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COMMON FIXED POINT THEOREMS IN $b-$FUZZY METRIC SPACES

Shaban Sedghi[1] and Nabi Shobe[2]

[1]Department of Mathematics, Qaemshahr Branch,
Islamic Azad University, Qaemshahr, Iran
e-mail: sedghi_gh@yahoo.com

[2]Department of Mathematics, Babol Branch,
Islamic Azad University, Babol, Iran
e-mail: nabi_shobe@yahoo.com

Abstract. In this paper, we consider complete $b-$fuzzy metric space and prove common fixed point theorems for a sequence of continuous functions converges uniformly in this spaces. Our results generalize the recent result many other known results.

1. Introduction and Preliminaries

The concept of fuzzy sets was introduced initially by Zadeh [24] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and application. George and Veeramani [6] , Kramosil and Michalek [10] have introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics, particularly in connections with both string and E-infinity theory which were given and studied by El Naschie [1, 2, 3, 4, 21]. Many authors [9, 11, 17, 18, 14] have proved fixed point theorem in fuzzy (probabilistic) metric spaces.

Definition 1.1. A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

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[0]The corresponding author: sedghi_gh@yahoo.com (Shaban Sedghi Ghadikolaei).

(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0,1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Two typical examples of a continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 1.2. A 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(5) $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Definition 1.3. A 3-tuple $(X, M, *)$ is called a $b-$fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$, $t, s > 0$ and $b \geq 1$ be a given real number,

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, \frac{t}{b}) * M(y, z, \frac{s}{b}) \leq M(x, z, t + s)$,
(5) $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

It should be noted that, the class of $b-$fuzzy metric spaces is effectively larger than that of fuzzy metric spaces, since a $b-$fuzzy metric is a fuzzy metric when $b = 1$.

We present an example shows that a $b-$fuzzy metric on X need not be a fuzzy metric on X.

Example 1.4. Let $M(x, y, t) = e^{\frac{-|x-y|^p}{t}}$, where $p > 1$ is a real number. We show that M is a $b-$fuzzy metric with $b = 2^{p-1}$.

Obviously conditions (1), (2), (3) and (5) of definition 1.3 are satisfied.

If $1 < p < \infty$, then the convexity of the function $f(x) = x^p$ $(x > 0)$ implies

$$\left(\frac{a + b}{2}\right)^p \leq \frac{1}{2}\left(a^p + b^p\right),$$

and hence, $(a+b)^p \leq 2^{p-1}(a^p+b^p)$ holds. Therefore,

$$\begin{aligned}\frac{|x-y|^p}{t+s} &\leq 2^{p-1}\frac{|x-z|^p}{t+s}+2^{p-1}\frac{|z-y|^p}{t+s}\\ &\leq 2^{p-1}\frac{|x-z|^p}{t}+2^{p-1}\frac{|z-y|^p}{s}\\ &= \frac{|x-z|^p}{t/2^{p-1}}+\frac{|z-y|^p}{s/2^{p-1}}.\end{aligned}$$

Thus for each $x, y, z \in X$ we obtain

$$\begin{aligned}M(x,y,t+s) &= e^{\frac{-|x-y|^p}{t+s}}\\ &\geq M(x,z,\frac{t}{2^{p-1}})*M(z,y,\frac{s}{2^{p-1}}),\end{aligned}$$

where $a*b=ab$. So condition (4) of definition 1.3 is hold and M is a $b-$ fuzzy metric.

It should be noted that in preceding example, for $p=2$ it is easy to see that $(X, M, *)$ is not a fuzzy metric space.

Before stating and proving our results, we present some definition and proposition in $b-$metric space.

Definition 1.5. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function. Then f is called $b-$nondecreasing, if $x > by$ this implies $f(x) \geq f(y)$ for each $x, y \in \mathbb{R}$.

