Nonlinear Functional Analysis and Applications
Vol. 22, No. 3 (2017), pp. 543-553
ISSN: 1229-1595(print), 2466-0973(online)

http://nfaa.kyungnam.ac.kr/journal-nfaa

COUPLED MULTIVALUED FIXED POINTS AND STABILITY

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Abstract. In this paper, we present a multivalued coupled fixed point results in metric spaces and discuss the stability of the fixed point sets of sequence of such multivalued mappings. For that purpose we introduce a notion of stability for coupled fixed point sets with the help of the Hausdorff metric on a product space. We illustrate our result with examples.

1. Introduction

We present in this paper a multivalued coupled fixed point theorem for multivalued coupled mappings which satisfies certain inequality which is a combinations of Banach and Kannan types. After its origination in the celebrated of work in 1922 ([1]), the metric fixed point theory has developed over the years in various directions. Banach's result was carried into the domain of multivalued analysis by Nadler, who in 1969 in his work [17] proved the multivalued analog of Banach's contraction mapping principle. The work was followed by a large number of papers dealing with different multivalued contractions and their fixed points. Several aspects of this development can be found in [11]. Coupled fixed point theorems were introduced by Guo et al. [10]

[0]Received October 6, 2016. Revised January 17, 2017.
[0]2010 Mathematics Subject Classification: 47H10.
[0]Keywords: Multivalued coupled fixed point, Housdorff metric, metric space, control function.

in the area of research. A problem related to coupled fixed points obtained a boost after the publication of the result by Bhaskar et al. in 2006 ([3]). Some works from this area are noted in [7, 9]. Multivalued extensions of coupled fixed points appeared in works like [2, 18].

We also discuss stability of the coupled fixed point sets associated with the multivalued coupled fixed point theorem which we prove here. It is a character of multivalued mappings that their fixed points, when they exists, are often, but not always non unique. It can be mentioned that the Nadler's multivalued contraction mappings theorem, unlike its counterpart on Banach's result is not a unique fixed point result. Thus stability analysis of fixed point sets find its natural place in multivalued analysis. There are several results dealing with the stability of fixed point sets as for instance [4]-[6], [8], [12]-[15], [16, 19]. We establish that a sequence of contraction mappings satisfying the condition of our theorem in complete metric spaces as stable coupled fixed point structure.

Before we present our findings in the next two sections, we highlight the following points.

• We use a control function in the existence theorem of multivalued coupled fixed point.
• We use Hausdorff metric in relation to the product of two metric spaces.
• We define the concept of stability for coupled fixed point sets.
• We illustrate both the existence of the coupled multivalued fixed point and stability of coupled fixed point sets with an example.

2. Preliminaries

The following are the concepts from setvalued analysis which we use in this paper. Let $(X,\ d)$ be a metric space. Then
$N_X = \{A : A \text{ is a non-empty subset of } X\}$,
$B_X = \{A : A \text{ is a non-empty bounded subset of } X\}$,
$CB_X = \{A : A \text{ is a non-empty closed and bounded subset of } X\}$ and
$C_X = \{A : A \text{ is a non-empty compact subset of } X\}$.

For $x \in X$ and $B \in N_X$, the function $D(x,\ B)$, and for $A,\ B \in CB_X$, the function $H(A,\ B)$ are defined as follows:

$$D(x,\ B) = \inf \{d(x,\ y) : y \in B\}$$

and

$$H(A,\ B) = \max \left\{ \sup_{x \in A} D(x,\ B),\ \sup_{y \in B} D(y,\ A) \right\}.$$

H is known as the Hausdorff metric induced by the metric d on CB_X ([17]). Further, if $(X,\ d)$ is complete then $(CB_X,\ H)$ is also complete.

Nadler [17] established the following Lemma.

Lemma 2.1. ([17]) *Let $(X,\ d)$ be a metric space and $A,\ B \in CB_X$. Let $q > 1$. Then for every $x \in A$, there exists $y \in B$ such that $d(x,\ y) \leq q\ H(A,\ B)$.*

Lemma 2.2. ([4]) *Let $(X,\ d)$ be a metric space and $A,\ B \in C_X$. Then for every $x \in A$, there exists $y \in B$ such that $d(x,\ y) \leq H(A,\ B)$.*

The following is a consequence of Lemma 2.2.

