, ecology, genetics engineering, metrology, medical sciences, bio-technology and networks. However many of these applications contain some random or uncertain

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environments, such as random game theory, random mathematical economics, random financial mathematics and related areas. To cover these problems, it is quite natural to study equilibrium problems that includes the probabilistic features, which will permit to save a greater generality from initial formulation (see [1, 2, 14, 20, 21]).

In this paper, inspired and motivated by recent works [8, 10, 11, 16, 17, 18, 20, 22, 23, 24, 25, 26], we discussed the existence of random solutions for the deterministic generalized vector equilibrium problems and as applications to studied random generalized vector optimization problems, random generalized vector variational inequality problems, random generalized vector best approximation problems and random fixed point problems.

Let C be a nonempty subset of the Hausdorff topological vector space E and let Y be a Hausdorff topological vector space. Assume that P is a nonempty closed convex pointed partially ordered cone in Y with apes at the origin and $intP \neq \emptyset$, where $intP$ denotes the topological interior of P.

Let $G : C \to 2^C$ be a set-valued mapping and $f : C \times C \to Y$ be a vector-valued mapping. Then the generalized vector equilibrium problem is to find an element $x \in C$ such that $x \in G(x)$ and

$$f(x, y) \notin -intP, \quad \forall\, y \in G(x).$$

Note that if, $Y = R$ and $P = [0, +\infty[$, then the generalized vector equilibrium problem reduces to generalized equilibrium problem for finding an element $x \in C$ such that $x \in G(x)$ and

$$f(x, y) \geq 0, \quad \forall\, y \in G(x).$$

Inspired by the deterministic case, we define a random generalized vector equilibrium problem (RGVEP). Let (Ω, Σ) be a measurable space, where Σ is a σ-algebra of the subsets of Ω. Let $G : \Omega \times C \to 2^C$ be a random set-valued mapping and $f : \Omega \times C \times C \to E$ be a random vector-valued function.

The random generalized vector equilibrium problem (RGVEP) is to find a function $\gamma : \Omega \to C$ such that

$$f(t, \gamma(t), y) \notin -intP, \quad \forall\, t \in \Omega, \;\; y \in G(x, t). \tag{1.1}$$

For each $t \in \Omega, \gamma(t)$ is called a deterministic solution of (1.1). The function γ will be a random solution whenever it is measurable. Suppose that (1.1) has a deterministic solution for each $t \in \Omega$. Then under suitable conditions, the multi-function S of equilibria sets of f defined by

$$S(t) = \bigcap_{x \in C} \{x \in G(t, x), y \in G(t, x) : f(t, x, y) \notin -intP\}, \;\; t \in \Omega \tag{1.2}$$

has a measurable selection. The problem (1.1) has at least one random solution.

Remark 1.1. Every deterministic solution of (1.1) is approached by a countable family of random solutions.

Special cases:

(a) **Random generalized vector optimization problem:**

Let $\phi : \Omega \times C \to E$ be a random vector-valued function and let $G : \Omega \times C \to 2^C$ be a random set-valued mapping. Then we find a measurable function $\gamma : \Omega \times C \to C$ such that

$$\phi(t, \gamma(t)) - \phi(t, y) \notin -intP, \ \ \forall\ (t, x) \in \Omega \times C, \ y \in G(t, x). \tag{1.3}$$

We know that if for all $x \in G(t, x), y \in G(t, x)$,

$$f(t, x, y) = \phi(t, y) - \phi(t, x), \ \ \forall\ (t, x, y) \in \Omega \times C \times C,$$

then (1.1) and (1.2) are equivalent to each others.

(b) **Random generalized vector variational inequality problem:**

We get measurable function $\gamma : \Omega \to C$ which is a solution to the following random generalized vector variational inequality problem for finding $x \in C, x \in G(t, x)$ such that

$$\langle A(t, \gamma(t)), y - \gamma(t)\rangle \notin -intP, \ \ \forall\ t \in \Omega, \ y \in G(t, x). \tag{1.4}$$

Note that the random solutions to the (1.1) and (1.4) are coincident for

$$f(t, x, y) = \langle A(t, x), y - x\rangle, \ \ \forall\ t \in \Omega, \ x \in G(t, x), \ y \in G(t, x).$$