Lemma 1.6. *Let $(X, M, *)$ be a $b-$fuzzy metric space. Then $M(x, y, t)$ is $b-$nondecreasing with respect to t, for all x, y in X. Also,*

$$M(x,y,b^n t) \geq M(x,y,t), \forall n \in \mathbb{N}.$$

Proof. Let $t > bs$. Then there exists a $\delta > 0$ such that $t = bs + \delta$. Therefore we have:

$$M(x,y,t) = M(x,y,bs+\delta) \geq M(x,x,\frac{\delta}{b})*M(x,y,\frac{bs}{b}) = M(x,y,s).$$

By condition (4) of definition 1.3 we have:

$$M(x,y,t) \geq M(x,x,\frac{\delta}{b})*M(x,y,\frac{t-\delta}{b}),$$

as $\delta \longrightarrow 0$ we get $M(x,y,t) \geq M(x,y,\frac{t}{b})$, that is $M(x,y,bt) \geq M(x,y,t)$. Hence, for every $n > 1$ we have:

$$M(x,y,b^n t) \geq M(x,y,b^{n-1}t) \geq \cdots \geq M(x,y,t).$$

□

Let $(X, M, *)$ be a $b-$fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}.$$

We recall the notions of convergence and completeness in a $b-$fuzzy metric space. Let $(X, M, *)$ be a $b-$fuzzy metric space. Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exists $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ is a topology on X (induced by the $b-$fuzzy metric M). A sequence $\{x_n\}$ in X converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$, for each $t > 0$. It is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$. The $b-$fuzzy metric space $(X, M, *)$ is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Lemma 1.7. *In a $b-$fuzzy metric space $(X, M, *)$, the following assertions hold:*

(i) If sequence $\{x_n\}$ in X converges to x, then x is unique,

(ii) If sequence $\{x_n\}$ in X is converges to x, then $\{x_n\}$ is a Cauchy sequence.

Proof. (i) It is easy to see that for each $0 < \epsilon < 1$, there exists a $0 < r < 1$ such that

$$(1 - r) * (1 - r) \geq 1 - \epsilon.$$

Let $x_n \longrightarrow y$ and $y \neq x$. Since $\{x_n\}$ converges to x and y, for $0 < r < 1$ and $t > 0$, there exists $n_1 \in \mathbb{N}$ and $n_2 \in \mathbb{N}$ such that for every $n \geq n_1$ and $n \geq n_2$,

$$M(x, x_n, t) > 1 - r$$

and

$$M(x_n, y, t) > 1 - r.$$

If $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by triangular inequality we have

$$M(x, y, t) \geq M(x, x_n, \frac{t}{2b}) * M(x_n, y, \frac{t}{2b}) > (1 - r) * (1 - r) \geq 1 - \epsilon.$$

Hence we get $M(x, y, t) = 1$ which is a contradiction. So, $x = y$.

(ii) As of above for each $0 < \epsilon < 1$, there exists a $0 < r < 1$ such that

$$(1 - r) * (1 - r) \geq 1 - \epsilon.$$

Since $x_n \longrightarrow x$ for this $0 < r < 1$ and $t > 0$, there exists $n_1 \in \mathbb{N}$ and $n_2 \in \mathbb{N}$ such that for every $n \geq n_1$ and $m \geq n_2$,

$$M(x, x_n, t) > 1 - r$$

and

$$M(x_m, x, t) > 1 - r.$$

If $n_0 = \max\{n_1, n_2\}$, then for every $n, m \geq n_0$ by triangular inequality we have

$$M(x_n, x_m, t) \geq M(x, x_n, \frac{t}{2b}) * M(x_m, x, \frac{t}{2b}) > (1-r) * (1-r) \geq 1-\epsilon.$$

Hence we get, $\{x_n\}$ is a Cauchy sequence. □

In $b-$fuzzy metric space we have the following proposition.

Proposition 1.8. *Let $(X, M, *)$ be a $b-$fuzzy metric space and suppose that $\{x_n\}$ and $\{y_n\}$ are b-convergent to x, y respectively then we have*

$$M(x, y, \frac{t}{b^2}) \leq \limsup_{n\longrightarrow\infty} M(x_n, y_n, t) \leq M(x, y, b^2 t)$$

and

$$M(x, y, \frac{t}{b^2}) \leq \liminf_{n\longrightarrow\infty} M(x_n, y_n, t) \leq M(x, y, b^2 t).$$

Proof. By Definition 1.3 we have

$$\begin{aligned} M(x, y, t) &\geq M(x, x_n, \frac{\delta}{b}) * M(x_n, y, \frac{t-\delta}{b}) \\ &\geq M(x, x_n, \frac{\delta}{b}) * M(x_n, y_n, \frac{t-\delta-\delta b}{b^2}) * M(y_n, y, \frac{\delta}{b}). \end{aligned}$$

