Nonlinear Funct. Anal. & Appl., Vol. 12, No. 4 (2007), pp. 681–691

CONVERGENCE THEOREMS OF ITERATIVE SEQUENCES WITH ERRORS FOR NONEXPANSIVE MAPPINGS

Liang-cai Zhao, Ding-ping Wu and Zheng-liang Zhang

Abstract. Let E be a real uniformly convex Banach space and C be a nonempty closed convex subset of E. It is shown that under a condition weaker than compactness, weak and strong convergence theorems of iterative sequences with errors to some common fixed points for three nonexpansive mappings. The results presented in this paper also extend and improve some important results(see,e.g.,[3, 4, 9, 11]).

1. Introduction and Preliminaries

Throughout this paper we assume that E is a real Banach space, C is a nonempty closed convex subset of E. Let $T : C \to C$ be a mapping, we use $F(T)$ to denote the set of fixed points of T; i.e., $F(T) = \{x \in E : Tx = x\}$. When $\{x_n\}$ is a sequence in E, then $x_n \rightharpoonup x$ and $x_n \to x$ will denote weak and strong convergence of the sequence $\{x_n\}$ to x, respectively.

We recall that a Banach space E is said to satisfy *Opial's condition* [6] if for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ implies that

$$\limsup_{n\to\infty} ||x_n - x|| < \limsup_{n\to\infty} ||x_n - y||$$

for all $y \in E$ with $y \neq x$.

Definition 1. Let $T : C \to C$ be a mapping. then T is said to be nonexpansive if

$$||Tx - Ty|| \leq ||x - y||$$

for all $x, y \in C$.

Received January 27, 2006. Revised March 10, 2006.

2000 Mathematics Subject Classification: 47H05, 47H09, 49M05.

Key words and phrases: Nonexpansive mapping, Opial's condition, demiclosed principle, condition (A*), common fixed point.

Definition 2. A mapping $T : C \to E$ is called demiclosed in $y \in E$ if for each sequence $\{x_n\}$ in C and each $x \in E$, $x_n \rightharpoonup x$ and $Tx_n \to y$ imply that $x \in C$ and $Tx = y$.

Now, we introduced the following iterative scheme with errors.

Let E be a uniformly convex Banach space and C its nonempty bounded closed convex subset. Let $S, T, R : C \to C$ be three nonexpansive mappings, for any given $x_1 \in C$,the sequence $\{x_n\}$ defined by

$$\begin{cases} x_{n+1} = a_n x_n + b_n S y_n + c_n u_n, \\ y_n = a'_n x_n + b'_n T z_n + c'_n v_n, \\ z_n = a''_n x_n + b''_n R x_n + c''_n w_n. \end{cases} \qquad n \geq 1 \tag{1.1}$$

where $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}, \{a''_n\}, \{b''_n\}, \{c''_n\}$ are real sequences in $[0, 1]$ satisfying $a_n + b_n + c_n = a'_n + b'_n + c'_n = a''_n + b''_n + c''_n = 1$ and $\{u_n\}, \{v_n\}, \{w_n\}$ are arbitrary given sequences in C.

Iterative techniques for convergence to fixed points of nonexpansive mappings have been studied by various authors in different frames. for example, Xu [11], Chidume and Moore [2], Das and Debata [3] and Takahashi and Tamura [9], they proved some convergence theorems for iterative scheme of fixed points of nonexpansive mappings. Recently, Khan and Fukhar-ud-din [4] studied the iterative scheme with errors for weak and strong convergence for two nonexpansive mappings in a uniformly convex Banach space.

The purpose of this paper is to study the weak and strong convergence of the iterative scheme with errors $\{x_n\}$ defined by (1.1) to a common fixed point for three nonexpansive mappings in Banach space. The results presented in this paper also extend and improve some important results(see,e.g.,[2, 3, 4, 9, 11]).

In order to prove the main results of this paper, we need the following lemmas:

Lemma 1. ([10]) *Let $\{s_n\}, \{t_n\}$ be two nonnegative sequences satisfying the following condition:*

$$s_{n+1} \leq s_n + t_n \qquad for\ all\ n \geq 1.$$

where $\sum_{n=1}^{\infty} t_n < \infty$. Then $\lim_{n\to\infty} s_n$ exists.