(c) **Random generalized vector best approximations and random fixed point problem:**

Let C be a nonempty subset of a normed vector space X and let P be a convex cone in R^m. Then for the mappings $g_k : \Omega \times C \to X, 1 \leq k \leq m$, we find the measurable function $\gamma : \Omega \to C$ such that for all $t \in \Omega, y \in G(t, x)$ and

$$(\|y - g_k(t, \gamma(t))\| - \|\gamma(t) - g_k(t, \gamma(t))\|)_{1\leq k\leq m} \notin -intP. \tag{1.5}$$

Note that if for all $t \in \Omega, x \in G(t, x), y \in G(t, x)$,

$$g_k(t, x) = a_k(t),$$

where $a_k : \Omega \to X$ for all $k \in \{1, 2, \cdots, m\}$, then (1.5) is a random version of the vector approximation problem studied in [23]. If

$m = 1, P = R^+$ and $g = g_1$, then (1.5) collapses to a random version of the standard best approximation problem (see [3]).

(d) **Random generalized best approximation problem:**

The random generalized best approximation problem is to find the measurable function $\gamma : \Omega \to C$ such that

$$\|\gamma(t) - g(t, \gamma(t))\| = \inf \|y - g(t, \gamma(t))\|, \quad y \in G(x, t). \tag{1.6}$$

For $g(t, x) = a(t)\ \forall t \in \Omega, x \in G(t, x)$, where $a : \Omega \to X$ is a mapping and $G : \Omega \times C \to 2^C$ is a random set-valued mapping, we recover a random version of the approximation problem (see [7]).

We remark that, if $g : \Omega \times C \to C$ is a self-mapping in its second argument, then problem (1.6) is collapses to a random fixed point problems for finding measurable function $\gamma : \Omega \to C$ such that

$$\gamma(t) = g(t, \gamma(t)), \quad \forall\ t \in \Omega. \tag{1.7}$$

The general problem (1.5) is a particular case of (1.1), by letting $f : \Omega \times C \times C \to R^m$ as follows:

$$f(t, x, y) = (\|y - g_k(t, x)\| - \|x - g_k(t, x)\|)_{1 \le k \le m},$$

for all $t \in \Omega$, $x \in G(t, x)$, $y \in G(t, x)$.

2. Preliminaries

Throughout in this paper, we assume that (Ω, Σ) is a measurable space. Let C be a nonempty subset of a topological vector space X.

A function $f : \Omega \to X$ is said to be $\Sigma-$measurable if $f^{-1}(A) \in \Sigma$ for every Borel subset A in X.

A map $f : \Omega \times X \to X$ is said to be random continuous if it is measurable in the first argument and continuous in the second one.

A multi-function $F : \Omega \to 2^X$ is said to have a measurable selection if there exists a measurable function $f : \Omega \to X$ such that

$$f(t) \in F(t), \quad \forall\ t \in \Omega.$$

F has a casting representation, whenever there is a countable family of measurable selections $\{f_i\}_{i \ge 1}$ such that $\{f_i(t)\}_{i \ge 1}$ is dense in $F(t)$, for each $t \in \Omega$.

Definition 2.1. A suslin space is a topological space which is the continuous image of a Polish space. A suslin set in a topological space is a subset which is a suslin space.

Remark 2.2. The Polish space is a homeomorphism of a complete metric space.

To recall the concepts of suslin families, first we define the suslin operator as follows. Denote by $\mathcal{V}$ and $\mathcal{V}'$ the respective sets of infinite and finite sequences of positive integers. For $\sigma = \{\sigma_i\}_{i\geq 1} \subset \mathcal{V}$, we denote $\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$ by $\sigma \mid n$. Let $A : \mathcal{V}' \to S$. Then

$$\bigcup_{\sigma\in\mathcal{V}} \bigcap_{n=1}^{\infty} A(\sigma \mid n)$$

is said to be obtained from S by the suslin operation.

Definition 2.3. Let S be a family of sets. S is said to be a suslin family if it is stable by the suslin operation.

Remark 2.4. If μ is an outer measure over Ω, then Σ is a suslin family. If (Ω, Σ) is a complete measurable space, then Σ is a suslin family.

Definition 2.5. Let $\phi : C \to E$ and $f : C \times C \to E$ be two given functions and assume that C is convex.

