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EXISTENCE AND LOCAL ATTRACTIVITY OF SOLUTIONS FOR A NONLINEAR FUNCTIONAL INTEGRAL EQUATION

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Abstract. In this paper, we study the existence and local attractivity of solutions for a nonlinear functional integral equation by using measures of noncompactness and Schauder fixed point theorem. The equation is considered in the Banach space consisting of real functions which are continuous and bounded on R_+.

1. Introduction

Nonlinear functional integral equations have wide applications in solving problems such as in vehicular traffic, biology, optimal control and economic, etc., see [2, 6, 8, 9].

Let R denote a real line and R_+ the set of nonnegative real numbers. Hu and Yan [1] studied the existence of solutions for the following nonlinear integral equations

$$x(t) = f\left(t, x(t), \int_0^t u(t,s,x(s))ds\right), \quad t \in R_+,$$

and

$$x(t) = g(t, x(t)) + x(t)\int_0^t u(t,s,x(s))ds, \quad t \in R_+.$$

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Maleknejad [3] considered the existence of solutions of the nonlinear integral equation

$$x(t) = g(t, x(t)) + f\left(t, \int_0^t u(t,s,x(s))ds, x(\gamma(t))\right), \quad t \in [0,a].$$

Motivated by the above work, the aim of this paper is to discuss the existence of solutions for the following generalized nonlinear functional integral equation

$$x(t) = g(t, x(\alpha(t))) + f\left(t, \int_0^{\eta(t)} u(t,s,x(\beta(s)))ds, x(\gamma(t))\right), \quad t \in R_+. \quad (1.1)$$

where $g{:}R_+{\times}R{\to}R$, $f{:}R_+{\times}R{\times}R{\to}R$, $u{:}R_+{\times}R_+{\times}R \to R$, $\alpha, \beta, \gamma, \eta : R_+ \to R_+$. Other special cases of equation (1.1) have been studied in [2, 6, 7, 8].

By using the measures of noncompactness and Schauder fixed point theorem, we obtain the existence of solutions of equation (1.1) which are uniformly locally attractive.

The result obtained in this paper generalizes the results obtained in [1, 2, 3, 7, 8].

2. Preliminaries

Suppose E is a real Banach space with the zero element θ. We write $B(x,r)$ to denote the closed ball centered at x with radius r and B_r to denote the ball $B(\theta, r)$. If X is a subset of E, we write $\bar{X}$ and $ConvX$ in order to denote the closure and convex closure of X, respectively. Moreover, the family of all nonempty bounded subsets of E is denoted by M_E and its subfamily consisting of all relatively compact sets is denoted by N_E.

Definition 2.1. ([4]) A mapping $\mu : M_E \to R_+$ is said to be a measure of noncompactness in E if it satisfies the following conditions:
(1) The family $ker\mu = \{X \in M_E : \mu(X) = 0\}$ is nonempty and $ker\mu \subset N_E$.
(2) $X \subset Y \Rightarrow \mu(X) \le \mu(Y)$.
(3) $\mu(\bar{X}) = \mu(X)$.
(4) $\mu(ConvX) = \mu(X)$.
(5) $\mu(\lambda X + (1-\lambda)Y) \le \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0,1]$.
(6) If $\{X_n\}$ is a sequence of closed sets from M_E such that $X_{n+1} \subset X_n$ ($n = 1, 2, \cdots$) and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection $X_\infty = \cap_{n=1}^{\infty} X_n$ is nonempty.

Remark 2.2. The family $ker\mu$ described in (1) of Definition 2.1 is said to be the kernel of the measure of noncompactness μ. The intersection X_∞ in (6) is a member of $ker\mu$. In fact, since $\mu(X_\infty) \le \mu(X_n)$ for any n then we infer that $\mu(X_\infty) = 0$.

Other facts concerning measures of noncompactness and their properties may be found in [4].