Lemma 2. ([7]) *Let E be a uniformly convex Banach space and $\{t_n\}$ a sequence in $[\varepsilon, 1-\varepsilon]$ for some $\varepsilon \in (0,1)$. Suppose that $\{x_n\}, \{y_n\}$ are two sequence in E such that for some $r \geq 0$*

$$(i) \limsup_{n\to\infty} ||x_n|| \leq r,$$
$$(ii) \limsup_{n\to\infty} ||y_n|| \leq r,$$
$$(iii) \lim_{n\to\infty} ||t_n x_n + (1-t_n)y_n|| = r.$$

Then $\lim_{n\to\infty} ||x_n - y_n|| = 0$.

Lemma 3. ([1]) *Let E be a uniformly convex Banach space satisfying Opial's condition, C be a nonempty closed convex subset of E and $T : C \to C$ be a nonexpansive mapping with $F(T) \neq \emptyset$. Then $I-T$ is demiclosed at zero, i. e., for each sequence $\{x_n\}$ in C, if the sequence $\{x_n\}$ converges weakly to $q \in C$ and $\{(I-T)x_n\}$ converges strongly to 0, then $(I-T)q = 0$.*

2. Main results

Lemma 4. *Let E be a uniformly convex Banach space, C be a nonempty bounded closed convex subset of E. Let $S, T, R : C \to C$ be three nonexpansive mappings and $\{x_n\}$ be the sequence defined by (1.1) satisfying the following conditions:*

$$\begin{cases} 0 < \delta \leq b_n, b_n', b_n'' \leq 1-\delta < 1 \quad for\ some\ \delta \in (0,1), \\ \sum_{n=1}^{\infty} c_n < \infty,\ \sum_{n=1}^{\infty} c_n' < \infty,\ \sum_{n=1}^{\infty} c_n'' < \infty. \end{cases} \tag{2.1}$$

If $F(S) \cap F(T) \cap F(R) \neq \emptyset$, then we have

$$\lim_{n\to\infty} ||Sx_n - x_n|| = \lim_{n\to\infty} ||Tx_n - x_n|| = \lim_{n\to\infty} ||Rx_n - x_n|| = 0.$$

Proof. Since C is bounded,there exist $M > 0$, such that

$$M = \sup_{x\in C} ||x||.$$

Assume that $F(S) \cap F(T) \cap F(R) \neq \emptyset$, Let $x^* \in F(S) \cap F(T) \cap F(R)$, it follows from (1.1) that

$$\begin{aligned}
&||x_{n+1} - x^*|| \\
&= ||a_n x_n + b_n S y_n + c_n u_n - x^*|| \\
&= ||b_n(S y_n - x^* + c_n(u_n - x_n)) + (1 - b_n)(x_n - x^* + c_n(u_n - x_n))|| \\
&\leq b_n ||S y_n - x^*|| + b_n c_n ||u_n - x_n|| + (1 - b_n)||x_n - x^*|| \\
&\quad + (1 - b_n) c_n ||u_n - x_n|| \\
&\leq b_n ||y_n - x^*|| + (1 - b_n)||x_n - x^*|| + c_n ||u_n - x_n|| \\
&\leq b_n ||y_n - x^*|| + (1 - b_n)||x_n - x^*|| + c_n M.
\end{aligned} \tag{2.2}$$

Now, we consider the first term on the right side of (2.2). It follows from (1.1) that

$$\begin{aligned}
&||y_n - x^*|| \\
&= ||a'_n x_n + b'_n T z_n + c'_n v_n - x^*|| \\
&= ||b'_n(T z_n - x^* + c'_n(v_n - x_n)) + (1 - b'_n)(x_n - x^* + c'_n(v_n - x_n))|| \\
&\leq b'_n ||T z_n - x^*|| + b'_n c'_n ||v_n - x_n|| + (1 - b'_n)||x_n - x^*|| \\
&\quad + (1 - b'_n) c'_n ||v_n - x_n|| \\
&\leq b'_n ||z_n - x^*|| + (1 - b'_n)||x_n - x^*|| + c'_n M.
\end{aligned} \tag{2.3}$$