(i) ϕ is said to be P-convex if for every $x, y \in C$,

$$\phi(tx + (1-t)y) \in t\phi(x) + (1-t)\phi(y) - P, \ \ \forall\, t \in (0,1).$$

(ii) ϕ is said to be P-quasiconvex if the set

$$\{x \in C : \phi(x) \notin -intP\}$$

is convex.

(iii) ϕ is said to be P-concave (resp. P-quasiconcave) if $-\phi$ is P-convex (resp. P-quasiconvex).

Definition 2.6. f is P-diagonally quasiconvex in y if for any finite subset $\{y_1, y_2, \cdots, y_n\}$ in C and any $y_0 \in co\{y_1, y_2, \cdots, y_n\}$, there exists $i \in \{1, 2, \cdots, n\}$ such that

$$f(y_0, y_i) \notin -intP.$$

Definition 2.7. ϕ is said to be P-lower (resp. upper) semi-continuous ($P{-}lsc$) if for any $x \in C$ and any neighborhood V of $\phi(x)$ in Y, there is a neighborhood U_x of x in X such that

$$\phi(u) \in V + P \text{ (resp. } \phi(u) \in V - P), \ \ \forall\, u \in U_x \cap C.$$

Definition 2.8. f is P-transfer lower (resp. upper) semi-continuous in X if for any $(x,y) \in C \times C$ and for any neighborhood V of $f(x,y)$, there exists a point $y' \in C$ and a neighborhood U_x of x in X such that

$$f(u,y') \in V + P \text{ (resp. } f(u,y') \in V - P), \ \ \forall\, u \in U_x \cap C.$$

Remark 2.9. If f is P-transfer lower (upper) semi-continuous in x, then the function $f(\cdot, y)$ is P-lower (upper) semi-continuous for each $y \in C$.

3. Existence theoems

Lemma 3.1. *Assume that Σ is a suslin family, X is a suslin space and $F : \Omega \to 2^X$ is a map such that*

$$GrF \in \Sigma \oplus \mathfrak{B}(X).$$

Then F has a casting representation.

Let Σ be a suslin family and let C be a closed convex and separable suslin set. For each $t \in \Omega$, $G : \Omega \times C \to 2^C$ is a random set-valued mapping, we define a set of deterministic generalized vector equilibria of $f(t,\cdot,\cdot)$ as

$$S(t) = \bigcap_{x \in C} \{x \in G(t,x), y \in G(t,x) : f(t,x,y) \notin -intP\}.$$

Theorem 3.2. *Suppose that*

- (Q1) *for each $i \in \Omega, x \in C, y \in G(t,x), f(\cdot,\cdot,y)$ is $\Sigma \oplus \mathfrak{B}(C)$-measurable;*
- (Q2) *for each $(t,x) \in \Omega \times C, x \in G(t,x), f(t,x,\cdot)$ is randomly $P - lsc$;*
- (Q3) *$G : \Omega \times C \to 2^C$ is continuous and measurable.*

If (1.1) *has at least one deterministic solution, then there exists a countable family of measurable functions $\{\gamma_i\}_{i \geq 1}$ which are solutions to* (1.1) *from Ω to C such that it is dense in the equilibrium sets.*

Proof. Assume that for each $t \in \Omega, S(t)$ is a nonempty subset of C. We show that assumptions $(Q1)$ and $(Q2)$ imply that the multi-function $S : \Omega \to 2^C$ which takes $S(t)$ as values satisfies

$$GrS \in \Sigma \oplus \mathfrak{B}(C).$$

First observe that for each $t \in \Omega$,

$$S(t) = \bigcap_{n \geq 1} \{x \in C, x \in G(t,x) : f(t,x,y_n) \notin -intP\}, \ \ \forall\, y_n \in G(t,x_n), \quad (3.1)$$

where $\{y_n\}_{n\geq 1}$ is a countable sequence in C. Since $\{y_n\}_{n\geq 1}$ is a countable dense sequence in C such that $t \in \Omega, x \in C, x \in G(t,x)$ and

$$f(t,x,y_n) \notin intP, \ \ \forall\, n \geq 1, \ y_n \in G(t,x_n),$$

for any arbitrary $y \in C$, we may assume that a subsequence (if necessarily) $\{y_n\}$ converges to y. Then from the random P-lower semi-continuity of $f(t,x,\cdot)$ implies that

$$f(t,x,y) \notin intP, \ \ \forall\, x \in G(t,x), \ y \in G(t,x).$$

Hence $x \in S(t)$ which prove (3.1). Now we can write

$$GrS = \bigcap_{n\geq 1}\{(t,x) \in \Omega \times C, x \in G(t,x) : f(t,x,y_n) \notin intP\}, \ \ y_n \in G(t,x_n).$$