Taking the upper limit as $n \to \infty$ we obtain

$$M(x, y, t) \geq \limsup_{n\longrightarrow\infty} M(x_n, y_n, \frac{t-\delta-\delta b}{b^2}),$$

so as $\delta \longrightarrow 0$ we get

$$M(x, y, t) \geq \limsup_{n\longrightarrow\infty} M(x_n, y_n, \frac{t}{b^2}).$$

On the other hand

$$\begin{aligned} M(x_n, y_n, t) &\geq M(x_n, x, \frac{\delta}{b}) * M(x, y_n, \frac{t-\delta}{b}) \\ &\geq M(x_n, x, \frac{\delta}{b}) * M(x, y, \frac{t-\delta-\delta b}{b^2}) * M(y, y_n, \frac{\delta}{b}). \end{aligned}$$

And taking the upper limit as $n \to \infty$ we obtain

$$\limsup_{n\longrightarrow\infty} M(x_n, y_n, t) \geq 1 * M(x, y, \frac{t-\delta-\delta b}{b^2}) * 1,$$

so as $\delta \longrightarrow 0$ we get

$$\limsup_{n\longrightarrow\infty} M(x_n, y_n, t) \geq M(x, y, \frac{t}{b^2}).$$

So we have

$$M(x, y, \frac{t}{b^2}) \leq \limsup_{n\longrightarrow\infty} M(x_n, y_n, t) \leq M(x, y, b^2t).$$

Similarly

$$M(x, y, \frac{t}{b^2}) \leq \liminf_{n\longrightarrow\infty} M(x_n, y_n, t) \leq M(x, y, b^2t).$$

□

Remark 1.9. In general, a $b-$fuzzy metric is not continuous.

2. Main Results

Definition 2.1. Let X be any nonempty set and $(Y, M, *)$ be a $b-$fuzzy metric space. Then a sequence $\{f_n\}$ of functions from X to Y is said to converge uniformly to a function f from X to Y if given $r, t > 0$, $0 < r < 1$, there exists $n_0 \in \mathbb{N}$ such that $M(f_n(x), f(x), t) > 1 - r$ for all $n \geq n_0$ and for all $x \in X$.

Theorem 2.2. *([7]) Let $f_n : X \longrightarrow Y$ be a sequence of continuous functions from a topological space X to a fuzzy metric space Y. If $\{f_n\}$ converges uniformly to f, then f is continuous.*

In the next theorem we show that the above Theorem is hold, where Y is a $b-$fuzzy metric space.

Theorem 2.3. *Let $f_n : X \longrightarrow Y$ be a sequence of continuous functions from a topological space X to a $b-$fuzzy metric space Y. If $\{f_n\}$ converges uniformly to f, then f is continuous.*

Proof. It is easy to see that for each $0 < \epsilon < 1$, there exists $0 < r < 1$ such that

$$(1 - r) * (1 - r) * (1 - r) \geq 1 - \epsilon.$$

Let $x_m \longrightarrow x$ we show that $f(x_m) \longrightarrow f(x)$. Since $\lim_{n\to\infty} f_n(x) = f(x)$ converges uniformly to f, for $0 < r < 1$, $t > 0$ and $x \in X$, there exists $n_1 \in \mathbb{N}$ and $n_2 \in \mathbb{N}$ such that for every $n \geq n_1$ and $n \geq n_2$,

$$M(f_n(x), f(x), t) > 1 - r$$

and

$$M(f_n(x_m), f_n(x), t) > 1 - r.$$

If $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by triangular inequality we have

$$\begin{aligned} & M(f(x_m), f(x), t) \\ \geq \; & M(f(x_m), f_n(x_m), \frac{\delta}{b}) * M(f_n(x_m), f_n(x), \frac{t-\delta-b\delta}{b^2}) * M(f_n(x), f(x), \frac{\delta}{b}) \\ > \; & (1-r)*(1-r)*(1-r) \geq 1-\epsilon. \end{aligned}$$

Hence we get $M(f(x_m), f(x), t) \longrightarrow 1$. □

Theorem 2.4. *Let $(X, M, *)$ be a complete $b-$fuzzy metric space and let $f_n : X \longrightarrow X$ be a sequence of continuous functions converges uniformly to f and satisfying the following conditions:*