Again, it follows from (1.1) that

$$\begin{aligned}
&||z_n - x^*|| \\
&= ||a''_n x_n + b''_n R x_n + c''_n w_n - x^*|| \\
&= ||b''_n(R x_n - x^* + c''_n(w_n - x_n)) + (1 - b''_n)(x_n - x^* + c''_n(w_n - x_n))|| \\
&\leq b''_n ||R x_n - x^*|| + b''_n c''_n ||w_n - x_n|| + (1 - b''_n)||x_n - x^*|| \\
&\quad + (1 - b''_n) c''_n ||w_n - x_n|| \\
&\leq b''_n ||x_n - x^*|| + (1 - b''_n)||x_n - x^*|| + c''_n M \\
&= ||x_n - x^*|| + c''_n M.
\end{aligned} \tag{2.4}$$

Substituting (2.4) into (2.3) and simplifying, then we have

$$\begin{aligned}
||y_n - x^*|| &\leq b'_n ||x_n - x^*|| + (1 - b'_n)||x_n - x^*|| + b'_n c''_n M + c'_n M \\
&\leq ||x_n - x^*|| + c'_n M + c''_n M.
\end{aligned} \tag{2.5}$$

Again, substituting (2.5) into (2.2), then we have

$$\begin{aligned}
&||x_{n+1} - x^*|| \\
&\leq b_n||x_n - x^*|| + (1 - b_n)||x_n - x^*|| + b_n(c'_n M + c''_n M) + c_n M \\
&\leq ||x_n - x^*|| + M(c_n + c'_n + c''_n).
\end{aligned}$$

It follows from the condition (2.1) that

$$\sum_{n=1}^{\infty} M(c_n + c'_n + c''_n) < \infty.$$

Hence, by Lemma 1, we know that the limit $\lim_{n\to\infty} ||x_n - x^*||$ exists. Suppose

$$\lim_{n\to\infty} ||x_n - x^*|| = r. \tag{2.6}$$

for some $r \geq 0$. If $r = 0$, the result is obvious. So we assume $r > 0$, then, from (2.5), we have

$$||y_n - x^*|| \leq ||x_n - x^*|| + c'_n M + c''_n M.$$

By condition (2.1), we have $\lim_{n\to\infty} c_n = \lim_{n\to\infty} c'_n = \lim_{n\to\infty} c''_n = 0$. It follows from (2.6) that

$$\limsup_{n\to\infty} ||y_n - x^*|| \leq r. \tag{2.7}$$

By using the similar method, from (2.4), we can also prove that

$$\limsup_{n\to\infty} ||z_n - x^*|| \leq r. \tag{2.8}$$

Again, since

$$\begin{aligned}
||Sy_n - x^* + c_n(u_n - x_n)|| &\leq ||Sy_n - x^*|| + c_n||u_n - x_n|| \\
&\leq ||y_n - x^*|| + c_n M,
\end{aligned}$$

it follows from (2.7) that

$$\limsup_{n\to\infty} ||Sy_n - x^* + c_n(u_n - x_n)|| \leq r.$$

Again, since

$$\begin{aligned}||x_n - x^* + c_n(u_n - x_n)|| &\le ||x_n - x^*|| + c_n||u_n - x_n|| \\ &\le ||x_n - x^*|| + c_n M,\end{aligned}$$

we have that

$$\limsup_{n\to\infty} ||x_n - x^* + c_n(u_n - x_n)|| \le r.$$

Since, $\lim_{n\to\infty} ||x_{n+1} - x^*|| = r$ this implies that

$$\lim_{n\to\infty} ||b_n(Sy_n - x^* + c_n(u_n - x_n)) + (1 - b_n)(x_n - x^* + c_n(u_n - x_n))|| = r.$$

By Lemma 2, we obtain that

$$\lim_{n\to\infty} ||Sy_n - x_n|| = 0. \tag{2.9}$$

From

$$\begin{aligned}||x_n - x^*|| &\le ||x_n - Sy_n + Sy_n - x^*|| \\ &\le ||x_n - Sy_n|| + ||y_n - x^*||,\end{aligned}$$

it yields that

$$\liminf_{n\to\infty} ||y_n - x^*|| \ge r. \tag{2.10}$$

By (2.7) and (2.10), we obtain

$$\lim_{n\to\infty} ||y_n - x^*|| = r.$$

Next,

$$\begin{aligned}||Tz_n - x^* + c'_n(v_n - x_n)|| &\le ||Tz_n - x^*|| + c'_n||v_n - x_n|| \\ &\le ||z_n - x^*|| + c'_n M,\end{aligned}$$