Assumption $(Q1)$ asserts that

$$\big\{(t,x) \in \Omega \times C, x \in G(t,x) : f(t,x,y_n) \notin intP\big\} \in \Sigma \oplus \mathfrak{B}(C), \ \ \forall\, y_n \in G(t,x_n).$$

In this way, $GrS \in \Sigma \oplus \mathfrak{B}(C)$ is a countable intersection of elements in $\Sigma \oplus \mathfrak{B}(C)$. Thus from Lemma 3.1, S has a casting representation. This leads clearly to our assertion. □

Theorem 3.3. *Let C be a closed convex subset of X. Let $G : \Omega \times C \to 2^C$ be a random set-valued mapping and let $f, g : \Omega \times C \times C \to E$ be mappings such that for all $(x,y) \in C \times C$*

- (i) $g(t,x,y) \notin -intP \Rightarrow f(t,x,y) \notin -intP, \ \ \forall\, x \in G(t,x), \ y \in G(t,x);$
- (ii) *g is randomly P-diagonally quasiconvex in y;*
- (iii) *f is randomly P-transfer usc in x;*
- (iv) *G is randomly continuous and convex;*
- (v) *there is a nonempty compact subset B in C such that for each $A \in \mathfrak{F}(C)$, there is a compact convex $B_A \subset C$ containing A such that for every $x \in B_A \setminus B$, there exists $y \in B_A$ with*

$$x \in int\{t \in \Omega, u \in X : g(t,u,y) \in -intP\}, \ \ \forall\, y \in G(t,x).$$

Then (1.1) *has at least one solution in B.*

Theorem 3.4. *Let C be a closed convex subset of X and $G : \Omega \times C \to 2^C$ be a random set-valued mapping. Assume that two bifunctions $f, g : \Omega \times C \times C \to E$ satisfying*

- (S1) *for each $t \in \Omega$, $x \in C$, $y \in G(t,x)$, $f(\cdot,\cdot,y)$ is $\Sigma \oplus \mathfrak{B}(C)$-measurable;*
- (S2) $g(t,x,y) \notin -intP \Rightarrow f(t,x,y) \notin -intP$, $\forall\, t \in \Omega$, $x \in G(t,x)$, $y \in G(t,x)$;
- (S3) *$g(t,\cdot,\cdot)$ is randomly P-diagonally quasiconvex in y;*
- (S4) *for each $t \in \Omega$, $x \in C$, $x \in G(t,x)$, $f(t,x,\cdot)$ is randomly $P-lsc$;*
- (S5) *$f(t,\cdot,\cdot)$ is randomly P-transfer usc in x;*

(S6) *G is continuous and measurable;*

(S7) *there is a nonempty compact subset $\mathfrak{B}_t$ in C such that for each $A \in \mathfrak{F}(C)$, there is a compact convex set $\mathfrak{B}_A \subset C$ contain A such that for every $x \in \mathfrak{B}_A \setminus \mathfrak{B}_t$, there exists $y \in \mathfrak{B}_A$ with*

$$x \in int\{u \in X, g(t,u,y) \in -intP\}, \ \ \forall\, t \in \Omega, \ y \in G(t,x).$$

Then there exist a countable family of measurable functions $\{\gamma_i\}_{i\geq 1}$ which are solutions to (1.1) *from Ω to C such that for each $i \geq 1$ and each $t \in \Omega$,*

(i) $\gamma_i(t) \in \mathfrak{B}_t$,

(ii) $\{\gamma_i(t)\}_{i\geq 1}$ *is dense in $S(t)$.*

Proof. From Theorem 3.3, since $S(t)$ is a nonempty subset of C for all $t \in \Omega$, the conclusion follows from Theorem 3.2. □