(i)$M(f_ix, f_jy, t) \geq \gamma(a(t)M(x, y, b^2t) + d(t)M(x, f_ix, b^2t) + c(t)M(y, f_jy, b^2t))$

for every $x, y \in X$, $\forall i, j \in \mathbb{N}$, where a, d and c are functions of $[0, \infty)$ into $(0, 1)$ such that

$$a(t) + d(t) + c(t) = 1, \; for\ any\ t > 0,$$

and $\gamma : \; (0, 1] \rightarrow \; (0, 1]$ is an increasing and continuous function such that $\gamma(a) > a$ for each $a \in (0, 1)$.
Then f have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Then we define

$$x_{n+1} \; = \; f_{n+1}x_n, \qquad n = 0, 1, 2, \cdots .$$

Now, for an integer n, we have

$$\begin{aligned} M(x_{n+1}, x_{n+2}, t) \; &= \; M(f_{n+1}x_n, f_{n+2}x_{n+1}, t) \\ &\geq \; \gamma(a(t)M(x_n, x_{n+1}, b^2t) + d(t)M(x_n, x_{n+1}, b^2t) \\ &\quad +c(t)M(x_{n+1}, x_{n+2}, b^2t)) \qquad (*) \end{aligned}$$

Now, we prove that sequence $d_n(t) = M(x_n, x_{n+1}, t)$ is an increasing sequence in $[0, 1]$. Suppose that $\{d_n(t)\}$ is not an increasing sequence in $[0, 1]$, that is,

$$d_n(t) = M(x_n, x_{n+1}, t) > M(x_{n+1}, x_{n+2}, t) = d_{n+1}(t)$$

for some $t > 0$ and for some $n \in \mathbb{N}$. Then by the condition (i), we have

$$\begin{aligned} d_{n+1}(t) &= M(x_{n+1}, x_{n+2}, t) \\ &= M(f_{n+1}x_n, f_{n+2}x_{n+1}, t) \\ &\geq \gamma(a(t)M(x_n, x_{n+1}, b^2t) + d(t)M(x_n, x_{n+1}, b^2t) \\ &+ c(t)M(x_{n+1}, x_{n+2}, b^2t)) \\ &\geq \gamma(a(t)d_{n+1}(b^2t) + d(t)d_{n+1}(b^2t) + c(t)d_{n+1}(b^2t)) \\ &= \gamma(d_{n+1}(b^2t)) \\ &> d_{n+1}(b^2t) \\ &\geq d_{n+1}(t), \end{aligned}$$

which is a contradiction. Thus $\{M(x_n, x_{n+1}, t); n \geq 0\}$ is increasing sequence in $[0, 1]$. Therefore, tends to a limit $\alpha(t) \leq 1$ for all $t > 0$. We claim that $\alpha(t) = 1$. Let $\alpha(t) < 1$ for some $t > 0$. Then by making $n \longrightarrow \infty$ in the inequality $(*)$ we get

$$\alpha(t) \geq \gamma(\alpha(b^2t)) > \alpha(b^2t) \geq \alpha(t),$$

which is a contradiction. Hence $\alpha(t) = 1$ for every $t > 0$, that is,

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1.$$

Now, we prove that $\{x_n\}$ is a Cauchy sequence in X. Suppose that $\{x_n\}$ is not a Cauchy sequence in X. For convenience, let $x_{n+1} = f_n x_n$ for $n = 0, 1, 2, 3, \cdots$. Then there is an $\epsilon \in]0, 1[$ such that for each integer k, there exist integers $m(k)$ and $n(k)$ with $m(k) > n(k) \geq k$ such that

$$c_k(t) = M(x_{n(k)}, x_{m(k)}, t) \leq 1 - \epsilon \quad \text{for} \quad k = 1, 2, \cdots . \tag{2.1}$$

We may assume that

$$M(x_{n(k)}, x_{m(k)-1}, t) > 1 - \epsilon, \tag{2.2}$$

by choosing $m(k)$ which is the smallest number exceeding $n(k)$ for which (2.1) holds. Hence we have

$$\begin{aligned} 1 - \epsilon &\geq c_k(t) \geq M(x_{n(k)}, x_{m(k)-1}, \frac{t}{2b}) * M(x_{m(k)-1}, x_{m(k)}, \frac{t}{2b}) \\ &\geq (1 - \epsilon) * d_{m(k)-1}(\frac{t}{2b}) \end{aligned} \tag{2.3}$$