it follows from (2.8) that

$$\limsup_{n\to\infty} ||Tz_n - x^* + c'_n(v_n - x_n)|| \le r.$$

It follows from

$$\begin{aligned}||x_n - x^* + c'_n(v_n - x_n)|| &\le ||x_n - x^*|| + c'_n||v_n - x_n|| \\ &\le ||x_n - x^*|| + c'_n M,\end{aligned}$$

that

$$\limsup_{n\to\infty} ||x_n - x^* + c'_n(v_n - x_n)|| \le r.$$

Since $\lim_{n\to\infty} ||y_n - x^*|| = r$, this means that

$$\lim_{n\to\infty} ||b'_n(Tz_n - x^* + c'_n(v_n - x_n)) + (1 - b'_n)(x_n - x^* + c'_n(v_n - x_n))|| = r.$$

By Lemma 2, we have

$$\lim_{n\to\infty} ||Tz_n - x_n|| = 0 \tag{2.11}$$

It follows from

$$\begin{aligned} ||x_n - x^*|| &\le ||x_n - Tz_n + Tz_n - x^*|| \\ &\le ||x_n - Tz_n|| + ||z_n - x^*||, \end{aligned}$$

that

$$\liminf_{n\to\infty} ||z_n - x^*|| \ge r. \tag{2.12}$$

By (2.8) and (2.12),we know that

$$\lim_{n\to\infty} ||z_n - x^*|| = r.$$

Again, since

$$\begin{aligned} ||Rx_n - x^* + c''_n(w_n - x_n)|| &\le ||Rx_n - x^*|| + c''_n||w_n - x_n|| \\ &\le ||x_n - x^*|| + c''_n M, \end{aligned}$$

hence from (2.6), we have

$$\limsup_{n\to\infty} ||Rx_n - x^* + c''_n(w_n - x_n)|| \le r.$$

Also, from

$$\begin{aligned} ||x_n - x^* + c''_n(w_n - x_n)|| &\le ||x_n - x^*|| + c''_n||w_n - x_n|| \\ &\le ||x_n - x^*|| + c''_n M, \end{aligned}$$

we have

$$\limsup_{n\to\infty} ||x_n - x^* + c''_n(w_n - x_n)|| \le r.$$

Since, $\lim_{n\to\infty} ||z_n - x^*|| = r$, this means that

$$\lim_{n\to\infty} ||b''_n(Rx_n - x^* + c''_n(w_n - x_n)) + (1 - b''_n)(x_n - x^* + c''_n(w_n - x_n))|| = r.$$

By Lemma 2, we get

$$\lim_{n\to\infty} ||Rx_n - x_n|| = 0. \tag{2.13}$$

Now we observe that

$$\begin{aligned} ||Tx_n - x_n|| &= ||Tx_n - Tz_n + Tz_n - x_n|| \\ &\le ||x_n - z_n|| + ||Tz_n - x_n|| \\ &= ||x_n - (a''_n x_n + b''_n Rx_n + c''_n w_n)|| + ||Tz_n - x_n|| \\ &= ||b''_n(x_n - Rx_n) + c''_n(x_n - w_n)|| + ||Tz_n - x_n|| \\ &\le b''_n||x_n - Rx_n|| + c''_n||x_n - w_n|| + ||Tz_n - x_n|| \\ &\le (1 - \delta)||x_n - Rx_n|| + ||Tz_n - x_n|| + c''_n M. \end{aligned}$$

Implies together with (2.11), (2.13)and $c''_n \to 0 (n \to \infty)$, we know that

$$\lim_{n\to\infty} ||Tx_n - x_n|| = 0. \tag{2.14}$$

Similarly, we have

$$\begin{aligned} ||Sx_n - x_n|| &= ||Sx_n - Sy_n + Sy_n - x_n|| \\ &\le ||x_n - y_n|| + ||Sy_n - x_n|| \\ &= ||x_n - (a'_n x_n + b'_n Tz_n + c'_n v_n)|| + ||Sy_n - x_n|| \\ &= ||b'_n(x_n - Tz_n) + c'_n(x_n - v_n)|| + ||Sy_n - x_n|| \\ &\le b'_n||x_n - Tz_n|| + c'_n||x_n - v_n|| + ||Sy_n - x_n|| \\ &\le (1 - \delta)||x_n - Tz_n|| + ||Sy_n - x_n|| + c'_n M. \end{aligned}$$