Theorem 3.5. *Let C be a closed convex subset of X. Let $G : \Omega \times C \to 2^C$ be a random set-valued mapping and let $f : \Omega \times C \times C \to E$ be a random vector valued function such that*

(T1) *for each $t \in \Omega, x \in G(t,x), y \in G(t,x), f(\cdot,x,y)$ is measurable and continuous;*

(T2) $f(t,x,x) \notin -intP, \ \ \forall\, t \in \Omega, \ x \in C$;

(T3) *$f(t,\cdot,\cdot)$ is randomly P-quasiconvex in y;*

(T4) *for each $t \in \Omega, y \in G(t,x), f(t,\cdot,y)$ is randomly continuous;*

(T5) *for each $t \in \Omega, x \in G(t,x), f(t,x,\cdot)$ is randomly $P - lsc$;*

(T6) *G is a continuous and convex valued mapping;*

(T7) *there is a compact subset $\mathfrak{B}_t$ in C such that for each $A \in \mathfrak{F}(C)$ there is a compact convex set $\mathfrak{B}_A \subset C$ containing A such that for every $x \in \mathfrak{B}_A \setminus \mathfrak{B}_t$, there exists $y \in \mathfrak{B}_A$ with*

$$f(t,x,y) \in -intP, \ \ x \in G(t,x), \ \ y \in G(t,x).$$

Then (1.1) *has a countable family of random solutions which is dense in the set of deterministic solutions.*

Proof. For each $t \in \Omega$, set $g = f$. Since G is continuous and measurable, $f(\cdot,\cdot,y)$ is also measurable and continuous in x, and so, it is $(\Sigma, \mathfrak{B}(C))$-measurable (see, [16]). Next, assumptions $(T2)$ and $(T3), (T6), (T7)$ imply that f is randomly P-diagonally quasiconvex in y. Therefore the conclusion follows from Theorem 3.4. □

4. Applications

Next we will give some applications as follows:

First we will give a random generalized vector optimization problem.

Theorem 4.1. *Let C be a closed convex set and let $G : \Omega \times C \to 2^C$ be a random set-valued mapping. Assume that ϕ is a random continuous mapping and randomly P-quasiconvex. Suppose that for each $t \in \Omega$, there is a compact subset $\mathfrak{B}_t$ in C such that for each $A \in \mathfrak{F}(C)$, there is a compact convex set $\mathfrak{B}_A \subset C$ containing A such that for every $x \in \mathfrak{B}_A \setminus \mathfrak{B}_t$, there exists $y \in \mathfrak{B}_A$ with*

$$\phi(t,x) - \phi(t,y) \in intP, \quad x \in G(t,x), \quad y \in G(t,x).$$

Then (1.3) *has a countable family of random solutions which is dense in the set of deterministic weak vector optimum points.*

Proof. Let for $(t,x,y) \in \Omega \times C \times C$,

$$f(t,x,y) = \phi(t,y) - \phi(t,x), \quad \forall\, x \in G(t,x), \ y \in G(t,x).$$

Then, we can get the result from Theorem 3.5. □

Corollary 4.2. *Let C be a closed convex compact set and let $G : \Omega \times C \to 2^C$ be a random set-valued mapping. Let ϕ be a random continuous and randomly P-quasiconvex mapping. Then* (1.3) *has a countable family of random solutions which is dense in the set of deterministic weak vector optimum points.*

Next, we will give a random generalized vector variational inequality problem.

Theorem 4.3. *Let C be a closed convex compact subset of X and let $G : \Omega \times C \to 2^C$ be a random set-valued mapping. Suppose that the duality pairing $\langle \cdot, \cdot \rangle$ is continuous on $C \times L(X,E)$. Assume that $A : \Omega \times C \to L(X,E)$ is a random continuous mapping, for each $t \in \Omega$ there is a compact subset $\mathfrak{B}_t$ in C such that for each $A \in \mathfrak{F}(C)$ there is a compact convex set $\mathfrak{B}_A \subset C$ containing A such that for every $x \in \mathfrak{B}_A / \mathfrak{B}_t$, there exists $y \in \mathfrak{B}_A$ with*

$$\langle A(t,x), y - x \rangle \in -intP, \quad x \in G(t,x), \quad y \in G(t,x).$$

Then there is a countable family of measurable functions $\{\gamma_i\}_{i \ge 1}$ from Ω to C which satisfies (1.4).

