**Nonlinear Funct. Anal. & Appl., Vol. 7, No. 3 (2002), pp. 495–496**

# A NOTE ON: DIFFERENTIAL INCLUSIONS ON BANACH SPACES WITH NONLOCAL STATE CONSTRAINTS

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Abstract. In the paper : Differential Inclusions on Banach Spaces with Nonlocal State Constraints: (see [1]) the choice of the multifunction to prove the existence of solutions of Differential inclusions using Bohenblast-Karlin fixed point theorem is not appropriate. In this note we give an alternative construction.

## Revised Proof of Theorem 7

Define the set $K$ , see [1, p404] as follows

$$K \equiv \mathcal{B}_o \equiv \{x \in X \equiv C(I, E) \ : \ x(t) \in B_{r_o}(E), t \in I\}$$

where $B_{r_o}$ is the closed ball of radius $r_o$ in $E$ around the origin as in [1, Lemma 5, p403]. Now define the multi function

$$G(x) \equiv L_\xi \hat{F}(x) \cap \mathcal{B}_o$$

where

$$\hat{F}(x) \equiv \{f \in L_1(I, E) : f(t) \in F(t, x(t)) \ \text{ a.e}\}$$

and the operator $L_\xi$ is as defined in [1, Theorem 2, pp 398]. Note that $x \longrightarrow L_\xi \hat{F}(x)$ maps $X \equiv C(I, E)$ into closed convex subsets of $X$ and it is upper semi continuous. Suppose $G(x) \neq \emptyset$ for $x \in \mathcal{B}_o = K$ (in case it is empty there is no solution). Since $X$, with the sup norm topology, is a normal topological space it follows from [2, Proposition 2.45, p 53] that

$$G \ : \ K \longrightarrow cc(K)$$

Received July 25, 2002.
2000 Mathematics Subject Classification: 34G20, 34K30, 35A05, 93C25.
Key words and phrases: Differential inclusions, existence of solutions.

is also upper semi continuous. Clearly $G(K) \subset K$ and, as seen in [1, Theorem 7], $G(K)$ is relatively compact. Thus by BohenBlast-Karlin Theorem [1, Lemma 6, p 404], $G$ has at least one fixed point in $K$ and hence the differential Inclusion (1-2) has a nonempty set of solutions. This completes the proof. $\square$

## References

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