Lemma 2.3. ([4]) *Let $(X,\ d)$ be a metric space and $T : A \to C_B$ be a multivalued mapping. Let $q \geq 1$. Then for $a,\ b \in A$ and $x \in Ta$, there exists $y \in Tb$ such that $d(x,\ y) \leq q\ H(Ta,\ Tb)$ where A and B are two nonempty compact subsets of X.*

Definition 2.4. Let $T : X \to CB_Y$ be a multivalued mapping, where $(X,\ \rho)$, $(Y,\ d)$ are two metric spaces and H is the Hausdorff metric on CB_Y. The mapping T is said to be continuous at $x \in X$ if for any sequence $\{x_n\}$ in X, $H(Tx,\ Tx_n) \to 0$ whenever $\rho(x,\ x_n) \to 0$ as $n \to \infty$.

Definition 2.5. ([18]) Let $T : X \times X \to N_X$ be a multivalued mapping, a point $z \in X$ is said to be fixed point of T, whenever $z \in Tz$. Let $T : X \to N_X$ be any mapping, a point $(x,\ y) \in X \times X$ is said to be a coupled fixed point of T, whenever $x \in T(x, y)$ and $y \in T(y, x)$.

Let (X, d) be metric space. Then $\eta : X \times X \to [0, \infty)$ by

$$\eta((x,\ y),\ (u,\ v)) = d(x,\ u) + d(y,\ v), \text{ for } (x,\ y),\ (u,\ v) \in X \times X.$$

We denote the Hausdorff metric with respect to this metric as follows.

Let (X, d)be any metric space, we denote by Q the Hausdorff metric on $X \times X$ with respect to the metric η on $X \times X$ defined above, that is for $A,\ B \in CB_{X\times X}$,

$$Q(A,\ B) = \max\left\{\sup_{(x,\ y)\in A} R((x,\ y),\ B), \quad \sup_{(u,\ v)\in B} R((u,\ v),\ A)\right\}.$$

where $R((x,\ y),\ B) = \inf\{\eta((x,\ y),\ (u,\ v)) : (u, v) \in B\}$.

The following notion are related to stability study of coupled fixed points. Let $\{T_n : T_n : X \to N_X\}$ be a sequence of multivalued mappings with respective fixed point sets $Fix(T_n)$. Let $T_n \to T$ as $n \to \infty$ and $Fix(T)$ be the fixed point sets of T. The fixed point sets of T_n are said to be stable if $Fix(T_n) \to Fix(T)$ as $n \to \infty$, where the convergence is with respect to the Hausdorff metric, that is, $H(Fix(T_n),\ Fix(T)) \to 0$ as $n \to \infty$.

In the same vein, we introduce the following notion of stability for coupled fixed point sets.

For any $F : X \times X \to N_X$ the set of coupled fixed points is denoted by

$$CFix(F) = \{(u,\ v) : (u,\ v) \text{ is a coupled fixed point of } F\}.$$

Let $\{F_n : F_n : X \times X \to N_X\}$ be a sequence of multivalued mappings with respective fixed point sets $CFix(F_n)$. Let $F_n \to F$ as $n \to \infty$ and $CFix(F)$ be the coupled fixed point sets of F. The coupled fixed point sets of F_n are said to be stable if $CFix(T_n) \to CFix(T)$ as $n \to \infty$, where the convergence is with respect to the Hausdorff metric, that is, $Q(CFix(F_n),\ CFix(F)) \to 0$ as $n \to \infty$.

3. Main results

Theorem 3.1. *Let (X, d) be a complete metric space. $F : X \times X \longrightarrow CB_X$ be a continuous coupled multivalued function. Suppose F satisfy the following:*

$$\begin{aligned} &H(F(x,y), F(u,v)) \\ &\le \frac{1}{2}\psi\Bigg(\max\Bigg\{ d(x,u) + d(y,v),\ D(x, F(x,y)) + D(y, F(y,x)), \\ &\qquad\qquad D(u, F(u,v)) + D(v, F(v,u)) \Bigg\} \Bigg), \end{aligned} \tag{3.1}$$

where $\psi : [0,\infty) \to [0,\infty)$ is any non decreasing function with $\sum \psi^n(t) < \infty$ and $\psi^n(t)$ is the n^{th} iterate of ψ and $\psi(t) < t$ for each $t > 0$. Then F has a coupled fixed point.