By (2.9), (2.11) and $c'_n \to 0 (n \to \infty)$, that

$$\lim_{n\to\infty} ||Sx_n - x_n|| = 0. \tag{2.15}$$

By (2.13), (2.14) and (2.15), so we know that

$$\lim_{n\to\infty} ||Sx_n - x_n|| = \lim_{n\to\infty} ||Tx_n - x_n|| = \lim_{n\to\infty} ||Rx_n - x_n|| = 0.$$

This completes the proof of Lemma 4. □

Now we give the main results of this paper.

Theorem 1. *Let E be a uniformly convex Banach space satisfying Opial's condition and C, S, T, R and $\{x_n\}$ be the same as in Lemma 4. If $F(S) \cap F(T) \cap F(R) \neq \emptyset$, then $\{x_n\}$ converges weakly to a common fixed point of the mappings S, T and R in C.*

Proof. Let $x^* \in F(S) \cap F(T) \cap F(R)$. Then $\lim_{n\to\infty} ||x_n - x^*||$ exists as proved in Lemma 4 . Now we prove that $\{x_n\}$ has a unique weak subsequential limit in $F(S) \cap F(T) \cap F(R)$. To prove this, let u and v be weak limits of the subsequences $\{x_{n_i}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. By Lemma 4, $\lim_{n\to\infty} ||x_n - Sx_n|| = 0$ and $I - S$ is demiclosed with respect to zero by Lemma 3, therefore we obtain $Su = u$. Similarly, $Tu = u$ and $Ru = u$. Again by the same method, we can prove that $v \in F(S) \cap F(T) \cap F(R)$. Next, we prove the uniqueness. For the purpose, we suppose that $u \neq v$, then by the Opial's condition

$$\begin{aligned}
\lim_{n\to\infty} ||x_n - u|| &= \lim_{i\to\infty} ||x_{n_i} - u|| \\
&< \lim_{i\to\infty} ||x_{n_i} - v|| \\
&= \lim_{n\to\infty} ||x_n - v|| \\
&= \lim_{j\to\infty} ||x_{n_j} - v|| \\
&< \lim_{j\to\infty} ||x_{n_j} - u|| \\
&= \lim_{n\to\infty} ||x_n - u||.
\end{aligned}$$

This is a contradiction. Hence $\{x_n\}$ converges weakly to a point in $F(S) \cap F(T) \cap F(R)$. This completes the proof of Theorem 1. □

Our next goal is to prove a strong convergence theorem.

Recall that a mapping $T : C \to C$ (where C is a subset of E), is said to satisfy condition (A) if there exists a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for all $r \in (0, \infty)$ such that $||x - Tx|| \geq f(d(x, F(T)))$ for all $x \in C$ where $d(x, F(T)) = \inf\{||x - x^*|| : x^* \in F(T)\}$.

Senter and Dotson [8] approximated fixed points of a nonexpansive mapping T by Mann iterates. Later on, Tan and Xu [10] and Maiti and Ghosh [5] studied the approximation of fixed points of a nonexpansive mapping T by Ishikawa iterates under the same condition (A) which is weaker than the requirement that T is demicompact. Recently, Khan and Fukhar-ud-din [4] studied the iterative scheme with errors for weak and strong convergence for two nonexpansive mappings under similar condition. We modify this condition for three mappings $S, T, R : C \to C$ as follows:

Three mappings $S, T, R : C \to C$ where C is a subset of E, are said to satisfy condition (A*) if there exists a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for all $r \in (0, \infty)$ such that $\frac{1}{3}(||x - Sx|| + ||x - Tx|| + ||x - Rx||) \geq f(d(x, F))$ for all $x \in C$ where $d(x, F) = \inf\{||x - x^*|| : x^* \in F = F(S) \cap F(T) \cap F(R)\}$.

We shall use condition (A*) instead of compactness of C to study the strong convergence of $\{x_n\}$ defined in (1.1). It is worth nothing that in case of nonexpansive mappings $S, T, R : C \to C$, condition (A*) is weaker than the compactness of C.