Proof. For each $t \in \Omega, (t,x,y) \in \Omega \times C \times C$, let

$$f(t,x,y) = \langle A(t,x), y - x \rangle, \quad \forall\, x \in G(t,x), \ y \in G(t,x).$$

Then conditions $(T2)-(T7)$ are holds. It remains to check condition $(T1)$. To do this, for each $t \in \Omega$ and $x, y \in C$, let $x \in G(t,x), y \in G(t,x)$ and $K \in \mathfrak{B}(C)$.

Then we have

$$\begin{aligned} f^{-1}(K,x,y) &= \{t\in\Omega, x\in G(t,x), y\in G(t,x) : f(t,x,y)\in K\} \\ &= \{t\in\Omega, x\in G(t,x), y\in G(t,x) : \langle A(t,x), y-x\rangle \in K\} \\ &= \left\{ t\in\Omega, x\in G(t,x), y\in G(t,x) : A(t,x)\in \prod_{y-x}^{K} \right\} \\ &= A^{-1}\left(\prod_{y-x}^{K}, x\right) \in \Sigma. \end{aligned}$$

Since

$$\prod_{y-x}^{K} = \{s\in L(X,E) : \langle s, y-x\rangle \in K\},\ \ \forall\, x\in G(t,x),\ y\in G(t,x)$$

is clearly a Borel subset of $L(X,E)$, we conclude that $f(\cdot,x,y)$ is measurable. This completes the proof. □

Corollary 4.4. *Suppose that C is a convex compact set and A is a randomly continuous mapping. Then there is a countable family of measurable functions satisfying* (1.4).

Now, we will give a random generalized vector best approximations and random fixed point problem:

Theorem 4.5. *Suppose that $g_k, 1\le k\le m$ are randomly continuous mappings and for each $t\in\Omega$ there is a compact subset $\mathfrak{B}_t$ in C such that for each $A\in\mathfrak{F}(C)$ there is a compact convex set $\mathfrak{B}_A\subset C$ containing A such that for every $x\in\mathfrak{B}_A\setminus\mathfrak{B}_t$, there exists $y\in\mathfrak{B}_A$ with*

$$(\|y-g_k(t,x)\| - \|x-g_k(w,x)\|)_{1\le k\le m} \in -intP,\ \ \forall\, x\in G(t,x),\ y\in G(t,x).$$

Then (1.5) *has a countable family of solutions.*

Proof. Define a random vector valued function $f:\Omega\times C\times C\to R^m$ by

$$f(t,x,y) = (\|y-g_k(t,x)\| - \|x-g_k(w,x)\|)_{1\le k\le m}$$

for $(t,x,y)\in\Omega\times C\times C$, $x\in G(t,x)$, $y\in G(t,x)$. From the assumptions, Theorem 3.5 is clearly fulfilled for $E=R^m$. This leads to the conclusions. □

Corollary 4.6. *Let C be a closed convex compact set. Let $G:\Omega\times C\to 2^C$ be a random set-valued mapping and $g_k, 1\le k\le m$ be randomly continuous mappings. Then* (1.5) *has a countable family of solutions.*

Corollary 4.7. *Let C be a closed convex compact set. Let $G : \Omega \times C \to 2^C$ be a random set-valued mapping and let $g : \Omega \times C \to X$ be a randomly continuous mapping satisfying for each $t \in \Omega$, there is a compact subset $\mathfrak{B}_t$ in C such that for each $A \in \mathfrak{F}(C)$ there is a compact convex subset $\mathfrak{B}_A \subset C$ containing A such that for every $x \in \mathfrak{B}_A \backslash \mathfrak{B}_t$, there exists $y \in \mathfrak{B}_A$ with*

$$\|y - g(t,x)\| < \|x - g(t,x)\|, \ \ \forall\, x \in G(t,x), \ y \in G(t,x).$$

Then there is a countable family $\{\gamma_i\}_{i\geq 1}$ of random solutions to (1.6).

Corollary 4.8. *If C is a closed convex compact set and $g : \Omega \times C \to C$ is a randomly continuous mapping, then g has a countable family of random fixed points.*

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