Proof. Let $(x_0, y_0) \in X \times X$, $x_1 \in F(x_0, y_0)$ and $y_1 \in F(y_0, x_0)$. By Lemma 2.2 there exists $x_2 \in F(x_1, y_1)$ and $y_2 \in F(y_1, x_1)$ such that

$$d(x_1, x_2) \le H(F(x_0, y_0), F(x_1, y_1)), \tag{3.2}$$

$$d(y_1, y_2) \le H(F(y_0, x_0), F(y_1, x_1)). \tag{3.3}$$

Since F satisfy (3.1), therefore, we have from (3.2)

$$\begin{aligned} d(x_1, x_2) &\le H(F(x_0, y_0), F(x_1, y_1)) \\ &\le \frac{1}{2}\psi\Bigg(\max\Bigg\{ d(x_0, x_1) + d(y_0, y_1), \\ &\qquad D(x_0, F(x_0, y_0)) + D(y_0, F(y_0, x_0)), \\ &\qquad D(x_1, F(x_1, y_1)) + D(y_1, F(y_1, x_1)) \Bigg\} \Bigg), \end{aligned} \tag{3.4}$$

$$d(x_1,x_2) \le \frac{1}{2}\psi\Bigg(\max\Bigg\{d(x_0,x_1)+d(y_0,y_1), d(x_1,x_2)+d(y_1,y_2)\Bigg\}\Bigg). \quad (3.5)$$

Again since F satisfy (3.1) therefore we have from (3.3)

$$\begin{aligned} d(y_1,y_2) &\le H(F(y_0,x_0),F(y_1,x_1)) \\ &\le \frac{1}{2}\psi\Bigg(\max\Bigg\{d(y_0,y_1)+d(x_0,x_1), \\ &\qquad D(y_0,F(y_0,x_0))+D(x_0,F(x_0,y_0)), \\ &\qquad D(y_1,F(y_1,x_1))+D(x_1,F(x_1,y_1))\Bigg\}\Bigg), \end{aligned} \quad (3.6)$$

$$d(y_1,y_2) \le \frac{1}{2}\psi\Bigg(\max\Bigg\{d(x_0,x_1)+d(y_0,y_1), d(x_1,x_2)+d(y_1,y_2)\Bigg\}\Bigg). \quad (3.7)$$

Adding (3.5) and (3.7) we get,

$$\begin{aligned} &d(x_1,x_2)+d(y_1,y_2) \\ &\le \psi(\max\{d(x_0,x_1)+d(y_0,y_1), d(x_1,x_2)+d(y_1,y_2)\}). \end{aligned} \quad (3.8)$$

Now,

$$\max\{d(x_0,x_1)+d(y_0,y_1), d(x_1,x_2)+d(y_1,y_2)\} = d(x_0,x_1)+d(y_0,y_1),$$

because if

$$\max\{d(x_0,x_1)+d(y_0,y_1), (d(x_1,x_2)+d(y_1,y_2)\} = d(x_1,x_2)+d(y_1,y_2),$$

then

$$d(x_1,x_2)+d(y_1,y_2) \le \psi(d(x_1,x_2)+d(y_1,y_2)) < d(x_1,x_2)+d(y_1,y_2).$$

If $d(x_1,x_2)=0=d(y_1,y_2)$, then $x_1=x_2\in F(x_1,y_1)$ and $y_1=y_2\in F(y_1,y_2)$, that is (x_1,y_1) is a coupled fixed point of F. We take $d(x_1,x_2)\ne 0\ne d(y_1,y_2)$, and we arrive at a contradiction. Therefore,