Theorem 2.3. ([5], *Schauder fixed point theorem*) *If S is a compact convex subset of Banach space E and $H : S \to S$ is a continuous mapping, then H has a fixed point in the set S.*

In what follows, we will work in the Banach space $BC(R_+)$ consisting of all real functions which are continuous and bounded on R_+ and furnished with the standard norm $\|x\| = \sup\{|x(t)| : t \in R_+\}$.

Further, we recall the construction of the measure of noncompactness which was introduced in [4]. Let X be a nonempty bounded subset of $BC(R_+)$ and T be a positive number. For $x \in X$ and $\varepsilon \geq 0$ denote by $w^T(x, \varepsilon)$ the modulus of continuity of the function x on the interval $[0, T]$, i.e.

$w^T(x, \varepsilon) = \sup\{|x(t) - x(s)| : t, s \in [0, T], |t - s| \leq \varepsilon\}$.

Further, let us put

$w^T(X, \varepsilon) = \sup\{w^T(x, \varepsilon) : x \in X\}$, $w_0^T(X) = \lim_{\varepsilon \to 0} w^T(X, \varepsilon)$, $w_0(X) = \lim_{T \to \infty} w_0^T(X)$.

If $t \in R_+$ is a fixed number, let us denote

$X(t) = \{x(t) : x \in X\}$ and $diam X(t) = \sup\{|x(t) - y(t)| : x, y \in X\}$.

Finally, consider the function μ defined on the family $M_{BC(R_+)}$ by

$$\mu(X) = w_0(X) + \limsup_{t \to \infty} diam X(t). \tag{2.1}$$

It can be shown that the function μ is a measure of noncompactness in the space $BC(R_+)$ [4] . The kernel $ker\mu$ of this measure consists of nonempty and bounded sets X such that functions from X are locally equicontinuous on R_+ and the thickness of the bundle formed by functions belonging to X tends to zero at infinity. This property will permit us to characterize solutions of the integral equation (1.1) in next section.

Now, let Ω be a nonempty subset of the space $BC(R_+)$. Let $Q : \Omega \to BC(R_+)$ be an operator and consider the following operator equation

$$x(t) = (Qx)(t), \qquad t \in R_+. \tag{2.2}$$

Definition 2.4. ([6, 8]) We say that the solutions of equation (2.2) are locally attractive if there exists a closed ball $B(x_0, r)$ in the space $BC(R_+)$ for some $x_0 \in BC(R_+)$ such that for arbitrary solutions $x(t)$ and $y(t)$ of equation (2.2) belonging to $B(x_0, r) \bigcap \Omega$, we have that

$$\lim_{t \to \infty} (x(t) - y(t)) = 0. \tag{2.3}$$

In the case, when the limit (2.3) is uniform with respect to the set $B(x_0, r) \bigcap \Omega$, i.e., when for each $\varepsilon > 0$ there exists $T > 0$ such that

$$|x(t) - y(t)| \leq \varepsilon$$

for all $x(t), y(t) \in B(x_0, r) \bigcap \Omega$ being solutions of equation (2.2) and for $t \geq T$, we say that solutions of (2.2) are uniformly locally attractive.

3. Main result

In this section, the functional integral equation (1.1) will be investigated in the Banach space $BC(R_+)$ described previously.

We will consider equation (1.1) under the following hypotheses.

(H_1) The function $g: R_+ \times R \to R$ is continuous and there exists a constant $P \geq 0$ such that

$$|g(t,x) - g(t,y)| \leq P|x-y|$$

for all $t \in R_+$ and $x, y \in R$.

(H_2) $f: R_+ \times R \times R \to R$ is a continuous function and there exist nonnegative constants $M_i (i=1,2)$ such that

$$|f(t,y_1,x_1) - f(t,y_2,x_2)| \leq M_1|y_1 - y_2| + M_2|x_1 - x_2|$$

for all $t \in R_+$ and $x_i, y_i \in R (i=1,2)$.

(H_3) There exist nonnegative constants a and b such that

$$|f(t,0,x)| \leq a + b|x|$$

for all $t \in R_+$ and $x \in R$.