Hence, $c_k(t) \longrightarrow 1-\epsilon$ for every $t>0$ as $k \longrightarrow \infty$. Also notice

$$\begin{aligned}
& c_k(t) = M(x_{n(k)}, x_{m(k)}, t) \\
\geq\ & M(x_{n(k)}, x_{n(k)+1}, \frac{t}{3b}) * M(x_{n(k)+1}, x_{m(k)+1}, \frac{t}{3b^2}) \\
& *M(x_{m(k)+1}, x_{m(k)}, \frac{t}{3b^2}) \\
=\ & d_{n(k)}(\frac{t}{3b}) * M(f_{m(k)+1}x_{m(k)}, f_{n(k)+1}x_{n(k)}, \frac{t}{3b^2}) * d_{m(k)}(\frac{t}{3b^2}) \\
\geq\ & d_{n(k)}(\frac{t}{3b}) \\
& *\gamma\begin{pmatrix} a(t)M(x_{m(k)}, x_{n(k)}, \frac{t}{3}) \\ +d(t)M(x_{m(k)}, f_{m(k)+1}x_{m(k)}, \frac{t}{3}) + c(t)M(x_{n(k)}, f_{n(k)+1}x_{n(k)}, \frac{t}{3}) \end{pmatrix} \\
& *d_{m(k)}(\frac{t}{3b^2}).
\end{aligned}$$

Thus, as $k \longrightarrow \infty$ in the above inequality we have

$$\begin{aligned}
1-\epsilon &\geq 1 * \gamma(a(t)(1-\epsilon) + d(t) + c(t)) * 1 \\
&\geq \gamma(1-\epsilon) \\
&> 1-\epsilon,
\end{aligned}$$

which is a contradiction. Thus, $\{x_n\}$ is Cauchy and by the completeness of X, $\{x_n\}$ converges to x in X. Thus

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} f_{n+1}x_n = x.$$

As f_n is continuous functions on X and f_n converges uniformly to f, hence we have

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} f_{n+1}x_n = fx.$$

Thus x is a common fixed point of f .

Now to prove uniqueness, let if possible $x' \neq x$ be another common fixed point of f. Then there exists $t>0$ such that $M(x,x',t)<1$, and

$$
\begin{aligned}
M(x,x',b^2t) &\geq \limsup_{n\longrightarrow\infty} M(f_nx,f_{n+1}x',t)\\
&\geq \limsup_{n\longrightarrow\infty} (\gamma(a(t)M(x,x',b^2t)+d(t)M(x,f_nx,b^2t)\\
&\quad +c(t)M(x',f_{n+1}x',b^2t)))\\
&= \gamma(a(t)M(x,x',b^2t)+\limsup_{n\longrightarrow\infty} d(t)M(x,f_nx,b^2t)\\
&\quad +\limsup_{n\longrightarrow\infty} c(t)M(x',f_{n+1}x',b^2t))\\
&\geq \gamma(a(t)M(x,x',b^2t)+d(t)M(x,x,t)+c(t)M(x',x',t))\\
&\geq \gamma(M(x,x',b^2t))\\
&> M(x,x',b^2t),
\end{aligned}
$$

which is a contradiction. Therefore, x is a unique common fixed point of f. □

Corollary 2.5. *([19]). Let $(X,M,*)$ be a complete fuzzy metric space and let $f_n : X \longrightarrow X$ be a sequence of continuous functions which converges uniformly to f and satisfying the following condition:*

$$\text{(i)}M(f_ix,f_jy,t)\geq\gamma(a(t)M(x,y,t)+d(t)M(x,f_ix,t)+c(t)M(y,f_jy,t))$$

for every $x,y\in X$, $\forall i,j\in\mathbb{N}$, where a,d and c are functions of $[0,\infty)$ into $(0,1)$ such that

$$a(t)+d(t)+c(t)=1,\ \ for\ any\ t>0,$$

and $\gamma:\ (0,1]\rightarrow\ (0,1]$ is an increasing and continuous function such that $\gamma(a)>a$ for each $a\in(0,1)$. Then f have a unique common fixed point.

Proof. Take $b=1$ in Theorem 2.4. □

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