$$d(x_1,x_2)+d(y_1,y_2) \le \psi(d(x_0,x_1)+d(y_0,y_1)). \quad (3.9)$$

By Lemma 2.2, there exists $x_3\in F(x_2,y_2)$ and $y_3\in F(y_2,x_2)$ such that

$$\begin{aligned} d(x_2,x_3) &\le H(F(x_1,y_1),F(x_2,y_2)) \\ &\le \frac{1}{2}\psi(\max\{d(x_1,x_2)+d(y_1,y_2), \\ &\qquad D(x_1,F(x_1,y_1))+D(y_1,F(y_1,x_1)), \\ &\qquad D(x_2,F(x_2,y_2))+D(y_2,F(y_2,x_2))\}) \quad (3.10) \\ &= \frac{1}{2}\psi(\max\{d(x_1,x_2)+d(y_1,y_2), d(x_2,x_3)+d(y_2,y_3)\}) \\ &= \frac{1}{2}\psi(d(x_1,x_2)+d(y_1,y_2)). \quad (3.11) \end{aligned}$$

Similarly we have,

$$d(y_2, y_3) \leq \frac{1}{2}\psi(d(x_1, x_2) + d(y_1, y_2)). \tag{3.12}$$

Adding (3.11) and (3.12) we get,

$$\begin{aligned} d(x_2, x_3) + d(y_2, y_3) \leq& \ \psi(d(x_1, x_2) + d(y_1, y_2)) \\ &\leq \psi^2(d(x_0, x_1) + d(y_0, y_1)). \end{aligned} \tag{3.13}$$

Continuing and arguing as above we construct a sequence$\{(x_n, y_n)\}$, where $x_{n+1} \in F(x_n, y_n)$ and $y_{n+1} \in F(y_n, x_n)$.

Now we have,

$$d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \leq \psi^n(d(x_0, x_1) + d(y_0, y_1)).$$

Therefore,

$$\sum(d(x_n, x_{n+1}) + d(y_n, y_{n+1})) \leq \sum_n \psi^n(d(x_0, x_1) + d(y_0, y_1)) < +\infty.$$

This implies that, $d(x_n, x_{n+1}) < \infty$, and $d(y_n, y_{n+1}) < \infty$. Since (X, d) is a complete metric space, therefore, $x_n \to x$ and $y_n \to y$ as $n \to \infty$. Now, $d(x_{n+1}, F(x_n, y_n)) = 0$, taking limit $n \to \infty$ and using the continuity of F we get, $d(x, F(x, y)) = 0$, this implies that, $x \in F(x, y)$. Similarly we have, $y \in F(y, x)$. Hence F have a coupled fixed point at (x, y) . □

Example 3.2. Let $X = [1,\ 3]$, $\ F : X \times X \to CB_X$ be defined as follows

$$F(x,\ y) = \left[1, \min\left\{x + \frac{1}{x} - \frac{1}{3},\ y + \frac{1}{y} - \frac{1}{3}\right\}\right].$$

We also define

$$\psi(t) = \frac{t}{9}.$$

It satisfies all the condition of Theorem 3.1. F has a couple fixed point at $(3, 3)$.

4. Stability

Lemma 4.1. *Let (X, d) be a complete metric space. $F_i : X \times X \longrightarrow CB_X$ be two continuous coupled multivalued function. Suppose F_i satisfy the following:*

$$\begin{aligned} &H(F_i(x, y), F_i(u, v)) \\ &\leq \frac{1}{2}\psi\Bigg(\max\Bigg\{d(x, u) + d(y, v),\ D(x, F_i(x, y)) + D(y, F_i(y, x)), \\ &\qquad\qquad D(u, F_i(u, v)) + D(v, F_i(v, u))\Bigg\}\Bigg), \end{aligned} \tag{4.1}$$

where $\psi : [0,\infty) \to [0,\infty)$ *is any non decreasing function with* $\sum \psi^n(t) < \infty$ *and* $\psi^n(t)$ *is the* n^{th} *iterate of* ψ *and* $\psi(t) < t$ *for each* $t > 0$. *Then*

$$Q(CFix(F_1),\ CFix(F_2)) \leq \Phi(2k),$$

where $k = \sup_{(x,y)\in X\times X} H(F_1(x,y), F_2(x,y))$. *We assume that* $CFix(F_i)$, $i = 1,\ 2$ *are bounded.*