(H_4) The function $u: R_+ \times R_+ \times R \to R$ is continuous. There exist a continuous function $v: R_+ \times R_+ \to R_+$ and a continuous nondecreasing function $\psi: R_+ \to R_+$ such that

$$|u(t,s,x)| \leq v(t,s)\psi(|x|)$$

for all $t, s \in R_+$ and $x \in R$. Moreover, we assume that

$$\lim_{t \to \infty} \int_0^{\eta(t)} v(t,s)ds = 0.$$

(H_5) The functions $\alpha, \beta, \gamma, \eta : R_+ \to R_+$ are continuous.

(H_6) There exists a positive solution r_0 of the inequality

$$(P+b)r + M_1 \cdot Q \cdot \psi(r) + G_0 + a \leq r$$

such that

$$P + b < 1.$$

(H_7) $P + M_2 < 1$.

Remark 3.1. We denote G_0 by

$$G_0 = \sup\{|g(t,0)| : t \in R_+\}. \tag{3.1}$$

Obviously, $G_0 < \infty$ in view of hypothesis (H_1). Note that the function $q : R_+ \to R_+$ defined by

$$q(t) = \int_0^{\eta(t)} v(t,s)ds$$

is continuous on R_+. This together with hypothesis (H_4) imply that $Q = \sup\{q(t) : t \in R_+\}$ is finite.

Theorem 3.2. *Under the hypotheses* (H_1)-(H_7), *equation* (1.1) *has at least one solution $x = x(t)$ which belongs to the space $BC(R_+)$. Moreover, solutions of equation* (1.1) *are uniformly locally attractive.*

Proof. Define the operator F on the space $BC(R_+)$ by

$$\begin{aligned}(Fx)(t)& \\ &= g(t, x(\alpha(t))) + f\left(t, \int_0^{\eta(t)} u(t,s,x(\beta(s)))ds, x(\gamma(t))\right), \quad t \in R_+.\end{aligned} \tag{3.2}$$

First we show that F is continuous. As all the functions on the right hand side of equation (3.2) are continuous, the function Fx is continuous on R_+ for each $x \in BC(R_+)$. Moreover, in view of hypotheses (H_1)-(H_5) and Remark 3.1, for fixed $t \in R_+$, we get

$$\begin{aligned}|(Fx)| &\leq |g(t, x(\alpha(t)))| + \left| f\left(t, \int_0^{\eta(t)} u(t,s,x(\beta(s)))ds, x(\gamma(t)\right)\right| \\ &\leq |g(t, x(\alpha(t))) - g(t,0)| + |g(t,0)| \\ &\quad + \left| f\left(t, \int_0^{\eta(t)} u(t,s,x(\beta(s)))ds, x(\gamma(t))\right) - f(t,0,x(\gamma(t)))\right| + |f(t,0,x(\gamma(t)))| \\ &\leq P|x(\alpha(t))| + G_0 + M_1 \int_0^{\eta(t)} v(t,s)\psi(|x(\beta(s))|)ds + a + b|x(\gamma(t))| \\ &\leq (P+b)\|x\| + M_1 \cdot Q \cdot \psi(\|x\|) + G_0 + a.\end{aligned}$$

This shows Fx is bounded on R_+. Then $Fx \in BC(R_+)$ which means that the operator F transforms the space $BC(R_+)$ into itself.

On the other hand, from the above estimate and hypothesis (H_6), we deduce that there exists a number $r_0 > 0$ such that the operator F transforms the ball B_{r_0} into itself.

In the following, let us take a nonempty subset $X \subset B_{r_0}$. Then, for arbitrarily $x, y \in X$ and a fixed $t \in R_+$, in view of hypotheses (H_1), (H_2), (H_4) and (H_5)

we obtain

$$\begin{aligned}
&|(Fx)(t)-(Fy)(t)| \\
&\leq P|x(\alpha(t))-y(\alpha(t))|+M_2|x(\gamma(t))-y(\gamma(t))| \\
&\quad + M_1\left|\int_0^{\eta(t)} u(t,s,x(\beta(s)))ds-\int_0^{\eta(t)} u(t,s,y(\beta(s)))ds\right| \qquad (3.3)\\
&\leq P diam X(\alpha(t))+M_2 diam X(\gamma(t))+2M_1\psi(r_0)\int_0^{\eta(t)} v(t,s)ds.
\end{aligned}$$