Theorem 2. *Let E be a uniformly convex Banach space and C, $\{x_n\}$ be as taken in Lemma* 4. *Let $S, T, R : C \to C$ be three nonexpansive mappings satisfying condition (A*). If $F(S) \cap F(T) \cap F(R) \neq \emptyset$, then $\{x_n\}$ converges strongly to a common fixed point of S, T and R.*

Proof. By Lemma 4. $\lim_{n\to\infty} ||x_n - x^*||$ exists for all $x^* \in F = F(S) \cap F(T) \cap F(R)$. Let it be r for some $r \geq 0$. If $r = 0$, the result is obvious. Suppose $r > 0$. By Lemma 4,

$$\lim_{n\to\infty} ||Sx_n - x_n|| = \lim_{n\to\infty} ||Tx_n - x_n|| = \lim_{n\to\infty} ||Rx_n - x_n|| = 0.$$

Moreover, it follows from

$$||x_{n+1} - x^*|| \leq ||x_n - x^*|| + M(c_n + c'_n + c''_n),$$

that

$$\inf_{x^*\in F} ||x_{n+1} - x^*|| \leq \inf_{x^*\in F} ||x_n - x^*|| + M(c_n + c'_n + c''_n).$$

That is,

$$d(x_{n+1}, F) \leq d(x_n, F) + M(c_n + c'_n + c''_n).$$

By virtue of Lemma 1, this implies that $\lim_{n\to\infty} d(x_n, F)$ exists. Now by condition (A*), $\lim_{n\to\infty} f(d(x_n, F)) = 0$. Since f is a nondecreasing function and $f(0) = 0$, therefore

$$\lim_{n\to\infty} d(x_n, F) = 0.$$

Now we can take a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and a sequence $\{y_j\} \subset F$ such that $||x_{n_j} - y_j|| < 2^{-j}$. Then follow the method of proof of Tan and Xu [10], we get that $\{y_j\}$ is a Cauchy sequence in F and so it converges. Let $y_j \to y$. Since F is closed, therefore $y \in F$ and then $x_{n_j} \to y$. As $\lim_{n\to\infty} ||x_n - x^*||$ exists, $x_n \to y \in F = F(S) \cap F(T) \cap F(R)$. This completes the proof of Theorem 2. □

References

1. F. E. Browder, *Nonlinear operators and nonlinear equations of evolution in Banach spaces*, Proceedings of the Symposium on Pure Mathematics ,vol.18, Proc. Amer. Math. Soc., Providence RI (1976).
2. C. E. Chidume and Chika Moore, *Fixed points iteration for pseudocontractive maps*, Proc. Amer. Math. Soc. **127(4)** (1999), 1163–1170.
3. G. Das and J. P. Debata, *Fixed points of quasi-nonexpansive mappings*, Indian J. Pure. Appl. Math. **17** (1986), 1263–1269.
4. S. H. Khan and H. Fukhar-ud-din, *Weak and strong convergence of a scheme with errors for two nonexpansive mappings*, Nonlinear Anal. **61** (2005), 1295–1301.
5. M. Maiti and M. K. Ghosh, *Approximating fixed points by Ishikawa iterates*, Bull. Austral. Math. Soc. **40** (1989), 113–117.
6. Z. Opial, *Weak convergence of sequence approximated of successive for nonexpansive mappings*, Bull. Amer. Math. Soc. **73** (1967), 591–597.
7. J. Schu, *Weak and strong convergence to fixed points of asymptotically nonexpansive mappings*, Bull. Austral. Math. Soc. **43** (1991), 153–159.
8. H. F. Senter and W. G. Dotson, *Approximating fixed points of nonexpansive mappings*, Proc. Amer. Math. Soc. **44(2)** (1974), 375–380.
9. W. Takahashi and T. Tamura, *Convergence theorems for a pair of nonexpansive mappings*, J. Convex Analysis. **5(1)** (1998), 45–58.
10. K. K. Tan and H. K. Xu, *Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process*, J. Math. Anal. Appl. **178** (1993), 301–308.
11. Y. Xu, *Ishikawa and Mann Iteration process with errors for nonlinear strongly accretive operator equations*, J. Math. Anal. Appl. **224** (1998), 91–101.

Liangcai Zhao, Dingping Wu and Zhengliang Zhang
Department of Mathematics
Yibin University
Yibin,Sichuan 644007
P. R. China
E-mail address: zhaolcyb@yahoo.com.cn