Proof. By Theorem 3.1, $CFix(F_i),\ i = 1, 2$ are nonempty. Let

$$k = \sup_{(x,y)\in X\times X} H(F_1(x,y), F_2(x,y)).$$

Let $(x_0, y_0) \in CFix(F_1)$, that is $x_0 \in F_1(x_0, y_0)$ and $y_0 \in F_1(y_0, x_0)$, by Lemma 2.2, there exists $x_1 \in F_2(x_0, y_0)$ and $y_1 \in F_2(y_0, x_0)$ such that

$$\begin{aligned} d(x_0,\ x_1) &\leq H(F_1(x_0, y_0),\ F_2(x_0, y_0)),\\ d(y_0,\ y_1) &\leq H(F_1(y_0, x_0),\ F_2(y_0, x_0)). \end{aligned} \tag{4.2}$$

Which implies that,

$$\begin{aligned} &d(x_0,\ x_1) + d(y_0,\ y_1)\\ &\leq H(F_1(x_0, y_0), F_2(x_0, y_0)) + H(F_1(y_0, x_0),\ F_2(y_0, x_0)) \leq 2k. \end{aligned} \tag{4.3}$$

Let (x_0, y_0) be a coupled fixed point of F_1. Therefore $(x_0, y_0) \in Fix(F_1)$.

Arguing as Theorem 3.1 we can construct a Cauchy sequence $\{(x_n, y_n)\}$ by $x_{n+1} \in F_2(x_n, y_n)$ and $y_{n+1} \in F_2(x_n, y_n)$ with $x_n \to x^*, y_n \to y^*$.

By previous theorem we can prove (x^*, y^*) is a fixed point of F_2.

Now,

$$d(x_0, x^*) \leq \sum_{i=0}^{n} d(x_i, x_{i+1}) + d(x_{n+2}, x^*) \tag{4.4}$$

and

$$d(y_0, y^*) \leq \sum_{i=0}^{n} d(y_i, y_{i+1}) + d(y_{n+2}, y^*). \tag{4.5}$$

Adding (4.4) and (4.5), we have

$$\begin{aligned} &d(x_0, x^*) + d(y_0, y^*)\\ &\leq \sum_{i=0}^{n} (d(x_i, x_{i+1}) + d(y_i, y_{i+1})) + d(x_{n+2}, x^*) + d(y_{n+2}, y^*)\\ &\leq \sum_{i=0}^{n} \psi^i(d(x_0, x_1) + d(y_0, y_1)) + d(x_{n+2}, x^*) + d(y_{n+2}, y^*). \end{aligned}$$

Taking limit $n \to \infty$, by (4.3), we have,

$$d(x_0, x^*) + d(y_0, y^*) \le \sum_{i=0}^{\infty} \psi^i(2k) \le \phi(2k).$$

Thus, given arbitrary $(x_0, y_0) \in Fix(F_1)$, we can find $(x^*, y^*) \in Fix(F_2)$ for which

$$\eta((x_0, y_0),\ (x, y)) \le \Phi(2k).$$

Similarly, we can prove that for arbitrary $(x_0', y_0') \in CFix(F_2)$, there exists a $(x', y') \in CFix(F_1)$ such that $\eta((x_0', y_0'),\ (x', y')) \le \Phi(2k)$. Hence, we conclude that

$$Q(CFix(F_1),\ CFix(F_2)) \le \Phi(2k).$$

□

Lemma 4.2. *Let $(X,\ d)$ be a complete metric space. Let $\{F_n : X \times X \longrightarrow CB_X : n \in \mathbb{N}\}$ be a sequence of multivalued mappings, uniformly convergent to a multivalued mapping $F : X \times X \to CB_X$. If F_n satisfies* (4.1) *for every $n \in \mathbb{N}$, then F also satisfies* (4.1).