From estimate (3.3), we derive

$$diam(FX)(t)\leq P diam X(t)+M_2 diam X(t)+2M_1\psi(r_0)\int_0^{\eta(t)} v(t,s)ds.$$

Then, by hypothesis (H_4) we get

$$\limsup_{t\to\infty} diam(FX)(t)\leq (P+M_2)\limsup_{t\to\infty} diam X(t). \qquad (3.4)$$

Further, let us take arbitrary numbers $T>0$ and $\varepsilon>0$. Next, fix a function $x\in X$ and $t_1,t_2\in[0,T]$ such that $|t_1-t_2|\leq\varepsilon$. Without loss of generality, we can assume that $t_1<t_2$. Then, taking into account our hypotheses (H_1),(H_2), (H_4) and (H_5)we obtain

$$\begin{aligned}
&|(Fx)(t_2)-(Fx)(t_1)| \\
&\leq|g(t_2,x(\alpha(t_2)))-g(t_2,x(\alpha(t_1)))|+|g(t_2,x(\alpha(t_1)))-g(t_1,x(\alpha(t_1)))| \\
&\quad+\left|f\left(t_2,\int_0^{\eta(t_2)} u(t_2,s,x(\beta(s)))ds,x(\gamma(t_2))\right)-f\left(t_2,\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds,x(\gamma(t_2))\right)\right| \\
&\quad+\left|f\left(t_2,\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds,x(\gamma(t_2))\right)-f\left(t_1,\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds,x(\gamma(t_2))\right)\right| \\
&\quad+\left|f\left(t_1,\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds,x(\gamma(t_2))\right)-f\left(t_1,\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds,x(\gamma(t_1))\right)\right| \\
&\leq P|x(\alpha(t_2))-x(\alpha(t_1))|+w^T(g,\varepsilon)+M_1\left|\int_0^{\eta(t_2)} u(t_2,s,x(\beta(s)))ds-\int_0^{\eta(t_1)} u(t_1,s,x(\beta(s)))ds\right| \\
&\quad+w^T(f,\varepsilon)+M_2|x(\gamma(t_2))-x(\gamma(t_1))| \\
&\leq P|x(\alpha(t_2))-x(\alpha(t_1))|+w^T(g,\varepsilon)+M_1w^T(u,\varepsilon)\eta(T)+M_1\psi(r_0)w^T(\eta,\varepsilon)V_T \\
&\quad+w^T(f,\varepsilon)+M_2|x(\gamma(t_2))-x(\gamma(t_1))|,
\end{aligned} \qquad (3.5)$$

where

$$w^T(g,\varepsilon) = \sup\{|g(t_2,x) - g(t_1,x)| : t_1,t_2 \in [0,T], |t_1 - t_2| \le \varepsilon, x \in [-r_0,r_0]\},$$

$$w^T(u,\varepsilon) = \sup\{|u(t_2,s,x) - u(t_1,s,x)| : t_1,t_2 \in [0,T], s \in [0,\eta(T)], \\ |t_1 - t_2| \le \varepsilon, x \in [-r_0,r_0]\},$$

$$\eta(T) = \sup\{\eta(t) : t \in [0,T]\},$$

$$w^T(\eta,\varepsilon) = \sup\{|\eta(t_2) - \eta(t_1)| : t_1,t_2 \in [0,T], |t_1 - t_2| \le \varepsilon\},$$

$$V_T = \sup\{v(t,s) : t \in [0,T], s \in [0,\eta(T)]\},$$

$$w^T(f,\varepsilon) = \sup\{|f(t_2,y,x) - f(t_1,y,x)| : t_1,t_2 \in [0,T], y \in [-Q\psi(r_0), Q\psi(r_0)], \\ |t_1 - t_2| \le \varepsilon, x \in [-r_0,r_0]\}.$$

Since the function $g(t,x)$ is uniformly continuous on the set $[0,T]\times[-r_0,r_0]$, the function $u(t,s,x)$ is uniformly continuous on the set $[0,T] \times [0,\eta(T)] \times [-r_0,r_0]$, the function $\eta(t)$ is uniformly continuous on the set $[0,T]$ and the function $f(t,y,x)$ is uniformly continuous on the set $[0,T]\times[-Q\psi(r_0),Q\psi(r_0)] \times [-r_0,r_0]$, we have $w^T(g,\varepsilon)\to 0$, $w^T(u,\varepsilon)\to 0$, $w^T(\eta,\varepsilon)\to 0$ and $w^T(f,\varepsilon)\to 0$ as $\varepsilon\to 0$. Then, by estimate (3.5), we derive the following inequality

$$w_0^T(FX) \le (P + M_2)w_0^T(X).$$

This yields

$$w_0(FX) \le (P + M_2)w_0(X). \tag{3.6}$$

Now, using formula (2.1), linking the estimates (3.4) and (3.6) we get

$$\mu(FX) \le (P + M_2)\mu(X). \tag{3.7}$$

In the following, let us consider the sequence $\{B_{r_0}^n\}$, where $B_{r_0}^1 = ConvF(B_{r_0})$, $B_{r_0}^2 = ConvF(B_{r_0}^1), \cdots$. Then $B_{r_0}^{n+1} \subset B_{r_0}^n \subset B_{r_0}$ for all $n = 1,2,\cdots$. Moreover, all sets of this sequence are nonempty, bounded, convex and closed. Apart from this, in view of (3.7) we get

$$\mu(B_{r_0}^n) \le (P + M_2)^n \mu(B_{r_0})$$

for $n = 1,2,\cdots$. Then by hypothesis (H_7), we deduce that

$$\lim_{n\to\infty} \mu(B_{r_0}^n) = 0.$$

Hence, by Definition 2.1, we infer that the set $Y=\cap_{n=1}^{\infty}B_{r_0}^n$ is nonempty, bounded, convex and closed. Moreover, the set Y is a member of the kernel $ker\mu$ (Remark 2.2) and relatively compact. In particular, we have that

$$\limsup_{t\to\infty} diamY(t)=\lim_{t\to\infty} diamY(t)=0. \tag{3.8}$$

Let us also observe that the operator F maps the set Y into itself.

Next, we show that F is continuous on the set Y.

Let us fix a number $\varepsilon>0$ and take arbitrary functions $x,y\in Y$ such that $\|x-y\|\le\varepsilon$. Taking into account the fact that $FY\subset Y$ and using the estimate (3.8), we can infer that there exists $T>0$ such that for arbitrary $t\geq T$ we have that

$$|(Fx)(t)-(Fy)(t)|\le\varepsilon. \tag{3.9}$$

Further, take $t\in[0,T]$. Then, keeping in mind our hypotheses (H_1), (H_2), (H_5) and (H_7), we obtain

$$\begin{aligned}
&|(Fx)(t)-(Fy)(t)|\\
&\le P|x(\alpha(t))-y(\alpha(t))|+M_2|x(\gamma(t))-y(\gamma(t))|\\
&\quad+M_1\left|\int_0^{\eta(t)}u(t,s,x(\beta(s)))ds-\int_0^{\eta(t)}u(t,s,y(\beta(s)))ds\right|\\
&\le (P+M_2)\varepsilon+M_1\int_0^{\eta(T)}w_u(\cdot,\cdot,\varepsilon)ds\\
&<\varepsilon+M_1\eta(T)w_u(\cdot,\cdot,\varepsilon),
\end{aligned} \tag{3.10}$$

where, $\eta(T)$ was introduced earlier and $w_u(\cdot,\cdot,\varepsilon)=\sup\{|u(t,s,x)-u(t,s,y)|: t\in[0,T], s\in[0,\eta(T)], |x|,|y|\le r_0 \text{ and } \|x-y\|\le\varepsilon\}$. Then the estimates (3.9) and (3.10) enable us to conclude that the operator F is continuous on the set Y.