Proof. Since F_n satisfies (4.1) for every $n \in \mathbb{N}$, therefore, we have

$$\begin{aligned} &H(F_n(x, y), F_n(u, v)) \\ &\le \frac{1}{2}\psi\Bigg(\max\Bigg\{d(x, u) + d(y, v),\ D(x, F_n(x, y)) + D(y, F_n(y, x)), \\ &\qquad D(u, F_n(u, v)) + D(v, F_n(v, u))\Bigg\}\Bigg). \end{aligned}$$

Since the sequence F_n converges uniformly to F, taking limit $n \to \infty$ in the above inequality we get,

$$\begin{aligned} &H(F(x, y), F(u, v)) \\ &\le \frac{1}{2}\psi(\max\{d(x, u) + d(y, v),\ D(x, F(x, y)) + D(y, F(y, x)), \\ &\qquad D(u, F(u, v)) + D(v, F(v, u))\Bigg\}\Bigg). \end{aligned}$$

Hence the result. □

Now we present our stability result.

Theorem 4.3. *Let $(X,\ d)$ be a complete metric space. Let $\{F_n : X \times X \longrightarrow CB_X : n \in \mathbb{N}\}$ be a sequence of multivalued mappings, uniformly convergent to* *a multivalued mapping $F : X \times X \longrightarrow CB_X$. Suppose that F_n satisfies* (4.1) *for every $n \in \mathbb{N}$. Then*

$$\lim_{n\to\infty} Q(CFix(F_n),\ CFix(F)) = 0,$$

where the conditions upon ψ is same as in Theorem 4.1*, that is, the fixed point sets of F_n are stable.*

Proof. By the Lemma 4.2, F_n for every $n \in \mathbb{N}$ and F satisfy (4.1). Let

$$k_n = \sup_{(x,y)\in X\times X} H(F_n(x,\ y), F(x,\ y)).$$

Since the sequence $\{F_n\}$ is uniformly convergent to F on X,

$$\lim_{n\to\infty} k_n = \lim_{n\to\infty} \sup_{(x,y)\in X\times X} H(F_n(x,y), F(x,y)) = 0. \tag{4.6}$$

Now, from Theorem 4.1, we get

$$Q(CFix(F_n),\ CFix(F)) \leq \Phi(2k_n), \quad \text{for every } \ n \in \mathbb{N}.$$

Since ψ is continuous and $\Phi(t) \to 0 \quad$ as $t \to 0$, using (4.6) we have

$$\lim_{n\to\infty} Q(CFix(F_n),\ CFix(F)) \leq \lim_{n\to\infty} \Phi(2k_n) = 0,$$

that is,

$$\lim_{n\to\infty} Q(CFix(F_n),\ CFix(F)) = 0.$$

Hence the proof is completed. □

Example 4.4. Let $X = \mathbb{R},\ d(x,y) = |x-y|$. Define

$$\{F_n : X \times X \longrightarrow CB_X : n \in \mathbb{N}\}$$

by

$$F_n(x,y) = \begin{cases} \{1+\frac{1}{n}, \max\{\frac{1}{4x}+\frac{1}{n},\ \frac{1}{4y}+\frac{1}{n}\}\}, & \text{if } x,\ y > 1; \\ \{\frac{1}{n}, \max\{\frac{1}{n}+\frac{x}{16},\ \frac{1}{n}+\frac{y}{16}\}\}, & \text{if } 0 < x \leq 1; \\ \{0\}, & \text{otherwise;} \end{cases}$$

$F_n \to F$ as $n \to \infty$. The F is given by

$$F(x,\ y) = \begin{cases} \{1, \max\{\frac{1}{4x},\ \frac{1}{4y}\}\}, & \text{if } x,\ y > 1; \\ \{0, \max\{\frac{x}{16},\ \frac{y}{16}\}\}, & \text{if } 0 < x, y \leq 1; \\ \{0\}, & \text{otherwise.} \end{cases}$$

We define $\psi : [0,\infty) \to [0,\infty)$ by

$$\psi(t) = \frac{1}{2}t.$$

Hence

$$Q(CFix(F_n), CFix(F)) \to 0 \quad \text{as } n \to \infty.$$

Remark 4.5. In the Theorem 4.1 and Theorem 4.3 if we relax the boundedness assumption on $CFix(F)$, then also these Theorems are valid provided we define the Hausdorff distance on closed sets in which case the distance function can take up infinite valued, that is, it is then defined with range as the extended real number systems.

Remark 4.6. It may be noted that there are another view point where coupled fixed pint problems are considered as fixed point problems in product spaces. This view point is not appeared in this paper.

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