Finally, taking into account all the facts of the set Y and the operator $F: Y\to Y$ established above, by Theorem 2.3 we infer that the operator F has at least one fixed point x in the set Y. Obviously, the function $x=x(t)$ is a solution of equation (1.1). Moreover, keeping in mind the fact that $Y\in ker\mu$, the characterization of sets belonging to $ker\mu$ (see descriptions made after (2.1)) and Definition 2.4, we deduce that solutions of equation (1.1) are uniformly locally attractive. This assertion is the consequence of the fact that if x and y are solutions of equation (1.1) belonging to B_{r_0}, then $x,y\in Y$. The proof is now completed. □

4. An example

In this section, we present an example to demonstrate the applications of our main result obtained in Section 3.

Example 4.1. Consider the following functional integral equation

$$x(t) = \frac{t + t\sin(x(\frac{t}{2}))}{2+3t^2} + \frac{1}{3+t}\int_0^{\sqrt{t}} \sin(\frac{s^4}{3+t^3})\arctan(1+x^2(\frac{s}{4}))ds + \frac{\arctan t}{4}\sin(x(\frac{t}{4})), \tag{4.1}$$

where $t \in R_+$.

Notice that the above equation is a special case of equation (1.1), if we put

$$\alpha(t) = \frac{t}{2}, \quad \beta(t) = \frac{t}{4}, \quad \gamma(t) = \frac{t}{4}, \quad \eta(t) = \sqrt{t}.$$

$$g(t,x) = \frac{t + t\sin x}{2+3t^2}, \quad f(t,y,x) = \frac{1}{3+t}y + \frac{\arctan t}{4}\sin x. \tag{4.2}$$

In what follows, we show that the hypotheses of Theorem 3.2 are satisfied. First, it easy to check that $|g(t,x) - g(t,y)| \leq P|x-y|$ with $P = \frac{\sqrt{6}}{12}$, then hypothesis (H_1) is satisfied.

Next, $|f(t,y_1,x_1) - f(t,y_2,x_2)| \leq M_1|y_1-y_2| + M_2|x_1-x_2|$, with $M_1 = \frac{1}{3}$ and $M_2 = \frac{\pi}{8}$. Moreover, $|f(t,0,x)| \leq \frac{\pi}{8}|x|$, then $a = 0$ and $b = \frac{\pi}{8}$, the hypotheses (H_2) and (H_3) are satisfied.

Further, $u(t,s,x) \leq v(t,s)\psi(|x|)$ with $v(t,s) = \frac{s^4}{3+t^3}$ and $\psi(|x|) = 1 + x^2$. Apart from this, $\int_0^{\eta(t)} v(t,s)ds = \frac{1}{5}\frac{t^2\sqrt{t}}{3+t^3}$. This implies $\lim_{t\to\infty}\int_0^{\eta(t)} v(t,s)ds = 0$. Then the hypothesis ($H_4$) holds.

On the other hand, $Q = \sup\{q(t) : t \geq 0\} = \frac{15^{\frac{5}{6}}}{90}$.

Obviously the functions $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\eta(t)$ satisfy hypothesis (H_5).

Next, by (3.1) and (4.2), $G_0 = \frac{\sqrt{6}}{12}$, then in view of the above obtained estimates of P, M_1, a, b and Q, the inequality from hypothesis (H_6) has the form

$$(\frac{\sqrt{6}}{12} + \frac{\pi}{8})r + \frac{1}{3} \cdot \frac{15^{\frac{5}{6}}}{90}(1+r^2) + \frac{\sqrt{6}}{12} \leq r. \tag{4.3}$$

It is easy to check that $r_0 = 1$ is a solution of inequality (4.3). Obviously, the second inequality of hypothesis (H_6) holds in our situation. Then hypothesis (H_6) is satisfied.

Finally, $P + M_2 < 1$, then hypothesis (H_7) holds. Thus by Theorem 3.2, equation (4.1) has at least one solution $x(t)$ in $BC(R_+)$. Moreover, solutions of this equation are uniformly locally attractive.

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